STOCHASTIC MODELING OF DAILY PRECIPITATION IN ABEOKUTA

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Abstract
In this study, the Chapman-Kolmogorov Equation (CKE) was modified and applied to model the daily precipitation data of Abeokuta, Ogun State. The modified equation incorporated the initial distribution of the system as a feedback. The daily precipitation data converges at the 2nd iteration with the modified CKE. To ascertain the validity of the result, a diagnostic test was conducted with the limiting characteristic equation. The test result showed that the limiting distribution of the system approached the absolute probability distribution. In addition, the Bayesian Information Criterion technique was used to determine the order of the Markov chain which was observed to be of order one. This gave the best fit for precipitation pattern which is relevant in the development of new growth and yield models of major crops such as corn, sorghum and soya bean; enabling farmers estimate the distribution of crop yield as the growing season progressed.

Keywords: Markov chain; Stationary distribution; Stochastic model; Daily precipitation

Introduction

Precipitation means rainfall or snow. In this part of the world, precipitation refers to rainfall; therefore in this study precipitation implies rainfall usually measured by rain gauge in millimeters (mm). It has a major influence on all human activity. In particular agricultural operations and major engineering activities are strongly influenced by weather phenomena. New developments in modeling growth and yield of major crops such as corn, wheat, soya beans and, cotton increased the need for precipitation models so as to enable farmers estimate the distribution of crop yields as the growing season progresses (Woolhiser, 1992).

Attempts have been made by individuals from a wide range of disciplines with varying success to estimate the precipitation process for a long time. Extensive climatic data including precipitation have been collected for many years all over the world with the greatest density of stations in the developed nations. The information content of these data sets were summarized by standard statistical analyses and results presented in tabular or graphical forms.

Since precipitation is the result of complicated physical processes, the non-linear or sensitive process that governs it makes a purely deterministic physical description and forecast impossible. It is common to model precipitation as a stochastic process both in space and time, due to its complex time varying phenomena which can be measured by a finite number of observations.

The stochastic modeling of daily precipitation may be considered in two components namely; the occurrence of rainfall during a day and the depth of rainfall on rainy days. The pattern of each of these is of significant importance to agriculture and water resources in Nigeria.

Water Resources of Nigeria

Nigeria is endowed with abundant inland water resources. Ita et al (1985) reported that there were 149, 191Km^2 (about 15.9% of the total area of Nigeria) of inland waters made up of major lakes, rivers, ponds flood plans, running and stagnant pools. In the 1980s, there were 347 reservoirs and lakes, 389 flood plans and rivers, 5000 fish ponds, 89 cattle drinking ports and many earth wells and boreholes (Satia, 1990). A great proportion of the Country's extension mangrove ecosystem lies within the Niger-Delta and is situated mostly in Akwa-Ibom, Cross Rivers, Delta, Lagos, Ondo and Rivers states, covering a surface area of between 500,000 and 885,000 hectares (Saline wetland).

The major rivers, estimated at about 10,812,400 hectares make up about 11.5% of the total surface of Nigeria which is estimated to be approximately 94185000 hectares. Other water bodies include small reservoirs; fish ponds and miscellaneous wetland suitable for rice cultivation cover about 3221500 hectares.

Ayoade (1981) reported that the sources of these waters are both surface and underground with rainfall as the primary contribution. There are eight main Rivers and Delta, Imo-Anambra, Hadejia-Chad, Sokoto, Rima, Niger, Owena and Ogun-Oshun basins, Much engineering development have been carried out in these basins especially in the north for hydroelectric generation and irrigation, the first being Niger at Kainji in 1968.

Rainfall Patterns in Nigeria

The incidence of the rainfall as measured by its variability from year to year is as important as the seasonal distribution of the rainfall as it is known to be much more variable than evaporation both spatially and temporarily. Rainfall variability has been the subject of many comments, but in reality comparatively little quantitative analysis has been undertaken in the tropical area (Jackson 1989).

The most significant climatic feature in Africa is undoubtedly rainfall. In Nigeria, there is no apparent continuity of the rainfall patterns from day to day and even within the affected areas, the rainfall is seldom consistent. Nigeria exhibit a variation in mean annual rainfall that is, it varies from about two to three months in the sahellian parts (in the north) to ten months in the coastal areas. The rainfall ranges therefore from about 250mm at the farthest north to about 3000mm on the coast (Ayoade 1981).

