On the Comparison of VECH and BEKK in Modeling of Oil Prices, Stock Exchange, Exchange and Inflation Rates Volatility in Nigeria

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ABSTRACT

A crucial component of financial time series is the modeling of volatility and co-volatility. The variances and covariances among financial data are modeled by multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models. The generalized autoregressive conditional heteroscedasticity (GARCH) model has been used to describe the volatility of a variety of univariate time series data, but there has not been much research done on using multivariate GARCH models to model multivariate time series data. Thus, this study aimed at comparing the performance of vector error conditional heteroscedasticity (VECH) and Baba Engle, Kraft, and Kroner (BEKK) MGARCH in modeling of oil prices, stock exchange, inflation and exchange rates volatility in Nigeria. The data for the study were collected from Central Bank of Nigeria Website and World Bank Data base. The data collected were analyzed using Augmented Dickey Fuller (ADF) test, diagonal VECH and diagonal BEKK. The results of the analysis revealed that diagonal BEKK performed better than the diagonal VECH in terms of model selection criterion. Based on the conditional-covariance results, it was concluded that the volatility spillover effects were strong and significant for all the variables except for the shocks of the returns of inflation rate and persistence shocks for the returns of exchange rate. Also, the magnitude of the estimate is not homogeneous across the variables but remains within a relatively tight range. The study recommended that further research should consider comparing diagonal BEKK with other MGARCH models.

Keywords: BEKK, GARCH, MGARCH, VECH, Conditional Co-variance

1.0 Introduction

Since the autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH) and their extensions were introduced, modeling volatility has been a significant field of study. A single asset's volatility can be studied at a time using the univariates GARCH model. Although a portfolio may include several assets or even various kinds of assets, it is crucial to understand their volatility and co-volatility. To address these problems, multivariate GARCH were introduced. The MGARCH provides suitable framework for modeling the covariance matrix of the numerous time series data.

ISSN NUMBER: 1116-249X

In the late 1980s and early 1990s, multivariate generalized autoregressive conditional heteroscedasticity models were initially created. After a period of quiet in the second part of the 1990s, this field now seems to be experiencing another rapid expansion phase (Adenomon *et al.*, 2019). The initial proposal for the MGARCH core structure came from Bollerslev *et al*. (1988) by generating a vectorized conditional correlation matrix, it extends univariate generalized autoregressive conditional heteroscedasticity (GARCH). The estimate of numerous parameters of the extended model were challenging and thus, Bollerslev *et al*. (1988) proposed the diagonal vector error conditional heteroscedasticity (VECH) model to improve tractability of estimate.

However, this type of MGARCH model was unable to be used to study spill-over effects because it oversimplified the correlation between parameters. While empirical research indicates that the factor GARCH, introduced by Engle *et al*. (1988), performs poorly on low land negative correlations, it does reduce the number of parameters to an orthogonal model.

By improving on the VECH model and creating a general quadratic form for the covariance equation, Baba Engle, Kraft, and Kroner Engle and Kroner (1995) successfully resolved the original VECH model's positive definiteness problem. Orthogonal models with two parameters are used in the standard BEKK estimation process, while orthogonal models with four parameters are used in the full generic BEKK model. Scalar and diagonal BEKK are two more plausible variants of the BEKK model in which the parameters are limited to either scalars or diagonal matrices.
Empirically, Metsileng *et al*. (2020) used multivariate GARCH models to study the volatility of the BRICS exchange rate. The monthly time series data from January 2008 to January 2018 were used in the study. Every variable was determined to be statistically significant using the BEKK-GARCH model. According to the calculation of diagonal parameters, only South Africa and Russia were statistically significant. This suggested that both South Africa's and Russia's exchange rates' conditional variance is influenced by their respective historical conditional volatility as well as the conditional volatility of the other BRICS exchange rates.
In order to determine whether model is more effective in modeling variance covariance matrices, Belasri and Ellaia (2017) compared the BEKK and Dynamic Conditional Correlation (DCC) models. They used the models to analyze daily stock prices in Morocco over ten years. The findings demonstrate that when it comes to modeling variance covariance matrices, the BEKK model outperforms the DCC model.

ISSN NUMBER: 1116-249X

Bala and Takimoto (2017) investigated stock return volatility spillovers in developed and developing markets (DMs) using multivariate-GARCH (MGARCH) models and their adaptations. To assess their impact on spillovers and volatilities, they included financial crisis dummies to the BEKK-MGARCH-type models. They also looked at how interactions between stock market volatility and the global financial crisis (2007–2009) were affected. Their key findings indicate that while correlations between established markets (DMs) are lower during financial crises, those between emerging markets (EMs) increase. They also discovered proof of volatility spillovers.

Using the Baba-Engle-Kraft-Kroner (BEKK) model for estimate, Shittu *et al.* (2019) applied the Multivariate GARCH model to the co-volatility of the Nigerian stock market, the USD/Naira exchange rate, and the inflation rate. In terms of minimum model evaluation tools, the study's results showed that the BEKK (1, 2) model was the best fit model for the series. The variance-covariance models demonstrate that the volatility of stock price returns is influenced by the inflation and exchange rate volatilities. It also demonstrates how stock price shocks have a significant impact on stock price volatility.

Minovic (2007) analyzed the Serbian financial market using the MGARCH (limited BEKK, DVEC, and CCC) models. Time series models that were bivariate and trivariate were used. It was discovered that conditional covariances for the index (BELEX15) and stocks (Hemofarm and Energoprojekt) showed notable variations over time.

Giacometti *et al*. (2023) used Conditional Value at Risk (CoVaR) to assess the risk spillover among European banks based on equities log-return data. The study used a spatial diagonal conditional correlation (DCC)-GARCH to describe the joint dynamic of returns, allowing each bank's conditional variance of log returns to depend parsimoniously on previous volatility shocks to other banks and their previous squared returns. The MGARCH model with Student's t-distribution is shown to be more accurate than the Filtered Historical Simulation and the standard multivariate Gaussian model by back testing of the derived risk measures. Additionally, the assessment of risk profiles and market risk spillovers is improved by the addition of a spatial component.

By merging a collection of pre-existing MCMC methods in the literature, Livingston Jr. and Nur (2023) propose a Bayesian analysis of MGARCH (l, m) models that includes estimate of the model order and the coefficient parameters. The multivariate GARCH model's BEKK formulation is the main emphasis of the suggested algorithm. Extensive simulation tests and real-world data are used

ISSN NUMBER: 1116-249X

to validate the suggested Markov Chain Monte Carlo (MCMC) techniques. Based on the results of comprehensive simulation simulations, the study concluded that the suggested approach offers accurate estimations.

Based on measurements of realized variances and correlations constructed from intraday data, Bauwens and Xu (2023) presented the scalar DCC-High-frEquency-bAsed VolatilitY (HEAVY) and DECO-HEAVY models for conditional variances and correlations of daily returns. The scalar HEAVY models perform better in terms of forecasts than the scalar BEKK-HEAVY model based on realized covariances and the scalar BEKK, DCC, and DECO multivariate GARCH models based solely on daily data, according to the empirical investigation.

