COMPARISON OF ARIMA, GARCH AND NNAR MODELS FOR MODELLING EXCHANGE RATE IN NIGERIA

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ABSTRACT

In the pursuit to understand the importance of exchange rates to any economy, it becomes very expedient to build reliable models for the prediction of the volatility of exchange rates of home currency vis-à-vis the currencies of the developed nations, especially the nations with whom the home country have bilateral economic relationship; such as the United States of America (USA), China, Japan, to mention but few. Thus, this study investigates the characteristics or features of Nigeria exchange rate (Naira/USD), as well as the conventional facts of the exchange rate using Neural Network Autoregressive (NNAR) model and popular BJ-type models such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) *models. The study utilized secondarily sourced daily time series data from Central Bank of Nigeria websites that covers the period between January 2021 and December 2022. The return series was computed and the Box-Jenkins, GARCH and NNAR modelling methodologies were implemented in R environment. The study empirical results revealed that among this thirteen (13) candidate ARIMA models estimated,* $ARIMA(1,0,0)$ *returned as the most parsimonious ARIMA model with the lowest Akaike Information Criterion (AIC). Also,* $GARCH(1,1)$ *returned as the most parsimonious GARCH-type models for the series. Lastly,* $NNAR(24,1,12)$ *model returned as the appropriate fitted NNAR models for the series. Furthermore, the three (3) utilized accuracy functions i.e., Root Mean Square Error (RMSE), Mean Square Error (MSE) and Mean Square Error (MAE) criteria established* $NNAR(24,1,12)$ *with the minimum accuracy values across the three (3) evaluation criteria. Thus, this study concludes that* $NNAR(24,1,12)$ *is the optimal model for the examined exchange rate returns series and it outperformed the* $ARIMA(1,0,0)$ and $GARCH(1,1)$ *time series models.*

Keywords: Exchange Rate, ARIMA, GARCH, NNAR

1. Introduction

According to May and Farrell (2018), ever since the collapse of the Breton Woods international monetary system of fixed exchange rates among emerging market currencies to be précised, elevated volatility is a conspicuous attribute of exchange rates. No doubt, the exchange rate and its volatility are vivacious causes of economy instability of many nations particularly Nigeria. That is the reason the exchange rate fluctuations in Forex (FX) market have attracted significant attentions in recent studies. Exchange rate volatility can be referred to as the measure of fluctuations in an exchange rate which is usually measured on hour, daily, weekly, monthly or annual basis. It is a vital factor in option trading as well as in risk management as it provides a simple approach to calculating value at risk of a financial asset. Numerous studies such as Olakorede et al., (2018), Onyeka-Ubaka (2018), Gaddafi *et al.* (2021), Adeosun and Gbadamosi (2022) and so on had pointed out that exchange rate volatility is a vital subject of macroeconomic analysis and has received a great deal of interest from academics, financial economists and policy makers, particularly after the collapse of the Breton Woods agreement of fixed exchange rates among major industrial countries. Therefore, exchange rate volatility exposes economic agent to a greater exchange rate risk.

The modeling of exchange rate and forecasting future values from past values using suitable model are so important and necessary (Adeosun and Gbadamosi 2022), as it plays significant role on other variables like price of crude oil, goods, services, unemployment rate, wage rate, interest rate and equally tells whether an economy grows or not (Adeosun and Gbadamosi 2022). It provides decision makers with foresight information on the prices of international goods and openly displays the contribution of other external sector involvement in the international market (Juan & Tang, 2020). It also determines wealth distribution among countries, and determines the monetary policies adopted by countries. Exchange rate plays important and significant roles in the growth of Nigeria's economy as a country that solely depends on exportation of crude oil for her source of income (Lateef, 2020). Therefore, the exchange rate remains the foundation for all economic activities in the country, being a country that hugely supports the importation of foreign goods into the country with little or no restriction.

Besides, enormous variations have been observed in the foreign exchange market in the last few months and its effect on the economy of any country cannot be over emphasized. Most financial analysts, risk management and policy makers are specifically interested in obtaining worthy estimates of the conditional variance (a distinctive feature of volatility) in order to enhance portfolio shares or its risk management. While many advanced forecasting methods have previously been developed, this study focus on the more fundamental and most commonly used Box-Jenkins (BJ) based time-series models. The BJ models are remarkably flexible at handling a wide range of different time series patterns (Adenomon, 2017a & 2017b) and have been widely used in practice. Statistical models exploit the inherent characteristics of a time series, leading to a concise model. This is possible because the model makes strong assumptions about the data, such as the true order of the Auto-Regressive (AR), Moving Average (MA) or ARMA process. The order p of an AR(p) process is defined as the number of previous values of the time-series (lags) upon which the next value is dependent. AR processes with a high p-order are important for monitoring fine granularity data (e.g., minutes, seconds, milliseconds), and for long-range dependencies, where values long past still influence future outcomes.