The effectiveness of rainfall for agriculture depends primary on whether the rainfall received is enough to off set evaporation losses which are considerable in the tropics. The reliability of the annual rainfall as measured by its variably from year to year is as important as the seasonal distribution.

Modelling Precipitation

Many researchers have found Markov chain model very useful especially in modelling precipitation occurrence. Gabriel and Neuman (1962) used a first order stationary Markov chains to describe rainfall occurrence in Tel Aviv. He reported that a first-order model is adequate for describing the situation in Tel Aviv (Israel). Non-stationary Markov chains have also been used by several investigators (Caskey, 1963; Katz, 1981). The appropriate order for Markov chain models have been investigated by Chin (1977),and Gates and Tong (1976). Gates and Tong (1976) argued that a second-order model would have been selected if Akaike Information Criteria (AIC) were used to identify the optimum order for the Tel Aviv data. Chin (1977) used AIC to determine the optimum model order at over 100 stations in the USA. He concluded that the optimum model depends on the season and geographical location, and reported that the optimum order for summer is one and the order for winter is higher than one. Woolhiser (1992) and Woolhiser et al. (1993) presented a modified Markov model which accounts for inter-annual variation in the sequence of wet and dry days.

Laux, *et al* (2007) described the link between the West African monsoon's onsets with atmospheric circulation pattern. Neuman, *et al* (2007) studied the ultimate trends of temperature, precipitation and river discharge in the Volta basin of West Africa.

Stochastic modelling of rainfall in Nigeria

Stochastic modelling of daily rainfall in Nigeria using Markov chains has been undertaken by Stern (1980, who showed that first-order Markov chain models were adequate for describing daily rainfall for three (Sokoto, Kano, Samaru) stations. Jackson (1981), however, argued that the optimum order of the model (at Kano and Benin) varies with season and could be as high as three. Both Stern (1980) and Jackson (1981) assumed the model order and then used Chi-squared technique to evaluate the performance of each order.

According to Jimoh and Webster (1995), a Markov chains model of order one may be used to describe the occurrence of wet and dry in Nigeria. Such model feature two parameter set P_{01} to characterize the probability of a wet day following a dry day and P_{11} to characterize the probability of a wet day following a wet day. The model parameter sets, when estimated from historical records, are characterized by a distinctive seasonal behaviour. The Markov chain model was used by Bello (2001) to investigate the pattern of sequences of late, normal and early onset and cessation of the rains at selected stations in the four major ecological zones of Nigeria. The transition probabilities of these events showed that the probability of persistence of early onset was highest and this was followed by late and normal respectively. He discovered a steady state transition probability which showed that persistence of late onset of the rains was higer than normal and early.

Methodology

In this study, we introduced a structural model that incorporates feedback from history. The structural parameters of interest are modelled as functions of the latent signals of the history. These models allow flexible and robust feedback mechanisms, have clear interpretations and have a computationally efficient estimation procedure.

The Data

The precipitation data used in the analysis of this study are random sequences generated from the record of Meteorological Service Department, Federal Ministry of Aviation, Okemosan, Abeokuta. The record gives the daily rainfall in Abeokuta. The total for each month can also be retrieved from the record. The variable, which is rainfall, is considered, daily and monthly for these twenty years and the units of measurement are millimeters. The Department started its operation in Abeokuta in March, 1981. Hence, the data collected are for the year 1982 to 2001. A wet day is marked by a day with rainfall in excess of 0.1mm while a dry day is a day with rainfall amount of less than 0.1mm.

Markov Chains

Let $E_1, E_2, \ldots, E_j$ (i=0,1,2...) represent the exhaustive and mutual exclusive outcomes (states) of a system at any time. Initially, at time t_0, the system may be in any of these states. Let $a_j^{(0)}$ (j=1,2,...) be the absolute probability that the system is in state E_j at t_o. Assume further that the system is Markovian,

then

$$p_{ij}=P\{X_{tn} = j / X_{tn-1} = i\}$$

is defined as the one-step transition probability of going from state i at t_{n-1} to state j at t_n and assume that these probabilities are fixed over time. Thus the transition probabilities from state E_i to state E_j can be more conveniently arranged in a matrix form as follows.

$$P = \begin{bmatrix} p00 & p01 & p02 \\ & \cdots\cdots & \\ p_{10} & p_{11} & p12 \\ . & \cdots & \end{bmatrix}$$

The matrix P is the homogeneous transition or stochastic matrix because all the transition probabilities p_{ij} are fixed and independent of time. The probabilities p_{ij} must satisfy the conditions,

$$\Sigma P_{ij} = 1 \text{ for all i}$$

$$P_{ij} \geq 0 \text{ for all i and j}$$

A transition matrix P together with the initial probabilities associated with the states E_j completely defines a Markov chain. A Markov chain is therefore defined as the transitional behaviour of a system over interval of time.