Based on a new parametrization of correlation matrices, Archakov *et al*. (2020) suggest a unique class of multivariate Realized GARCH models that make use of realized measures of volatility and correlations. Three structures equi-correlation, block equi-correlation, and "free"—were used in the study to compare static and dynamic correlation models. Both in-sample and out-of-sample performance is improved by the dynamic block equi-correlation specification, according to the study's findings.

A class of Affine multivariate GARCH models was presented by Escobar-Anel *et al*. (2020). A covariance-dependent pricing kernel and stylized facts of asset returns such as DCC can be accommodated by the suggested model due to its flexibility. Along with the S&P 500 Index, the study used five assets for which volatility indexes are publicly available. When pricing two-assets options, it was shown that the suggested methodology is noticeably faster than Monte Carlo simulation.

In order to evaluate the dynamic dependency among return volatility for five Tunisian sectorial stock index series, Neifar (2020) introduced MGARCH volatility models. DVECH, DCC, and CCC are among the MGARCH models taken into consideration in the study. According to the DVECH model's conclusions, there is a significant and positive cross-shock of finance and bank stock returns on Tunindex return, volatility is predictable, and certain sectorial stock markets are interdependent. The DCC model's findings demonstrate that there are cross-border relationships between sectors and that macroeconomic instability factors significantly impact the evolution of the mean of returns.

Boman (2019) uses out-of-sample Value at Risk of several portfolios to analyze three distinct multivariate GARCH models. Sector portfolios with varying market capitalizations were included

ISSN NUMBER: 1116-249X

in the study. The Generalized Orthogonal (GO)-GARCH model, Constant Conditional Correlation (CCC), and DCC are the models that are being compared. The predicted VaR limit is forecasted one, five, and ten days in advance. The DCC outperforms the others in terms of conditional and unconditional violations of the VaR estimations, according to the empirical data.

For the multivariate GARCH (1,1) model, Ledoit *et al*. (2003) created an estimation process for the generic diagonal-VECH formulation. Using 25 years of weekly data on seven major national stock markets, the developed estimating procedure was compared to two well-known traditional multivariate GARCH (1,1) models: the diagonal BEKK model and the CCC model, as well as two popular but less complex estimators: the exponential smoothing estimator and the rolling-window estimator. A variety of metrics, including forecast accuracy, precision of value-at-risk estimations, persistence of standardized residuals, and optimal portfolio selection, were used to determine that the flexible multivariate GARCH method outperformed the other models.

Islam (2017) uses daily settlement prices to assess the hedging effectiveness of the Malaysian crude palm oil futures market by using four competing econometric models (the vector error correction model (VECM), the standard ordinary least square (OLS) regression model, and two variations of the multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) models: diagonal-VECH and diagonal-BEKK GARCH models). The empirical results show that the MGARCH models, in particular the diagonal-BEKK GARCH model, perform better than the other three models, suggesting that this model fits better when designing hedging strategies. Maharana *et al* (2024) examined how the COVID-19 pandemic impacted stock market volatility and interconnectedness between India and other selected global economies. The study employed BEKK, DCC, CCC and E-GARCH models. Using data from 2016 to 2024, the study revealed that BEKK outperformed the other models compared in the study.

Based on the empirical literatures reviewed, it was observed that there is paucity of studies on diagonal VECH and diagonal BEKK model. This study intends to add to the body of existing literatures by comparing the performance of VECH and BEKK in Modeling of Oil Prices, Stock Exchange, Exchange and Inflation Rate Volatility in Nigeria. The study is of significance as it helps in understanding how oil prices, stock exchange, exchange rates, and inflation rates interact and influence each other's volatility. This is particularly important for Nigeria, where oil exports significantly impact the economy. By understanding these relationships, policymakers and investors can make informed decisions.

ISSN NUMBER: 1116-249X

2.0 Materials and Methods

2.1 Source of Data and Variables of the Study

The source of data for this study was secondary collected from Central bank of Nigeria website and World Bank data based. Four variables were considered in this study. These variables include exchange rate (EXR), inflation rate (INFLA), oil price (OP) and stock exchange (SE). Monthly data were extracted on these variables from January 2003 to December, 2023.

2.2 Statistical Technique for Data Analysis

2.2.1 Unit Root Test

The Augmented Dickey Fuller (ADF) (Dickey & Fuller, 1981) was employed in testing the stationary of data. The ADF model is given as follows:

$$\Delta y_t = ay_{t-1} + x_t'\delta + B_1\Delta y_{t-1} + B_2\Delta y_{t-2} + \cdots + B_p\Delta y_{t-p} + v_t \quad (1)$$

Where x_t are optional exogenous regression which may consist of constant or constant and trend. The ADF t-test the null hypothesis which states that:

$\boldsymbol{H_0}: \theta = 0$, implying that the data needs to be difference to make it stationary. Against the alternative hypothesis: $\boldsymbol{H_1}: \theta < 0$, Implying that the data is trend stationary and needs to be analyzed by means of using time trend in the regression model instead of differencing the data.

The test statistics is conventional to t-ratio for θ:

$$t_\theta = \frac{\widehat{\theta}}{se(\widehat{\theta})} \quad (2)$$

Where $\widehat{\theta}$ is the estimate of a, and $se(\widehat{\theta})$ is the coefficient standard error.

2.2.2 Vector Error Conditional Heteroscedastic (VECH) Multivariate GARCH Model

Engle et al. (1988) introduced the VECH model. This model is a simple extension of the univariate GARCH model. Lagged conditional variances and covariances cause the conditional variance and covariance to change.

$$VECH(M_t) = K + \sum_{j=1}^{q} C_j VECH(r_{t-j}r_{t-j}') + \sum_{j=1}^{p} D_j VECH(M_{t-1}) \quad (3)$$

Where $VECH(.)$ is a matrix operator, K is a $n(n+1)/2 \times 1$ vector, and C_j and D_j are parameter matrices. However, the computation of the VEC-model is unpleasant since conditions has to be made in order to make M_t positive definite. A VECH-model is also very computationally demanding. As model that was developed as an improvement of the original VEC-model is the BEKK-model, which has the property that M_t is positive definite by definition.

ISSN NUMBER: 1116-249X

2.2.3 Baba, Engle, Kraft and Kroner (BEKK) Model

For an estimated MGARCH model to be considered plausible, the parameter Σ_t must be positive definite at all degrees of disturbance. Engle & Kroner (1995) published a quadratic formulation for the parameters that ensured positive definiteness. This model is relatively economical and suitable for a variety of assets since the number of parameters rises linearly with the number of assets (De Goeij et al., 2004). The BEKK model is provided in the format that follows.

$$\Sigma_t = C_0 C_0' + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ki}'\varepsilon_{t-1}\varepsilon_{t-1}' A_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} B_{ki}'\Sigma_{t-1}B_{ki} \quad (4)$$

where C_0 is a lower triangular matrix,

A_{ki} and B_{ki} are $N \times N$ parameters matrices

Based on the symmetric parameterization of the model, Σ_t is almost surely positive definite provided that $C_0 \times C_0'$ is positive definite.