Nevertheless, in recent years, there are growing concerns among time series researchers (i.e. Bala & Asemota, 2013; Agbailu *et al.* 2020) that Box-Jenkins models may not always lead to accurate forecasting especially in economic and business applications where the level of randomness is high and the constancy of patterns or parameters cannot be guaranteed. On the other hand, the Neural Network modelling approaches such the Neural Network Autoregressive (NNAR) of time series analysis has numerous advantages in recent period (Okeke, Yahaya and Agbailu 2022, Oskar *et al.*, 2019, Matteo and Gerasimos 2019). There are two features which make general neural networks attractive for time-series modeling. Firstly, neural networks have general nonlinear function mapping capability which can approximate any continuous function. It is therefore capable of solving many complex problems, given adequate data. Secondly, a neural network is a non-parametric data driven model and it does not require restrictive assumptions on the underlying process from which data are generated. Hence, it is on background this study pursues to compare empirically the performance of NNAR model and BJ-type model such as ARIMA and GARCH models in forecasting the Nigeria foreign exchange rate (Naira/USD).

2. Source of Data

The study utilizes daily time series data and covers a period between January 2021 to December 2022. The returns/volatility are calculated and are represented as the differences in exchange rates as $R_t = \log\left(\frac{R_t}{R_{t-1}}\right)$. This study pursues to estimate and examine the performances of the BJ-types and NNAR models using the R-software.

3. Model Specification

3.1 Autoregressive and Moving Average Process $ARMA(p, q)$

This is the combination of AR and MA processes of time series. Therefore ARMA (p, q), where p is the autoregressive order and q the moving average order can generally be represented as

$$y_t - \phi_1 y_{t-1} - \phi_2 y_{t-2} - \cdots - \phi_p y_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_{t-2} \varepsilon_{t-2} + \cdots + \theta_{t-q} \varepsilon_{t-q} \quad 1$$

This can be written as; $\emptyset(L) y_t = \theta(L) \varepsilon_t$ 2

where $\emptyset(L) = 1 - \emptyset_1 L - \cdots - \emptyset_p L^p$ and $\theta(L) = 1 + \theta_1 L + \cdots - \theta_q L^q$

Many real life time series are non-stationary. For such time series Box and Jenkins (1970) propose that differencing up to an order d could render it stationary.

If the time series of an ARMA model has to be differenced a certain number of times to become stationary, the model becomes what is known as an autoregressive integrated moving average model, or an ARIMA model. As mentioned previously, a time series which has to be differenced d number of times in order to become stationary, is integrated of order d, denoted $I(d)$. In its general form, the ARIMA model is denoted $ARIMA(p, d, q)$ which means that the AR is of the p^{th}-order, the time series is integrated d number of times, and the moving average is of the q^{th}-order. This further means that if the underlying AR and MA models are of the first-order, and the time series is stationary at the first difference, the ARIMA model is denoted $ARIMA(p, 1, q)$. It is important to note that an ARIMA model is not derived from any economic theory, that is, it is an atheoretic model. The Box-Jenkins methodology can be followed to determine $p, d, and\ q$ and estimate an ARIMA model (Box and Jenkins, 1970).

3.2 The Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model

Although the ARCH model is simple, it often requires many parameters to adequately describe the volatility process of an asset return. Bollerslev (1986) extended the ARCH model to allow for a more flexible lag structure. He introduced a conditional heteroskedasticity model that includes lags of the conditional variance as regressors in the model for the conditional variance (in addition to lags of the squared error term $e_{t-1}^2, e_{t-2}^2, \ldots, e_{t-q}^2$): the generalized ARCH (GARCH) model. The generalized autoregressive conditional Heteroscedastic (GARCH) model is used to investigate the volatility clustering and persistence. The model has only three parameters that allows for an infinite number of squared errors to influence the current conditional variance (volatility). The general framework of this model, GARCH (p, q), is expressed by allowing the current conditional variance to depend on the first p past conditional variances as well as the q past squared innovations. That is,

Mean equation:

$$r_t = \mu + \varepsilon_t \qquad 3$$

Variance equation:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \qquad 4$$

where r_t is return of the asset at time t, μ is average returns, ε_t is residual returns, defined as; $\varepsilon_t = \sigma_t \epsilon_t$ and ϵ_t is a sequence of independent and identically distributed (iid) random variables with mean zero and variance 1. In equation (4) the constraints $\alpha_i \geq 0$ and $\beta_i \geq 0$, are needed to ensure σ_t^2 is strictly positive (Ahmed et al, 2018). Also, p is the number of lagged σ^2 terms and q is the number of lagged ε^2 terms.