State Space

A structural model can be written as a state space model with the state of the system representing the various unobserved components and the parameters (structural parameters) having clear interpretations.

It can be shown for example that it is possible to put many types of time series model into a state space form. They include regression and auto-regression moving averages (ARMA) models for which exponential smoothing methods are thought to be appropriate.

The Model

Many practical applications of random sequences involve the important case where the underlying statistical properties are constant or invariate with respect to the parameter, which is namely time or space. This simplifies the random models in two ways: first, the model can be specified with many fewer parameters than would be the case if the statistics were varying with time, and secondly, these few parameters can then be more reliably estimated from the data. It is often desired to partially characterise a random sequence based on knowledge of only its first two moments, that is , its mean function and its covariance function.

To specify the distribution for the rainfall data, we considered a model such that the value of the process at time, t thus determines the conditional probabilities for future values of the process. The values of the process are thus called the state of the process and the conditional probabilities are thought of as a transition probability between the states. If only a finite uncountable set of value X_i is allowed the process is called a Markov Chain or a Markov random process.

A Markov random process satisfies the conditional probability mass function expression $P(X_n = x_n / X_{n-1} = X_{n-1}, \ldots X_0 = X_0)$
$= P(X_n = X_n / X_{n-1} = X_{n-1})$.

Where $X_0, X_1, \ldots X_n \in \{0,1\}$. In other words, it was assumed that the probability of wetness of any day depends only on whether the previous day was wet or dry. Given the event on previous day, the probability of wetness is assumed independent of further preceding days. The Markov chain is referred to as a two-state chain, as X_n is zero or one.

Precipitation occurrence model can be viewed as a sequence of random variables

$X_t : t = t_1, t_2, \ldots\ldots\ldots t_n$ where

$$X_{tn} = \begin{cases} 0, & \text{if the nth day is dry} \\ 1, & \text{if the nth day is wet.} \end{cases}$$

The collection quantities for the random process X_n are dependent on the past history of the process $(X_{n-1}, X_{n-2} \ldots X_0)$ but not upon the future of the process $(X_{n+1}, X_{n+2} \ldots)$

If for all n,

$P(X_n = j / X_{n-1} = i_{n-1}, \ldots X_0 = i_0)$...(1)

$= P(X_n = j / X_{n-1} = i) \qquad = P_{ij}$..(2)

(2) Describes the one-step transition probabilities from i to j.

Markov was led to develop Markov chains as a natural extension of sequences of independent random variables. In 1906, he proved that for a Markov chain with positive transition probabilities and numerical states, the average of the outcome converges to the expected value of the limiting distribution (the fixed vector).

In the short-run, we can specify the transition density from time n to time n+k where $k \geq 0$. However, we must make sure that this multi-step transition density is consistent. It must exceed a one-step density that would sequentially yield the same result.

The Chapman– Kolmogorov equation

$P_{ij}^{n+m} = \sum p_{ik}^{m} P^{n}_{kj}$..(3)

supplies both necessary and sufficient conditions for these more general transition densities.

Using a modified form of equation (3), we specified the n – step transition probabilities of lower order as

$P_{ij}^{(n)} \quad = P\{X_n = j / X_0 = i\}$

$\qquad = P_{ik}^{(n-1)} P_{kj}$

Where $P_{ij}^{(1)}$ is P_{ij}

It can be easily seen that $P_{ij}^{(0)} = \begin{cases} 1 & i = j \\ 0 & \text{Otherwise} \end{cases}$

Let $Pi^{(0)} = P[X_0 = i]$ be initial distribution of the system, Then the absolute probabilities of the outcomes at the nth trial is $P_j^{(n)}$ where $P_j^{(n)} = P[X_n = j]$