Engle and Kroner (1995) proved that the necessary condition for the covariance stationarity of the BEKK model is that the eigenvalues, that is the characteristic roots of, $\sum_{k=1}^{K}\sum_{i=1}^{q}(A_{ik}^{*} \otimes A_{ik}^{*}) + \sum_{k=1}^{K}\sum_{i=1}^{p}(B_{ik}^{*} \otimes B_{ik}^{*})$ should be less than one in absolute value. Thus, the process can still render stationary even if there exists an element with a value greater than one in the matrix. Obviously, this condition is different from the stationarity condition required by univariate GARCH model: that the sum of ARCH and GARCH terms has to be less than one (Pang *et al*, 2002).

The BEKK (1, 1, K) model is defined as:

$$\Sigma_t = C_0 C_0' + \sum_{k=1}^{K}\sum_{i=1}^{q} A_k'\varepsilon_{t-1}\varepsilon_{t-1}' A_k + \sum_{k=1}^{K}\sum_{i=1}^{p} B_k'\Sigma_{t-1}B_k \quad (5)$$

where C_0, A_k and B_k are $N \times N$ parameter matrices, but C_0 is upper triangular. We can also write $C_0 \times C_0'$. Positivity of Σ_t is guaranteed if $\Sigma_0 \geq 0$. Here, there are 11 parameters, against 21 in the VEC model (Bauwens, 2005). This model allows for dynamic dependence between the volatility series (Tsay, 2005).

The diagonal and scalar BEKK models can be defined as follows:

i. The diagonal BEKK model. Take, kA and kB as diagonal matrices. For this case, the BEKK model is a restricted version of the VECH model with diagonal matrices (Bauwens, 2005; Franke *et al*, 2005).

ISSN NUMBER: 1116-249X

ii. The scalar BEKK model $A_k = a_k \times U,\ B_k = b_k \times U,$ Where ***a*** and ***b*** scalars and ***U*** is a matrix of ones.

The diagonal BEKK model is given by the following equations:

$$\sigma_t = CC' + a\varepsilon_{t-1}\varepsilon'_{t-1}a' + b\sigma_t b' \tag{6}$$

$$\sigma_{11,t} = C_{11}^2 + a_{11}^2\varepsilon_{1,t-1}^2 + b_{11}^2\sigma_{11,t-1} \tag{7}$$

$$\sigma_{22,t} = C_{11}^2 + C_{22}^2 + a_{22}^2\varepsilon_{2,t-1}^2 + b_{22}^2\sigma_{22,t-1} \tag{8}$$

$$\sigma_{12,t} = \sigma_{21,t} = C_{11}C_{22} + a_{11}a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}\sigma_{12,t-1} \tag{9}$$

The issues with this model are essentially the same as those with the full BEKK model: no equation has a parameter that only controls a specific covariance equation. Hence, it is not clear whether the parameters for σ_{12} are just the result of the parameter estimates for σ_{11} and σ_{22} or if the covariance equation alters the parameter estimates of the variance equations. Furthermore, the model lacks flexibility, making it susceptible to misspecification. However, it is evident that either the volatility or the covariance process is mis-specified when the covariance shows a different level of persistence than the volatilities (Baur, 2004).

The BEKK (1,1,1) model $\Sigma_t = \Omega + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'\Sigma_{t-1}B$ can be written as a VEC mode (subject to restrictions) using the formula.

$$VEC(\Sigma_t) = VEC(\Omega) + (A\otimes A)VEC(\varepsilon_{t-1}\varepsilon'_{t-1}) + (B\otimes B)'VEC(\Sigma_{t-1}) \tag{10}$$

Hence, the BEKK model is weakly stationary if the eigenvalues of $((A\otimes A) + (B\otimes B))$ are smaller than one in modulus, and thus

$$VEC(\Sigma_t) = (I_{N^2} - (A\otimes A)' - (B\otimes B)')VEC(\Omega)) \tag{11}$$

ISSN NUMBER: 1116-249X

3.0 Results

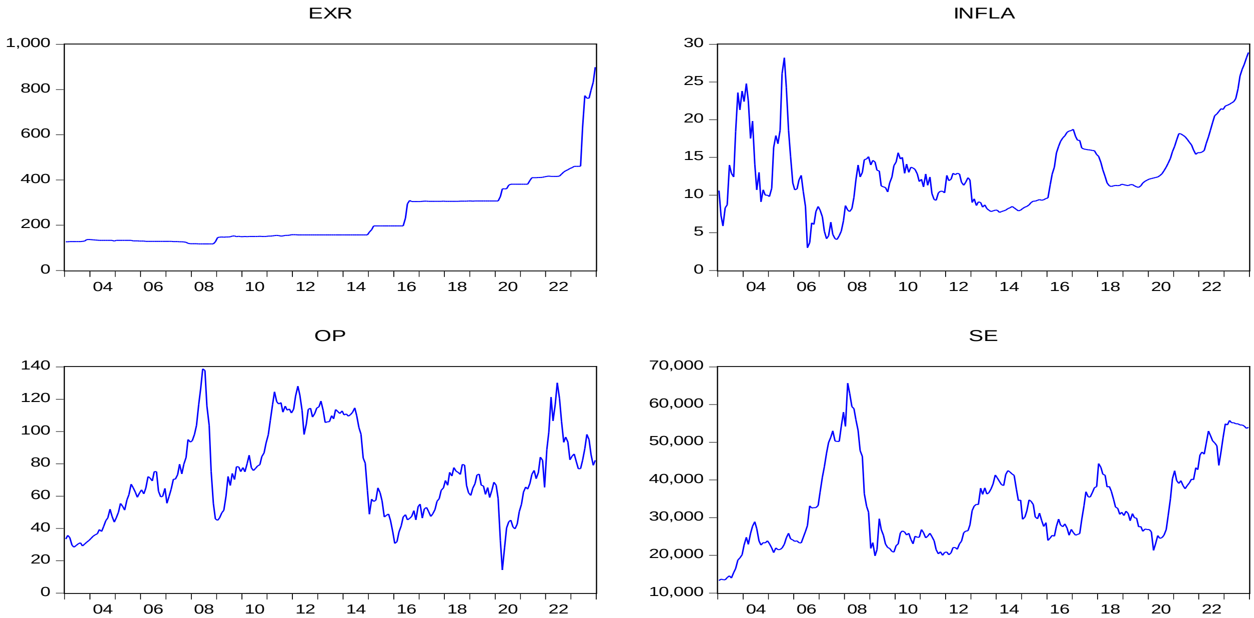

Figure 1: Time plot of Exchange Rate (EXR), Inflation Rate (INFLA), Oil Price (OP) and Stoch Exchange (SE)

Figure 1 present the graph of the study variables. From the graph, it was observed that exchange rate exhibits an increasing trend over time while oil price, inflation rate and stock exchange fluctuate over time with high rising and falling suggesting that these series are volatile. There was existence of trend in the series for example, in the inflation rate, the graph exhibits an upward trend between 2022 to 2023, upward trend exists in oil price and stock exchange between 2003 to 2008. Also, downward trends were also observed in some periods. The presence of these trends in the series provide evidence of non-constant mean over time.