3.3 Neural Network Autoregressive (NNAR) Model

NNAR is a hybrid model comprising a linear and a non-linear component has been employed in the experiments (Zhang, 2003):

$$y_t = L_t + N_t \qquad 5$$

where L_t is the linear AR component and N_t is the non-linear component. First, we model the linear part by fitting an AR function to the data series. Then, the residuals are modeled using neural networks. Let r be the residual of the linear component, then:

$$r_t = y_t - \hat{L}_t \quad 6$$

where $\hat{L}_t$ is the estimate of the linear AR component. For non-linear patterns, we use neural networks:

$$\hat{r}_t = f(r_{t-1}, r_{t-2}, \dots, r_{t-q}) \quad 7$$

where q is the number of input delays and f is the non-linear function. So the combined forecast will be

$$y_t = \hat{L}_t + \hat{r}_t + e_t \quad 8$$

where e_t is the error of the combined model. Since linear AR models cannot model non-linearity, we assume that the residuals of the linear component will contain non-linear patterns, which a non-linear component, such as a neural network, should be able to model. In this way, the hybrid model is exploiting the strength of both components

3.4 Forecasting Evaluation Criteria

Mean Squared Error (MSE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE) which are defined as follows;

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2 \quad 9$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2} \quad 10$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \quad 11$$

Where A_t is the actual value in time *t*, and F_t is the forecast value in time *t*.

4. Results and Discussion

Table 1 presents the descriptive statistics of the daily exchange rate (EXCHR) and its returns (EXCHRR). As observed from Table 1, the mean, median, maximum and minimum of the exchange rate (EXCHR) were 410.81, 413.05, 448.05 and 379.00 respectively for the time period examined. The standard deviation (Std. Dev.) reveals a fluctuation of about ±17.83 within the daily exchange rate. Considering the Jarque-Bera statistics (8.31) and its probability value of the data, it can be deduced that the p-value ($0.016 < 0.05$) suggests non-normality of the data at 5% level of significance.

Table 1: Descriptive Statistics

	EXCHR	EXCHRR
Mean	410.810	0.000
Median	413.050	0.000
Maximum	448.050	0.001
Minimum	379.000	-0.034
Std. Dev.	17.829	0.002
Skewness	-0.321	-20.773
Kurtosis	2.998	448.104
Jarque-Bera	8.313	4021850.000
Probability	0.016	0.000
Observations	484	483

Note: EXCHR and EXCHRR denote Nigeria Exchange Rate (Naira/USD) and Its Returns respectively
Source: Researchers' computations

Fig 1 presents the time series plot of the exchange rate (EXCHR). As observed in Fig 4.1, the exchange rate (NGR/USD) was relatively constant between 4th January 2021 and 10th May 2021 at official rate of about 379/USD. The exchange rate skyrocketed to 409/USD in 31st May 2021 and was relatively constant till end of the year 2021. The exchange rate slightly increased to 415.91/USD in February 2022 and was relatively constant till 29th July 2022. Lastly, the figure depicts an increasing trend between the August 2022 and December 2022.