But $P_j^{(n)} = P[X_n = j]$

$= \sum P(X_n = j / X_0 = i)P(X_0 = i)$

$= \sum P^{(n)}_{ij} P_i^{(0)}$

Therefore, $Pj(n) = \sum P^{(n)}_{ij} P_i^{(0)}$

$= P^{(n)} = P^{(0)}P^{(n)}$

Theorem:

Let $\{X_n, n \geq 0\}$ be a Markov chain with an initial distribution p_0, one-step transition probability, P_{ij} and n-step transition probabilities $P_{ij}^{(n)}$ be the probability that a process in state i will be in state j after n additional transition. Then

(i) Modified Chapman-Kolmogorov Equation

$$P[X_n = j] = \sum_{i\epsilon s} p_0 (i) P_{ij}^n \quad \text{.......(4)}$$

(ii) Chapman – Kolmogorov equation, earlier stated in equation (3)

$$P_{ij}^{n+m} = \sum p_{ik}^m P^n_{kj}$$

Proof:

i) $P[x_n = j] = \sum_{i\epsilon s} P[x_n = j, x_0 = i]$

$= \sum p(x_0 = i)\, p(x_n = j / x_0 = i)$

$= \sum p_0(i)\, P^n_{ij}$

ii) $P_{ij}^{n+m} = P(x_{n+m} = j / x_0 = i)$

$= \sum P[x_m = k, x_{n\_m} = j / x_0 = i)$

$= \sum P\,(x_m = k / x_0 = i)\, P(x_{n+m} = j / x_0 = i, x_m = k)$

$= \sum P^m_{(l,k)} P(x_{n+m} = j / x_m = k)$

$= \sum_{l=0} P_{ik}^n P_{kj}^m$

In terms of matrix multiplication, the Chapman – Kolmogorov equation becomes

$$P^{(n+m)} = P^{(n)} . P^{(m)} \quad \text{.......(5)}$$

Hence,

$$P.\, P^{(n-1)} = P.P.P^{n-0} = \ldots = P^n \quad \text{.......(6)}$$

and thus $P^{(n)}$ may be calculated by multiplying the matrix P by itself n times.

If $p_{jk}^{(n)} = P(X_n = k / X_o = j)$, then the Chapman-Kolmogorov equation

$$P_{jk}^{(n)} = \sum p_{jl}^{(m)} p_{lk}^{(n-m)}, \qquad 1 \leq m \leq n-1$$

In matrix notation,

$$P_n \equiv (p_{jk}^{(n)}) = P_{n-m} P_m$$

But

$P_1 = P$ **so** $P_n = P^n$. **Let** $p_n = (\ldots P(X_n = 0), \ldots, P(X_n = k) \ldots)$

Denote the probability distribution of X_n

$$P_n = P_{n-1}\; P$$

And

$$P_n = PoP^n$$

Where P_n is the absolute probability matrix

Many physical systems tend to settle down to an equilibrium state, where the state occupational probabilities are independent of the initial probabilities. The long-run behaviour of the system implies when the number of transitions tends to infinity. The classification of states in Markov chains is useful in studying the long-run behaviour of a system.

2.3 Classification of States in Markov Chains

Let $A \in S$. The hitting time T_A of A is

$$T_A = \begin{cases} \text{MIN } (N > 0 : Xn \in A) & \text{If } X_n \text{ ever hits} \\ & \text{otherwise} \end{cases}$$

If A = {a}, then Ta denoting the distribution of the chain, starting from the state x (i.e Po (x) =1 and Po(y) = 0 for any y ± x), by P^x, the distribution of the chain starting from the initial distribution p_o is P^{po} and then the formula

$$P^{po}(A) = \Sigma p_o(i) P^i(A)$$

This amount to first choosing the initial state i at random from po, and the chain starting from state i, Then,

$$P_{jk}(n) = \Sigma p^j (T_k = m) P_{kk}^{(n-m)}$$

The state is called to be absorbing if P_{kk} =1. If the chain ever reaches K it stays there forever. Therefore for an absorbing state K,

$$P_{jk}^{(n)} = P^j (T_k < n).$$

A Markov chain is said to be irreducible if every state E_j can be reached from every other state E_i after a finite number of transitions; that is, for $i \neq j$

$$P_{ij}^{(n)} > 0, \quad \text{for } 1 \leq n < \infty$$

In this case all the states of the chain communicate. In a Markov chain a set C of states is said to be closed if the system, or if one of the states of C, will remain in C indefinitely. A special example of a closed set is a single state E_j with transition probability $p_{jj} = 1$. In this case, E_j is called an absorbing state. All the states of an irreducible chain must form a closed set and no other subset can be closed