ISSN NUMBER: 1116-249X

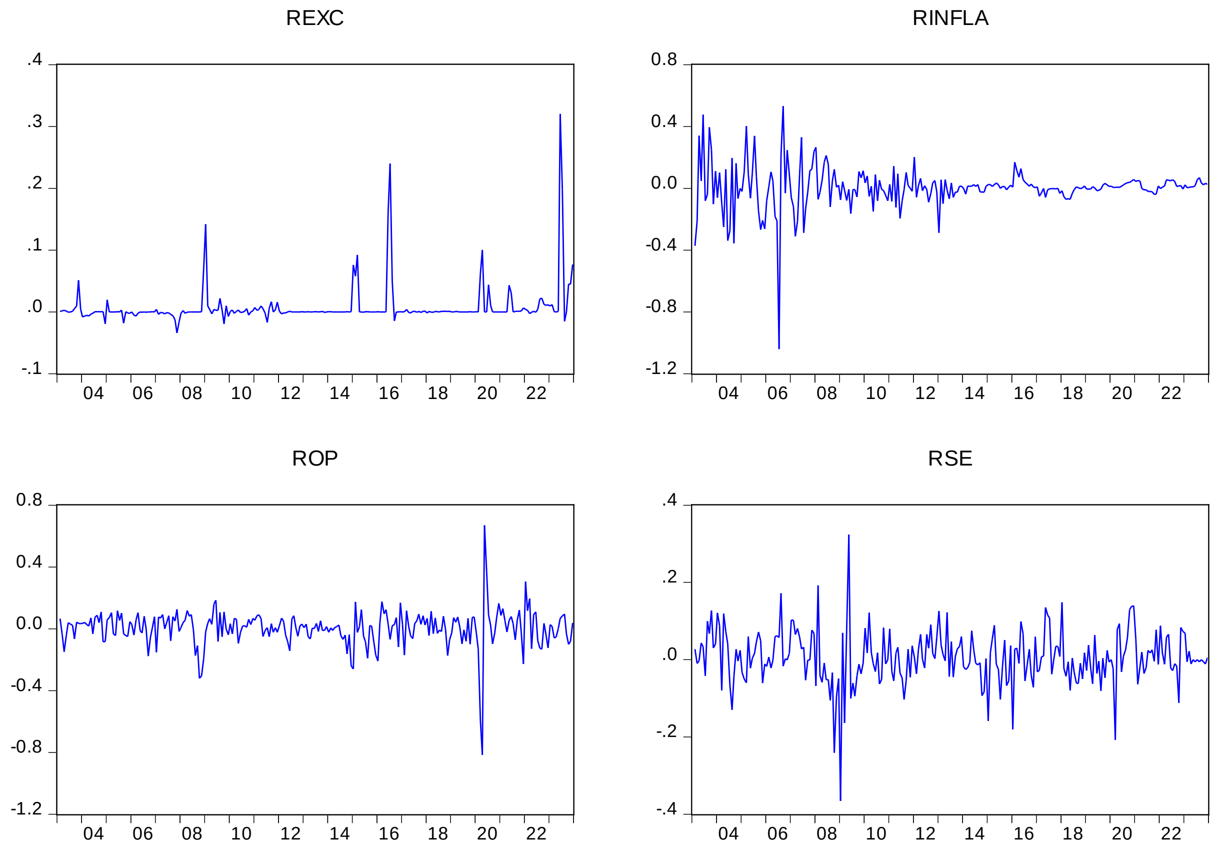

Figure 2: Time plot of Return on Exchange Rate (REXR), Return on Inflation Rate (RINFLA), Return on Oil Price (ROP) and Return on Stock Exchange (RSE)

Figure 2 presents the graph of return on exchange rate, return on inflation rate, return on oil price and return on stock exchange. The graph revealed evidence of volatility clustering that is, period of large changes followed by period of large changes and vice versa. There was also mean reversal as the volatility always reversed back to zero. Thus, indicating that the series is a stationary with volatility clustering and mean reversals.

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Table 1: Descriptive Statistics Results

	EXR	INFLATION RATE	OIL PRICE	STOCK EXCHANGE
Mean	232.8503	13.17611	73.66464	32757.40
Median	157.3100	12.32000	70.43000	29641.32
Maximum	898.8976	28.92000	138.7400	65652.38
Minimum	117.7200	3.000000	14.28000	13298.80
Std. Dev.	139.2620	5.028307	27.21496	11254.56
Skewness	1.973230	0.775813	0.299422	0.725632
Kurtosis	7.990965	3.521780	2.171789	2.715679
Jarque-Bera	425.0850	28.13785	10.96774	22.96355
Probability	0.000000	0.000001	0.004153	0.000010
Sum	58678.28	3320.380	18563.49	8254864.
Sum Sq. Dev.	4867873.	6346.252	185904.2	3.18E+10
Observations	252	252	252	252

Source: Authors' Compilation

Table 1 presents the descriptive statistics results of the study variables. The average exchange rate was 232.8503 USD/Naira with minimum and maximum values of 117.7200 and 989.8976 respectively. The average inflation rate was 13.17611 with minim and maximum values of 3 and 28.92 respectively. The average oil price was 73.66464 with minimum and maximum values of 14.28 and 138.74 respectively. The average stock exchange for the period under study was 32757.40 with minimum and maximum values of 13298.80 and 65652.38 respectively. The Jarque-Bera statistic results indicates that the series were not normally distributed ($p < 0.05$).

Table 2: Augmented Dickey Fuller (ADF) Unit Root Test Result

Variables	Test Critical Values			Test	P-value	Remark
	1%	5%	10%	Statistic		
EXR	-3.4568	-2.8731	-2.5730	2.7304	1.0000	Not stationary
REXR	-3.4566	-2.8730	-2.5730	-7.3085	0.0000	Stationary
INLA	-3.4580	-2.8736	-2.5733	-0.3080	0.9204	Not stationary
RINFLA	-3.4576	-2.8734	-2.5732	-6.9550	0.0000	Stationary
OP	-3.4564	-2.8729	-2.5729	-2.9037	0.0463	Stationary
ROP	-3.4565	-2.8730	-2.5730	-11.7153	0.0000	Stationary
SE	-3.4567	-2.8730	-2.5730	-2.3610	0.1540	Not stationary
RSE	-3.4567	-2.5730	-2.5730	-7.2104	0.0000	Stationary

Source: Authors' Compilation

Table 2 presents the ADF unit root test results of the variables under study. The results revealed that exchange rate, inflation rate and stock exchange were not stationary at level ($p > 0.05$). However, there returns series, that is, the returns on exchange rate, returns on inflation rate and returns on stock exchange were stationary are level ($p < 0.05$). On the other hand, oil prices and its returns series were stationary at level ($p < 0.05$).