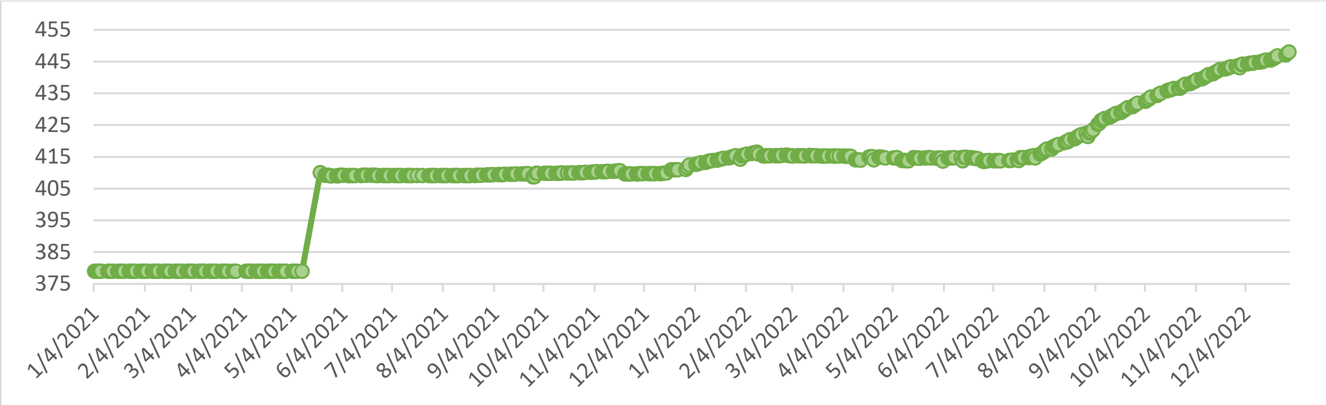

Fig 1. Exchange Rate (NGR/USD) Time Series Plot

The ARCH effect is associated with the concept of heteroscedasticity. It becomes apparent that there is clustering in the returns producing a pattern which is determined by some factor (see Fig 2).

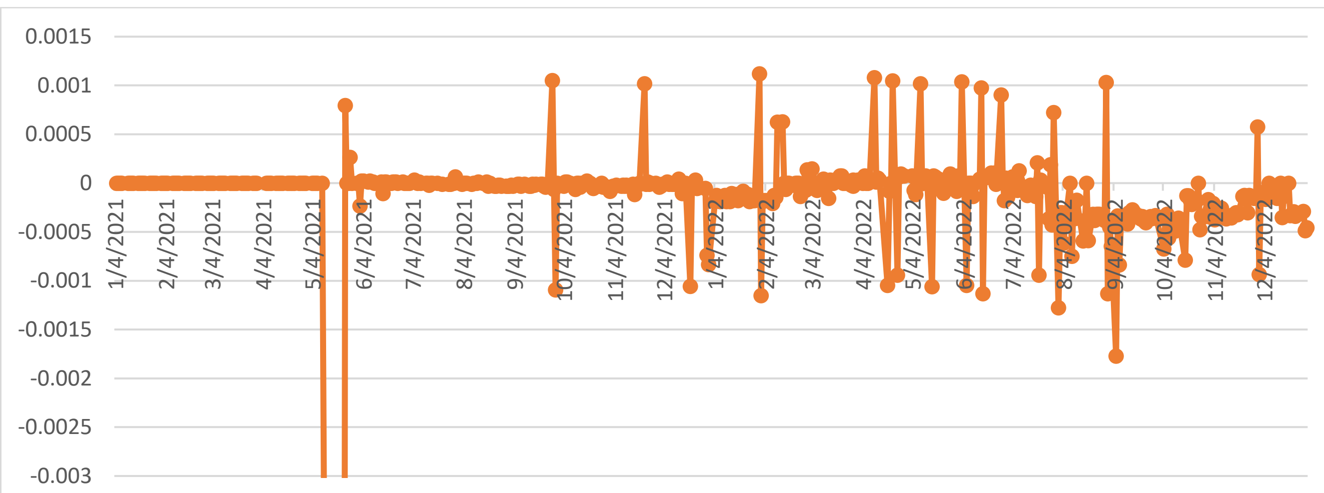

Fig 2. Time Series Plot of the Exchange Rate Returns (EXCHRR)

The exchange rate returns series is further subject to Heteroscedasticity test; Table 2 presents the test results.

Table 2: Heteroscedasticity Test

```
ARCH heteroscedasticity test for residuals
alternative: heteroscedastic

Portmanteau-Q test:
     order   PQ  p.value
[1,]     4 21.5 2.54e-04
[2,]     8 32.8 6.81e-05
[3,]    12 34.3 6.10e-04
[4,]    16 37.1 2.06e-03
[5,]    20 40.5 4.37e-03
[6,]    24 50.1 1.40e-03
Lagrange-Multiplier test:
     order    LM  p.value
[1,]     4 713.1 0.00e+00
[2,]     8 303.1 0.00e+00
[3,]    12 184.6 0.00e+00
[4,]    16 129.0 0.00e+00
[5,]    20  99.4 6.87e-13
[6,]    24  72.7 4.70e-07
```

Source: *R Output*

The results from Table 2 shows that we can reject the H_0 , hence, exchange rate return exhibit an ARCH effect. It becomes appropriate to apply ARIMA, GARCH and NNAR models that are sufficient to cope with the changing variance in EXCHRR since the return series meets the pre-conditions for the volatility modeling.