Ergodic Markov Chains

An irreducible Markov chain is ergodic if all its states are ergodic. In this case the absolute probability distribution

$$a^{(n)} = a^{(0)}P^n$$

always converges uniquely to a limiting distribution as n $\longrightarrow \infty$, where the limiting distribution is independent of the initial probabilities

Limit Theorems

It is easy to show that if state j is transient, then

$$\sum_{n=1}^{\infty} p_{ij}^{n} < \infty, \quad \text{for all i.}$$

It means that, starting in i, the expected number of transitions into state j is finite as a consequence it follows that for j transient $p^n_{ij} \to 0$ as $n \to \infty$

Let μ_{ij} denote the expected number of transitions needed to return to state j.

That is

$$\mu_{ij} = \begin{cases} \infty & \text{if j is transient} \\ \sum_{n=1} nf^n_{ij} & \text{if j is recurrent} \end{cases}$$

By interpreting transitions into state j as being renewals, we obtain the following theorem

The Optimum Order

The Bayesian Information Criterion (BIC) can be used for the optimum order of Markov chain. The criterion assumes the following conditions:

1. The total number of states, S of X_n is finite
2. The chain is stationary
3. The chain is ergodic (chin, 1977)

The optimum order, k, of the chain is the order that gives minimum BIC. That is the value at which the BIC attains its minimum value

$$BIC(d) = n \log \hat{} \sigma_\varepsilon^2 + d \log n$$

Where n = number of observation to which the model is fitted (n = N – K)

N = number of observation
K = the order, 0, 1, 2 ...

Result and Discussion

The Optimum Order

Using the Bayes Information Criterion (BIC), the optimum order of the system is given in the table below:

Table 1: Optimum Order using BIC

Order	0	1	2
d	1	2	3
BIC	887.729	882.762	883.397

Table 2: Combined Transition

Yesterday/today	Dry	Wet	Total
Dry	1450	928	2378
Wet	930	972	1902
Total	2380	1900	4280

Long run Behaviour of the System

Consider the transition Matrix

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} = P_{ij}(n)$$

Where pij = $P(X_1 = j / X_0 = i)$, i, j = 0,1 with $P_{00} + P_{01} = 1$ and $P_{10} + P_{11} = 1$. That is P_{01} = P (wet today/dry yesterday) and P_{10} = P(dry today/wet yesterday).

3.2.1 N – Step Transition Matrices

Let $X = \{X_n, n > 0\}$ be a (temporally homogeneous) Markov chain with state space S. The chain can be determined by its one-step transition matrix and an initial distribution.

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$$

$$= \begin{bmatrix} 0.6098 & 0.3902 \\ 0.4890 & 0.5110 \end{bmatrix}$$

Limiting or Steady State Distribution

The Characteristic Equation is given by

$\pi P = \pi$ in matrix form

Where $\pi = (\pi_1, \pi_2)$

$$P = \begin{bmatrix} 0.6098 & 0.3902 \\ 0.4890 & 0.5110 \end{bmatrix}$$

The transition matrix

The chain is irreducible and recurrent.

Therefore,

$$\pi P = \pi$$

$$(\pi_1 \quad \pi_2) \begin{bmatrix} 0.6098 & 0.3902 \\ 0.4890 & 0.5110 \end{bmatrix} = (\pi_1 \quad \pi_2)$$

$\pi_1 = 0.5562$

$\pi_2 = 0.4438$

The mean recurrence time for state 0 is 1.80 and the mean recurrence time for state 1 is 2.25

Short-Run Distribution

Table 3: Absolute Probabilities of Wet and Dry Days at time, t_n using the Modified Chapman – Kolmogorov Equation.

n	$P_1^{(n)}$	$P_2^{(n)}$
1	0.5561	0.4439
2	0.5562	0.4438
3	0.5562	0.4438
4	0.5562	0.4438
5	0.5562	0.4438
6	0.5562	0.4438
7	0.5562	0.4438
8	0.5562	0.4438
9	0.5562	0.4438
10	0.5562	0.4438