ISSN NUMBER: 1116-249X

Table 3: Model Selection Criterion

Model	Loglikelihood	AIC
Diagonal-VECH	1386.939	-10.78039
Diagonal-BEKK	1407.158	-11.03712

Source: Author's compilation

Table 3 presents the model selection criteria using Loglikelihood and Akaike Information Criteria (AIC). The diagonal BEKK has the largest value of loglikelihood and the small values value of AIC. The smaller AIC value suggest that the diagonal BEKK is the best multivariate GARCH model suitable for this data set, the higher loglikelihood value confirmed the BEKK suitability to the data set. This may be due to the fact that the VECH models are more straightforward, while BEKK models can handle complex relationships between variables (Stelzer, 2008). Thus, the diagonal BEKK model provides a robust and reliable framework for analysing volatility persistence and spillover effects of oil prices, stock exchange, exchange and inflation rates volatility in Nigeria and was adopted for this study.

Table 4: Estimation of Mean and Variance Equation of Diagonal BEKK Model

	Coefficient	Std. Error	z-Statistic	Prob.
		Mean Equation		
REXR (μ_1)	0.003770	0.005071	0.743522	0.4572
RINFLA (μ_2)	0.011354	0.003242	3.502281	0.0005
ROP (μ_3)	0.011636	0.006010	1.936032	0.0529
RSE (μ_4)	0.010632	0.005202	2.043597	0.0410
		Variance Equation		
C_{11}	0.026452	0.000922	28.70071	0.0000
C_{12}	0.001129	0.007084	0.159333	0.8734
C_{13}	-0.000471	0.018012	-0.026154	0.9791
C_{14}	-0.008640	0.006401	-1.349777	0.1771
C_{22}	-0.002458	0.003355	-0.732630	0.4638
C_{23}	-0.036257	0.061878	-0.585949	0.5579
C_{24}	-0.006298	0.032422	-0.194237	0.8460
C_{33}	0.004824	0.460444	0.010478	0.9916
C_{34}	0.022250	1.865970	0.011924	0.9905
C_{44}	-1.90E-09	21888429	-8.70E-17	1.0000
$\emptyset_{11}$	0.830823	0.197892	4.198363	0.0000
$\emptyset_{22}$	0.021535	0.032912	0.654312	0.5129
$\emptyset_{33}$	0.790726	0.097241	8.131614	0.0000
$\emptyset_{44}$	0.207456	0.050286	4.125517	0.0000
θ_{11}	-0.004198	0.361250	-0.011621	0.9907
θ_{22}	0.987678	0.000870	1134.636	0.0000
θ_{33}	0.633325	0.080570	7.860575	0.0000
θ_{44}	0.913178	0.057252	15.95028	0.0000

ISSN NUMBER: 1116-249X

Source: Author's compilation

Table 4 presents the estimates of mean equation coefficients and variance equation coefficients. The coefficient of the mean equation for returns on inflation rate and returns on stock exchange were significant ($p < 0.05$). However, the coefficient for returns on exchange rate and returns on oil price were not significant ($p > 0.05$). The estimated model as presented in Table 4 shows that $\emptyset_{11}$, $\emptyset_{33}$, $\emptyset_{44}$, θ_{22}, θ_{33} and θ_{44} were significant. The own volatility effect of return series for inflation rate ($\theta_{22} = 0.9877$) is largest as compared to other variables and the own volatility effect of return series for stock exchange ($\theta_{44} = 0.9132$) is the second largest while the own volatility effect of return series for oil price ($\theta_{33} = 0.6333$) is the third largest as compared to other variables. The own volatility for return on exchange rate ($\theta_{11} = -0.0042$) was negative and statistically insignificant suggesting that returns on exchange rate is highly volatile, alternating, and negatively linked to one period lagged returns. In addition, the lagged own-volatility effect for exchange rate ($\emptyset_{11} = 0.8308$), oil price ($\emptyset_{33} = 0.7907$) and stock exchange ($\emptyset_{44} = 0.2074$) are the first, second and third largest as compared to others, respectively.

Table 5: Estimate of coefficient for conditional variance-covariance Equation

	Coefficient	**Std. Error**	**z-Statistic**	**Prob.**
M(1,1)	0.000700	4.88E-05	14.35036	0.0000
M(1,2)	2.99E-05	0.000188	0.159035	0.8736
M(1,3)	-1.25E-05	0.000476	-0.026156	0.9791
M(1,4)	-0.000229	0.000171	-1.339417	0.1804
M(2,2)	7.32E-06	4.34E-06	1.687161	0.0916
M(2,3)	8.86E-05	9.88E-05	0.897012	0.3697
M(2,4)	5.73E-06	2.59E-05	0.220671	0.8253
M(3,3)	0.001338	0.000607	2.204301	0.0275
M(3,4)	0.000340	0.000171	1.983449	0.0473
M(4,4)	0.000609	0.000455	1.339648	0.1804
A1(1,1)	0.830823	0.197892	4.198363	0.0000
A1(2,2)	0.021535	0.032912	0.654312	0.5129
A1(3,3)	0.790726	0.097241	8.131614	0.0000
A1(4,4)	0.207456	0.050286	4.125517	0.0000
B1(1,1)	-0.004198	0.361250	-0.011621	0.9907
B1(2,2)	0.987678	0.000870	1134.636	0.0000
B1(3,3)	0.633325	0.080570	7.860575	0.0000
B1(4,4)	0.913178	0.057252	15.95028	0.0000

Source: Author's compilation

Table 5 presents the estimates coefficient for conditional variance-covariance of the diagonal BEKK multivariate GARCH model. The constants of the conditional variance-covariance

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equation represented by M(1,1) to M(4,4) were not significant ($p > 0.05$) with the exception of constant term for the returns on exchange rate which is statistically significant ($p < 0.05$). Also, the coefficient of the shock and the persistence of shocks (given by matrix A1(1,1) to A1(4,4) and matrix B1(1,1) to B1(4,4) were statistically significant ($p < 0.05$) with the exception of A1(2,2) and B1(1,1) which were not statistically significant ($p > 0.05$). The highest impact of shocks (given by elements of matrix A) was detected for the returns on exchange rate (A1(1,1) = 0.8308), followed by the returns on oil price (A1(3,3) = 0.7907) and this was followed by returns on stock exchange (A1(1,1) = 0.2075) with returns on inflation rate (A1(2,2) = 0.0215) been the variables with the least shocks. The highest persistence of shocks (given by elements of matrix B) was proved for the returns on inflation rate (B1(2,2) = 0.9877), succeeded by returns on stock exchange (B1(4,4) = 0.9132) and returns on oil price (B1(3,3) = 0.6333) with returns on exchange rate been the least persistence of shock (B1(1,1) = -0.0042).