Before the estimation of the ARIMA-GARCH and NNAR models, a nonlinearity test might still be necessary to describe the important features of the data at hand. Table 3 below, reports the results of the nonlinearity test (BDS) which can be found in Goldfeld and Quandt, (1976); Hamilton, (1989)

.

Table 3: BDS Test

```
         BDS Test

data:  EXCHRR.ts

Embedding dimension =  2 3

Epsilon for close points =  0.0008 0.0016 0.0023 0.0031

Standard Normal =
      [ 8e-04 ] [ 0.0016 ] [ 0.0023 ] [ 0.0031 ]
[ 2 ]    4.5790     0.2794    -0.0062    -0.0454
[ 3 ]    4.4597     0.2703    -0.0287    -0.0610

p-value =
      [ 8e-04 ] [ 0.0016 ] [ 0.0023 ] [ 0.0031 ]
[ 2 ]         0     0.7799     0.9950     0.9638
[ 3 ]         0     0.7869     0.9771     0.9514
```

Source: *R Output*

The BDS test results in Table 3 indicates that there is nonlinearity effect in EXCHRR series. The table shows that the p-values are less than 5%, consequently implying a rejection of the null hypothesis that the series is linearly dependent. This result is an indication of the messy behaviour of financial time series data therefore the data can be estimated using a nonlinear model.

Table 4 presents the summary of the estimated ARIMA models with their respective Akaike information criterion (AIC) values. Using the specification measure (i.e. AICs), among the thirteen (13) estimated ARIMA models, ARIMA (1,0,0) was selected after returning with the lowest AIC (-4635.06). Thus, ARIMA (1,0,0) returns as the most parsimonious ARIMA models for the exchange rate returns series. Table 5 presents the model estimation.

Table 4: ARIMA Candidate Models

```
ARIMA(2,0,2)(1,0,1)[12] with non-zero mean : -4942.36
ARIMA(0,0,0)            with non-zero mean : -4636.662
ARIMA(1,0,0)(1,0,0)[12] with non-zero mean : -4951.506
ARIMA(0,0,1)(0,0,1)[12] with non-zero mean : -4632.82
ARIMA(0,0,0)            with zero mean     : -4633.877
ARIMA(1,0,0)            with non-zero mean : -4965.495
ARIMA(1,0,0)(0,0,1)[12] with non-zero mean : -4963.463
ARIMA(1,0,0)(1,0,1)[12] with non-zero mean : Inf
ARIMA(2,0,0)            with non-zero mean : -4962.523
ARIMA(1,0,1)            with non-zero mean : -4963.507
ARIMA(0,0,1)            with non-zero mean : -4634.851
ARIMA(2,0,1)            with non-zero mean : Inf
ARIMA(1,0,0)            with zero mean     : -4962.934

Now re-fitting the best model(s) without approximations

ARIMA(1,0,0)            with non-zero mean : -4635.058

Best model: ARIMA(1,0,0)            with non-zero mean
```

Source: *R Output*

Table 5 presents the estimation results of ARIMA(1,0,0). The results reveal that the AR(1) with coefficient of -0.0408 was found to be insignificant at 0.05 level. This implies that AR(1) is negatively however insignificantly contributing to the yield of exchange rate returns at rate of 4%. Moreover, the ARIMA(1,0,0) model was diagnosed for goodness of fit. The ACF and PACF plots of the model residuals return that all the lags were insignificance (see Fig 3). Similarly, Ljung-Box test results in Table 6 depict no p-values are less than 0.05 level, hence no trace serial correlation in the model residuals.

Table 5. ARIMA(1,0,0) Model Estimation

```
> summary(ARIMA)
Series: EXCHRR.ts
ARIMA(1,0,0) with non-zero mean

Coefficients:
          ar1     mean
      -0.0408  -2e-04
s.e.   0.0629   1e-04

sigma^2 = 4.796e-06:  log likelihood = 2320.55
AIC=-4635.11   AICc=-4635.06   BIC=-4622.51

Training set error measures:
                        ME        RMSE          MAE MPE MAPE      MASE
Training set -2.749628e-06 0.002185424 0.0003828801 Inf  Inf 0.9240366
                   ACF1
Training set 0.02005683
> coeftest(ARIMA)

z test of coefficients:

            Estimate  Std. Error z value Pr(>|z|)
ar1       -0.04079472  0.06288337 -0.6487  0.51651
intercept -0.00021360  0.00010485 -2.0373  0.04162 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Source: *R Output*

Table 6. Ljung-Box Test of the ARIMA(1,0,0) Model