Table 4: Absolute Probabilities of Wet and Dry Days at time, t_n using the Chapman – Kolmogorov Equation.

n	$P_1^{(n)}$	$P_2^{(n)}$
1	-	-
2	-	-
3	-	-
4	-	-
5	0.5562	0.4438
6	0.5562	0.4438
7	0.5562	0.4438
8	0.5562	0.4438
9	0.5562	0.4438
10	0.5562	0.4438

Discussion

The short run behaviour of the system, using the Chapman-Kolmogorov equation, the probability distribution matrix of the system is given by

$$\begin{bmatrix} 0.5562 & 0.4438 \\ 0.5562 & 0.4438 \end{bmatrix}$$

Note that the two rows are identical and the system becomes stationary at n = 5. This gives the Probability distribution of the system as

(0.5562 0.4438)

Using the Modified Chapman-Kolmogorov equation, the system becomes stationary at n =2 with the distribution given as

(0.5562 0.4438)

The long run behaviour of the system using the limiting characteristic equation also gives the probability distribution of the system as

(0.5562 0.4438)

In this work, three major steps were addressed in the stochastic approach to precipitation data. First, we discovered that the probability distribution of the precipitation data is Markovian, defined as

$$P\{X_{tn} = x_n/X_{tn-1}=x_{n-1},\ldots,X_{t0}=x_0\} = P\{X_{tn}=x_n/X_{tn-1}=x_{n-1}\}$$

Secondly, we considered the appropriate model for describing the daily rainfall pattern in Abeokuta as a Markov chain of order one. Thirdly, we considered the model by identifying the behaviour of the system over a period of time; we developed a technique, modifying the existing Chapman-Kolmogorov equation to assess the stationary analysis: a set of reachable states, the transition probabilities matrix of the Markov chain and the stationary probabilities vector. With efficient representation of the reachable states, s, and the transition probabilities matrix, P, the work described a two-state Markov chain approach to daily rainfall occurrence data.

Bayesian information Criteria (BIC) technique was used to estimate the optimum order of chain that fits the sequence of wet and dry days. The technique showed that the optimum order of the model is the first-order model that is Markov chain of order one best fit the model.

The conditional probabilities were computed to get the probability distribution and the one step transition probability which is

$$P = \begin{bmatrix} 0.6098 & 0.3902 \\ 0.4890 & 0.5110 \end{bmatrix}$$

The limiting distribution as n approaches infinity was given.

Chapman-Kolmogorov equation was modified and compared

$$P_n = P_i^0 P^n$$

The chain converges as n increases to give the Stationary distribution. The Stationary distribution is given as (0.5562 0.4438). The interesting result is that the modified Chapman-Kolmogorov equation approaches stationary faster than the Chapman-Kolmogorov equation.

Conclusion

This work describes a two state Markov chain approach to rainfall occurrence data. In this approach, dynamics were introduced into a state space by considering a modification of the Chapman-Kolmogorov Equation.

The possible system state X_n, (n = 0,1) is the state of the process at time n. In the study, there are two states with no restriction on possible transitions. The transition table showed the transition from wet to dry and vice versa. The model looked at the behaviour of rainfall occurrence data in Abeokuta over a period of twenty years. In this case, though the absolute probability tends to be independent of the initial distribution, the initial distribution can assist easy computation.

The condition distribution of X_n given X_{n-1} is the one step transition. By applying the Chapman-Kolmogorov equation, the n-step transition probabilities raised to nth power using the Matrix multiplication.

Finally, the conclusions which are drawn from this model must be tampered by the fact that the transition probabilities are obtained from relative frequency. The state distribution at 0,1 approach the vector.

$$\begin{bmatrix} 0.5562 & 0.4438 \end{bmatrix}$$

In fact, from time n = 2 to time n = 20, the modified Chapman-Kolmogorov gives the same vector implying that n does not have to be large before the state distributions are nearly equal to the fixed vector. So at n = 2, the chain becomes stationary. In other word, the rainfall pattern follows a first order Markov Chain model. At n = 2, the chain is in a stationary state and the stationary probabilities distribution is given as

(0.5562 0.4438) in matrix form

This is to say that the probability of a dry day during a raining season is higher (56%) than the probability of a wet day (44%). The implication is that there is likely to have drier day than wet days even in raining season (April to October).

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