4.2.1 Diagonal ARCH Model Adequacy Checking

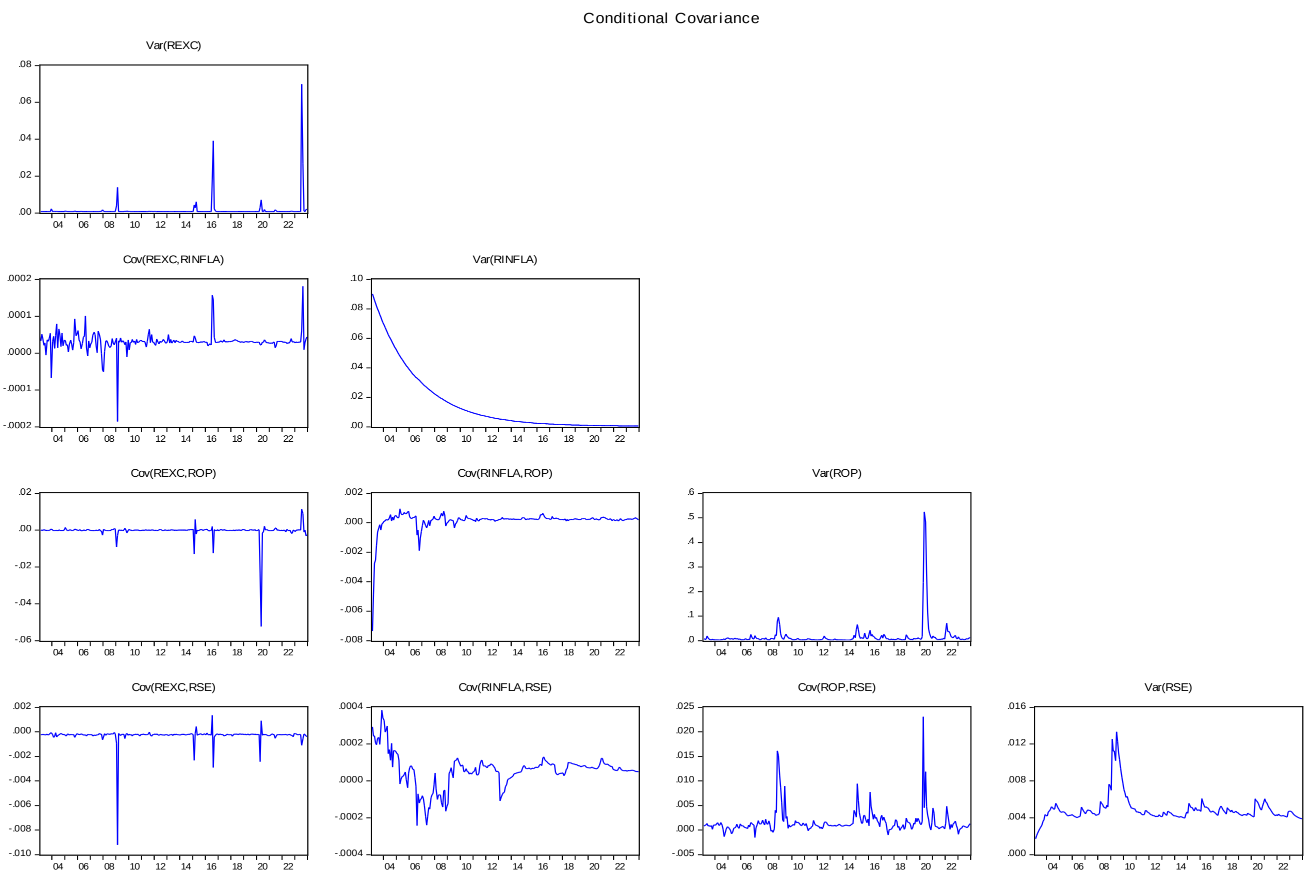

Figure 3: Conditional Covariance plots

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Figure 3 presents the results for conditional covariance of the estimated diagonal BEKK multivariate GARCH model. It is clear that the volatility persistence in all the four variables is generally high though in different time periods.

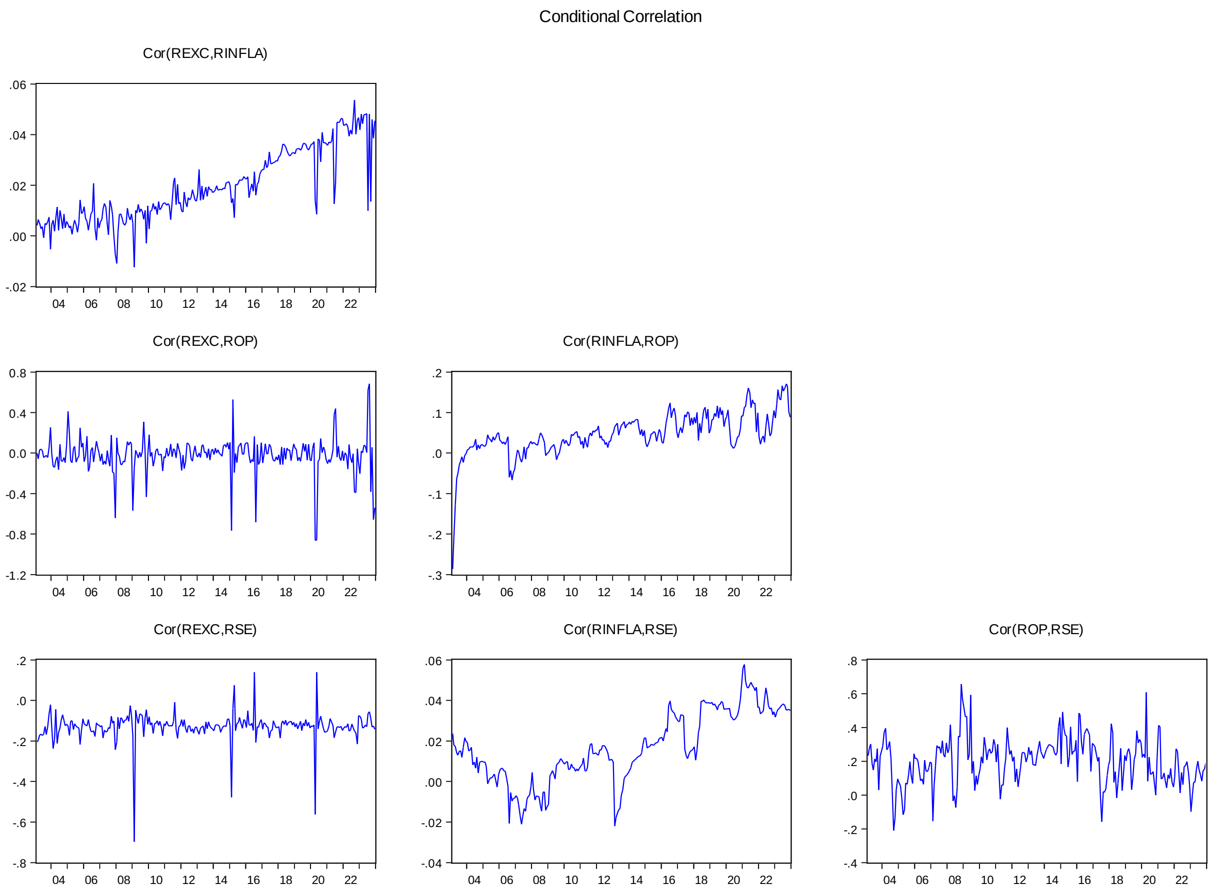

Figure 4: Conditional Correlation plots

Figure 4, present the conditional correlation plots of the variables under study. The volatility persistence is visible in all the variables considered in this study.