```
        Box-Ljung test

data:  ARIMA$residuals
X-squared = 0.19953, df = 1, p-value = 0.6551

> Box.test(ARIMA$residuals,lag=2,type="Ljung")

        Box-Ljung test

data:  ARIMA$residuals
X-squared = 0.20763, df = 2, p-value = 0.9014

> Box.test(ARIMA$residuals,lag=3,type="Ljung")

        Box-Ljung test

data:  ARIMA$residuals
X-squared = 0.21058, df = 3, p-value = 0.9759
```

Source: *R Output*

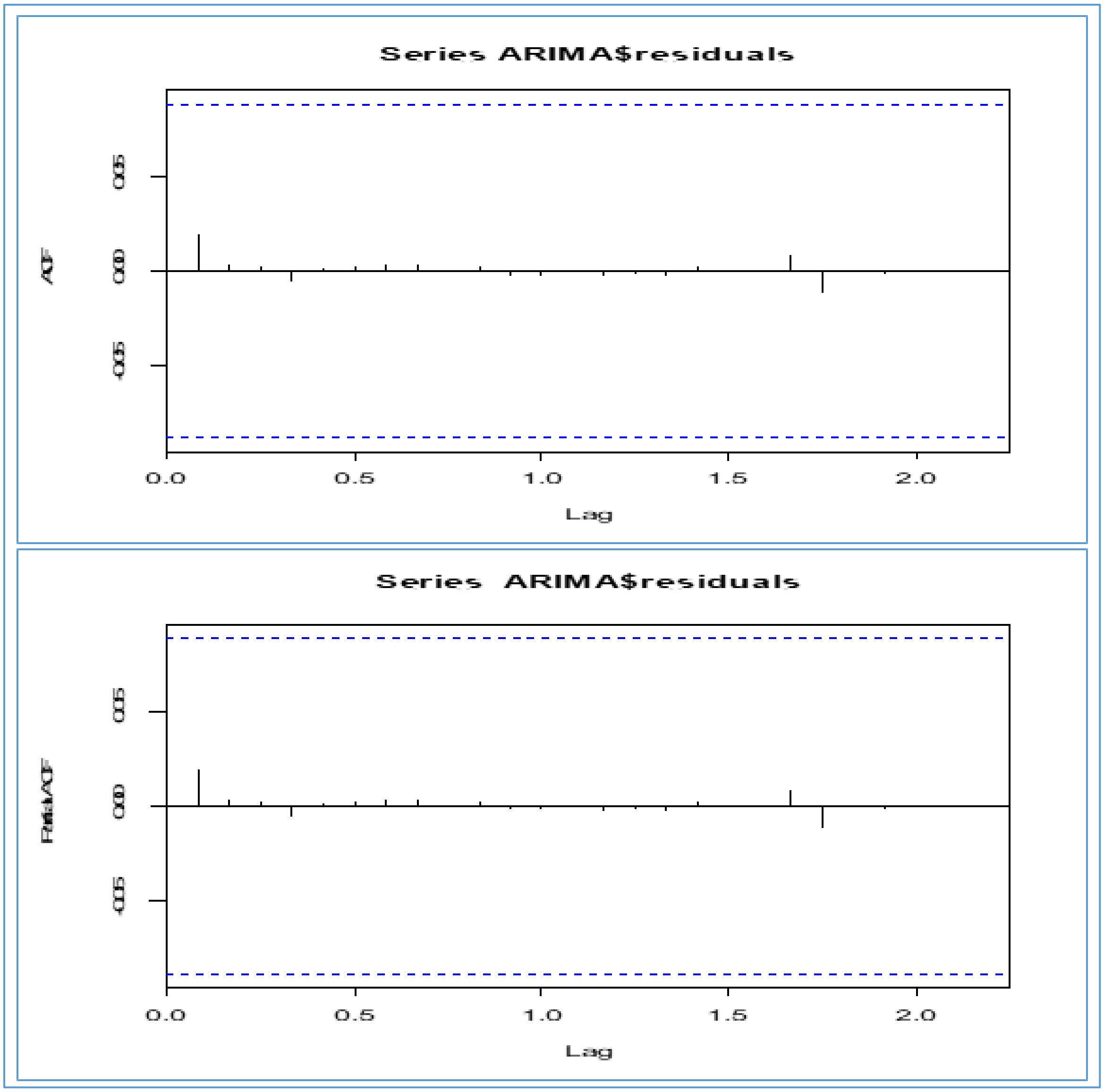

Fig 3 ACF and PACF Plots of the ARIMA(1,0,0) Residuals

Subsequently, the parameters of appropriate GARCH models were estimated through a search algorithm that tries a number of different coefficients before converging on the optimum values. The most parsimonious model and Q-statistics are plotted to determine linear dependence in the series as well as the order of the model to be fitted. Table 7 presents the results of five GARCH candidate models.

Table 7: GARCH Estimation and Selection

Model	ARMA Term	AIC value
GARCH-(1,1)	**(1,1)**	**-13.443**
GARCH-(2,1)	(1,1)	-7.6215
GARCH-(1,2)	(1,1)	-8.3341
GARCH-(2,2)	(1,1)	-7.6185
GARCH-(2,3)	(1,1)	-7.1860

Source: Researchers' computations using *R*

From Table 7, using the specification measures such as the Akaike information criteria (AIC), among the five estimated GARCH models. The selected model was GARCH-(1,1) with the lowest AIC of -13.443. Table 8 presents the GARCH-(1,1) model estimations. Furthermore, in the GARCH-(1,1) estimation, the estimate of the mean equation for returns series is statistically significant and the result of the variance (volatility) equation is represented in Table 8.

The persistence coefficient is 0.9587 (close to 1) which is required to have a mean reverting process. The closer the value to 1, indicates that shocks to volatility are very high and will remain as the variances are not stationary.

Table 8: GARCH (1,1) Model Estimation

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000039    0.000013  -2.93917 0.003291
omega   0.000000    0.000000   0.02112 0.983150
alpha1  0.054286    0.010730   5.05904 0.000000
beta1   0.904448    0.004334 208.67485 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu     -0.000039    0.006953 -0.005666  0.99548
omega   0.000000    0.000292  0.000014  0.99999
alpha1  0.054286    9.849275  0.005512  0.99560
beta1   0.904448   17.839205  0.050700  0.95957

LogLikelihood : 3156.371

Information Criteria
------------------------------------

Akaike       -13.443
Bayes        -13.408
Shibata      -13.443
Hannan-Quinn -13.429
```