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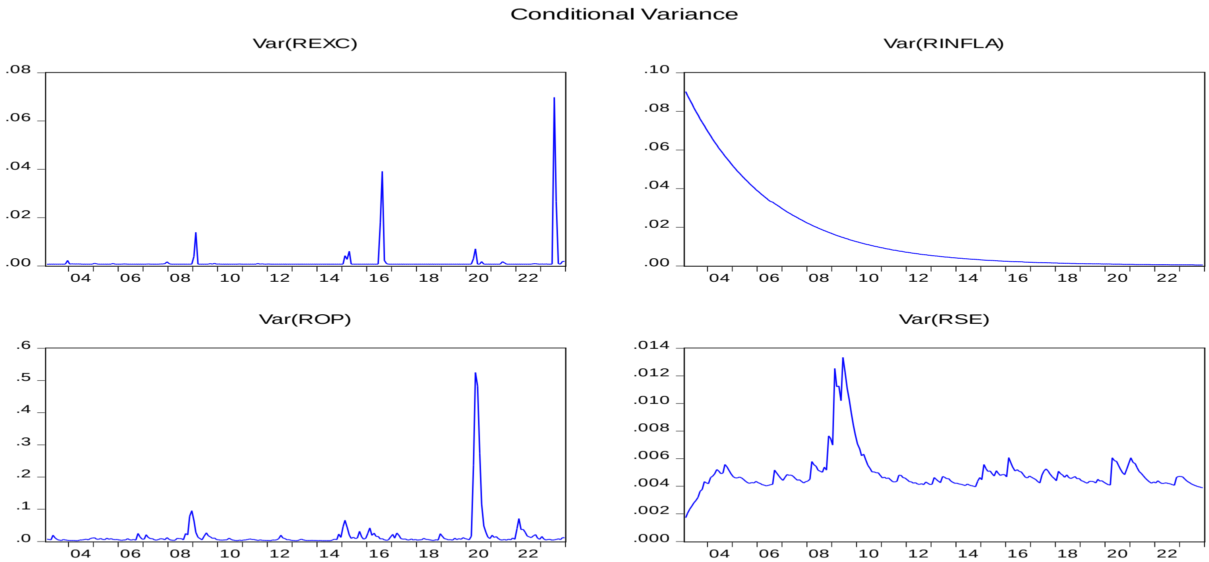

Figure 5: Conditional Variance plots

Figure 5 presents the plots of conditional variance. Similar to conditional covariance plots, the conditional variance plots displayed volatility persistence in the all variables.

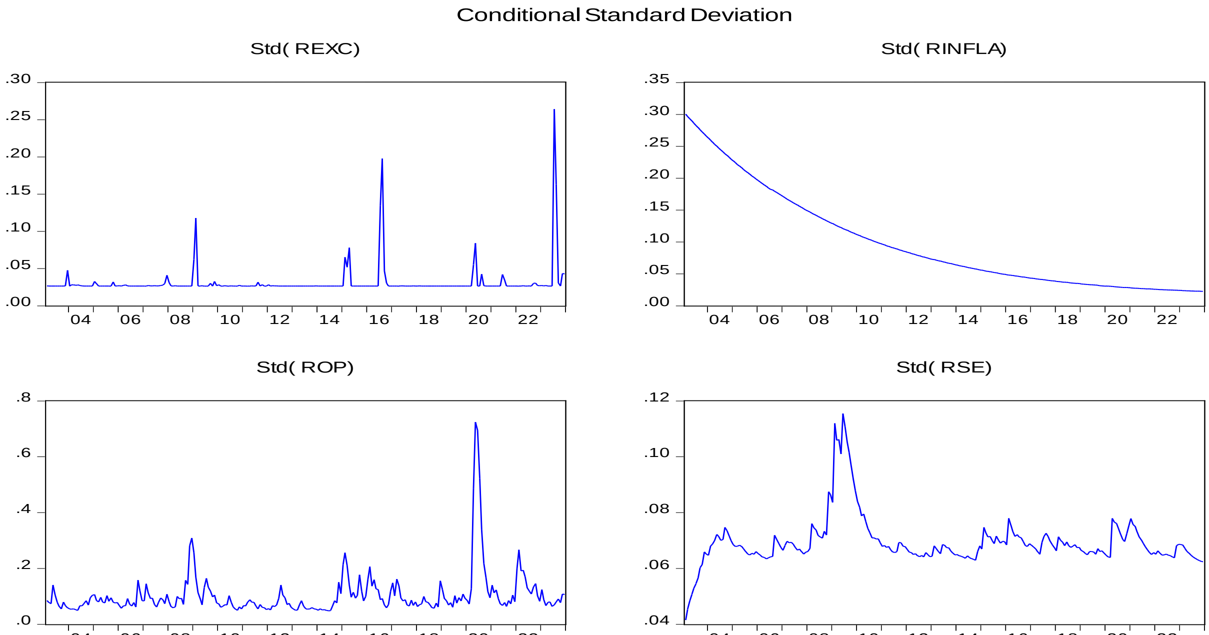

Figure 6: Conditional Standard Deviation plots

The conditional standard deviation plots presented in figure 4.6 revealed similar findings of volatility persistence shocks.