Note: $\alpha_1 + \beta_1 = 0.9587$
Source: *R Output*

Table 9: GARCH-(1,1) Model Diagnosis

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.03364 0.85446
Lag[2*(p+q)+(p+q)-1][2]   1.69657 0.31849
Lag[4*(p+q)+(p+q)-1][5]   9.75455 0.01064
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.209  0.2715
Lag[2*(p+q)+(p+q)-1][5]     2.882  0.4291
Lag[4*(p+q)+(p+q)-1][9]     4.071  0.5769
d.o.f=2

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[3]    0.1831 0.500 2.000  0.6687
ARCH Lag[5]    0.9784 1.440 1.667  0.7394
ARCH Lag[7]    1.7871 2.315 1.543  0.7623
```

Source: *R Output*

Moreover, in order to diagnose the goodness of fit of the mean equation and the GARCH-(1,1) model for the series, the serial correlation and homoscedasticity (ARCH effects) of the model residuals are considered, the result of which is presented in Table 9. From the diagnosis of the goodness of fit of the models for the EXCHRR, the model i.e. GARCH-(1,1) seem appropriate for the data at the 95% confidence level because the serial correlation–statistics show that there is no statistically significant trace of autocorrelation and heteroscedasticity left in the squared standardized residual indicating that the mean equation and variance equation are adequately specified.

Furthermore, Table 10 presents the $NNAR(p, P, k)$ model estimation for the EXCHRR series. The table results depict an estimation of NNAR $(24,1,12)_{12}$ model for the EXCHRR data. The model estimation reveals $p = 24$ as the number of lagged values of inputs, $k = 12$ as the total number of hidden nodes and $P = 1$ as the number of seasonal lag. An average of 20 networks was returned. Thus, $NNAR(24,1,12)$ returned as the best fit NNAR model.

Table 10. NNAR Model Estimation

```
> NNAR
Series: EXCHRR.ts
Model:  NNAR(24,1,12)[12]
Call:   nnetar(y = EXCHRR.ts)

Average of 20 networks, each of which is
a 24-12-1 network with 313 weights
options were - linear output units

sigma^2 estimated as 7.171e-09
```

Source: *R Output*

Additionally, the best fitted models $ARIMA(1,0,0)$, GARCH-(1,1) and $NNAR(24,1,12)$ performances were assessed using out-sample forecasts for the time period examined. Table 11 presents the models performances based on the following accuracy measure criteria: Root Mean Square Error (RMSE), Mean Square Error (MSE), and Mean Absolute Error (MAE). The results return minimum accuracy measures for $NNAR(24,1,12)$ model across the four criteria examined. This implies $NNAR(24,1,12)$ model evidently outperformed $ARIMA(1,0,0)$ and GARCH-(1,1) models. Thus, $NNAR(24,1,12)$ model returns as the optimal model for the examined exchange rate returns series (Okeke, Yahaya and Agbailu 2022, Oskar *et al.*, 2019, Matteo and Gerasimos 2019).

Table 11. Accuracy Evaluation of the Optimal Models

	ARIMA(1,0,0)	GARCH (1,1)	NNAR(24,1,12)12
RMSE	0.00022	0.000124	**0.000122**
MSE	1.012e-07	1.001e-08	**1.0011e-09**
MAE	0.00014	0.00012	**0.00003**

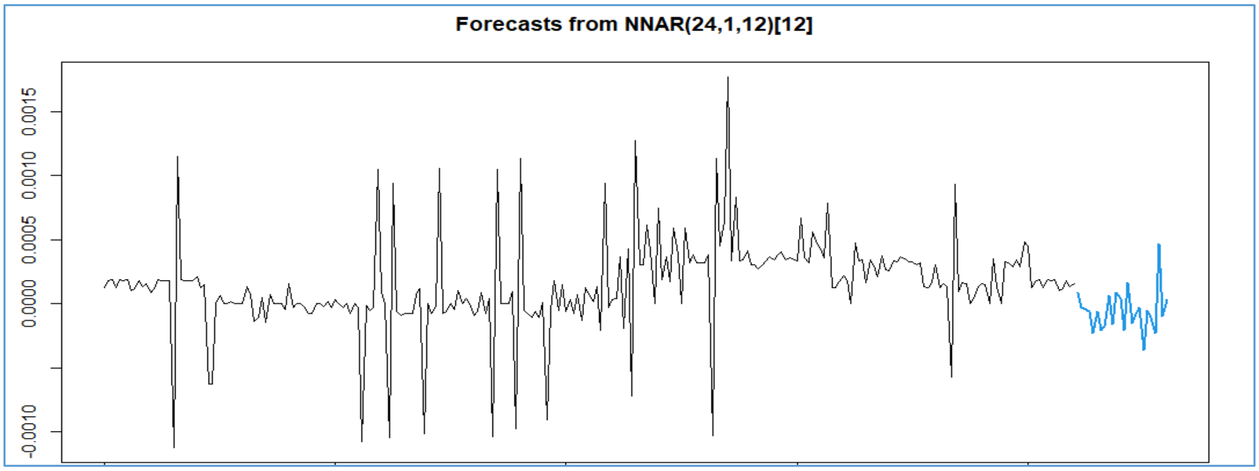

Fig 4 $NNAR(24,1,12)$ Forecast Graph

Last of all, the returned optimal model i.e. $NNAR(24,1,12)$ model was used to forecast the exchange rate returns for the period of 30 days, the results are presented in Fig 4. The Figure provides the details of the forecast which include the Point Forecast

5. Conclusion and Recommendations

This study examined the performance of three classes of time series models; The ARIMA models, the GARCH and the NNAR models. Empirical results revealed that among the thirteen (13) candidate ARIMA models estimated, $ARIMA(1,0,0)$ returned with the lowest AIC. Thus, $ARIMA(1,0,0)$ is the most parsimonious ARIMA models for the Nigeria exchange rate (Naira/USD) returns series. Also, GARCH-(1,1) returned as the most parsimonious GARCH-type models for the series. The empirical results for the symmetric model GARCH-(1,1) showed that the persistence coefficient is 0.9587 (i.e. close to 1) which is required to have a mean reverting process. The closer the value to 1, indicates that shocks to volatility are very high and persistent.

Lastly, $NNAR(24,1,12)$ model returned as the appropriate fitted NNAR models for the series under study. The diagnostic goodness of fit results for selected models revealed no any trace of serial correlation in the models. Based on the results of the analysis, the study deduced that BJ-type models such as ARIMA and GARCH models have the possibility of not appropriately modelling the financial time series returns such as exchange rate. Thus, this study recommends Neural Network Autoregressive modelling approach for modelling exchange rate returns.

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