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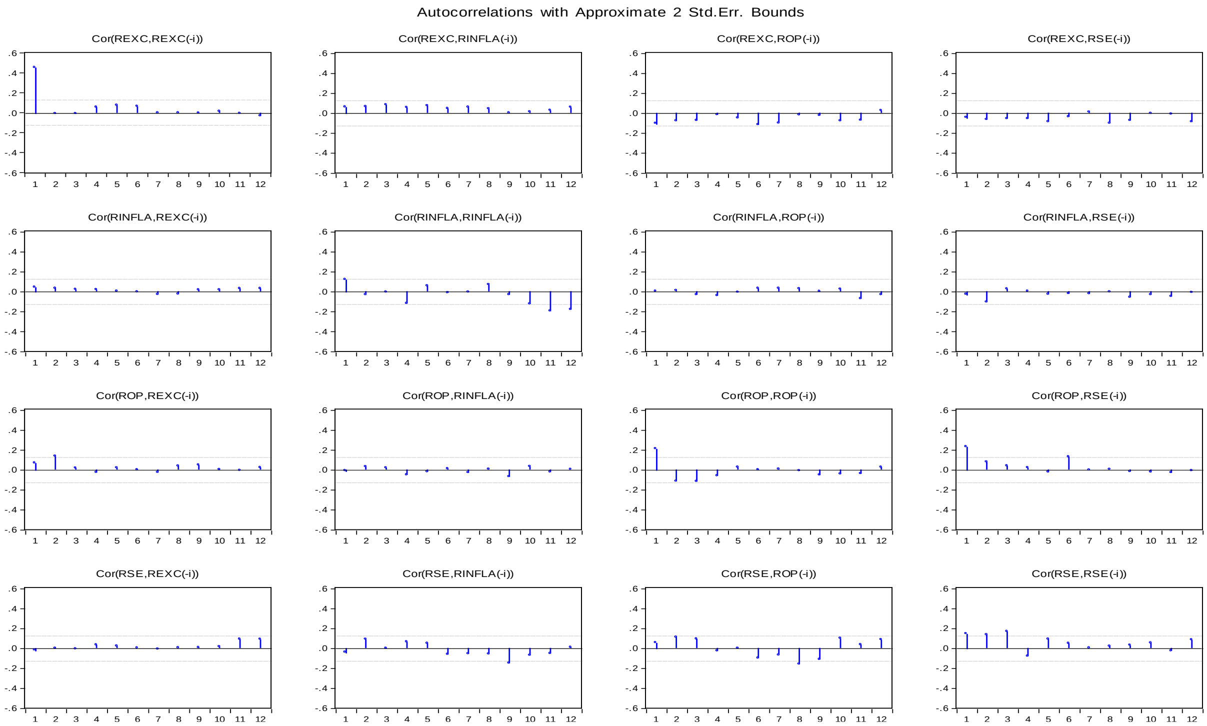

Figure 7: Autocorrection Plot with Approximate 2 standard error bound

Figure 7 presents the matrix autocorrelation plots with approximate standard bound of the estimated diagonal BEKK multivariate GARCH model. The results revealed some significant spikes in some of the autocorrelation plots especially in the leading diagonal. Few of the autocorrelation plots below and above the leading diagonal exhibits significant spikes.

Table 6: Residual Portmanteau Tests for Autocorrelations

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	DF
1	85.01373	0.0000	85.35379	0.0000	16
2	111.4953	0.0000	112.0481	0.0000	32
3	129.0006	0.0000	129.7651	0.0000	48
4	141.9056	0.0000	142.8791	0.0000	64
5	151.3164	0.0000	152.4812	0.0000	80
6	166.1286	0.0000	167.6562	0.0000	96
7	174.0102	0.0002	175.7639	0.0001	112
8	187.7387	0.0005	189.9443	0.0003	128
9	202.5379	0.0009	205.2939	0.0006	144
10	214.9907	0.0024	218.2634	0.0015	160
11	233.5858	0.0024	237.7108	0.0013	176
12	252.1219	0.0023	257.1775	0.0012	192

Source: Author’s Compilation

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Table 6 presents the residuals portmanteau test for autocorrelations. Here, we test the null hypothesis which states that there are no residual autocorrelations up to lag h. The result revealed that there is presence of autocorrelation up to lag 12 ($p < 0.05$).

4.0 Table 7: Residuals normal Distribution Test

Component	Test Statistics					
	Skewness	p-value	Kurtosis	p-value	Jarque-Bera	p-value
1	7.9263	0.0000	85.8511	0.0000	74416.81	0.0000
2	-0.7933	0.0000	8.324047	0.0000	322.7729	0.0000
3	-1.0852	0.0000	5.242488	0.0000	101.8556	0.0000
4	-0.6845	0.0000	7.005385	0.0000	187.3873	0.0000
Joint		0.0000		0.0000	75028.83	0.0000

Source: Author’s Compilation

Table 7 presents the residual test results for multivariate normal test using the skewness, kurtosis and Jarque-Bera. The results using the three cases revealed p-value less than 0.05 indicating that the residuals re not normally distributed.

5.0 Conclusion

This study compared the performance of diagonal VECH and diagonal BEKK MGARCH model. Based on the results of the data analysis, it was concluded that the diagonal BEKK model performed better than the diagonal VECK model. The mean equation coefficient of the diagonal BEKK model for return on inflation rate and return on stock exchange were significant. However, return on exchange rate and return on oil price were not significant. Moreover, own volatility effect of return series for inflation rate is the largest, followed by the own volatility effect of return series for stock exchange while the own volatility effect of return series for oil price is the third largest. However, the own volatility effect of return series coefficient was negative and insignificant only for exchange rate suggesting that exchange rate was highly volatile, alternating and negatively linked to one period lagged returns, though statistically insignificant. Based on the conditional-covariance results, it was concluded that the volatility spillover effects were strong and significant for all the variables except for the shocks of the returns of inflation rate and persistence shocks for the returns of exchange rate. The magnitude of the estimate is not homogeneous across the variables but remains within a relatively tight range. Influence of lagged covariance on future covariance is found to be positive in all estimations with the exception for returns of exchange rate which is negative. The positive estimates are relatively high with largest been lagged return for inflation rate which is 98.8%. Hence the Diagonal BEKK Model exhibits very large GARCH and

ISSN NUMBER: 1116-249X

relatively low ARCH effects. The empirical findings of this study suggests that investors in Nigerian markets can use the oil prices, stock exchange, exchange and inflation rates futures contract as an effective instrument to minimize risk. The study recommended that further research should consider comparing diagonal BEKK with other MGARCH models using same variables.

REFERENCES

Adenomon, M. O.; Mbuk, J. J.; Yahaya, H. U. Modeling Volatility of Nigeria Stock Exchange Using Multivariate GARCH Models. *Preprints* 2019, 2019020039. https://doi.org/10.20944/preprints201902.0039.v1

Archakov, I., Hansen, P. R., & Lunde, A. (2020). A multivariate realized GARCH model. *arXiv preprint arXiv:2012.02708*.

Bala, D.A and Takimoto, T (2017). Stock Markets Volatility Spillovers during Financial Crises: A DCC-MGARCH with skewed-t density approach. 17: 25-48.

Bauwens, L., & Xu, Y. (2023). DCC-and DECO-HEAVY: Multivariate GARCH models based on realized variances and correlations. *International Journal of Forecasting*, *39*(2), 938-955.

Belasri, Y. & Ellaia, R. (2017). Estimation of Volatility and Correlation with Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models: An Application to Moroccan Stock Markets. International Journal of Economics and Financial Issues, 7(2), 384-396.

Boman, V. (2019). A comparison of multivariate GARCH models with respect to Value at Risk.

Dickey, D. A. & W. A. Fuller (1981). Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica 49, 1057-1072.

Engle, R. F. & Kroner, K. F. (1995), Multivariate simultaneous generalized arch. Econometric Theory, 11(1), 122-150.

Engle, R. F. Wooldridge, J. M. & Bollerslev T. (1988). A Capital Asset Pricing Model with Time-Varying Covariances. *Journal of Political Economy*, 96(1), 116–131.

Escobar-Anel, M., Rastegari, J., & Stentoft, L. (2020). Affine multivariate GARCH models. *Journal of Banking & Finance*, *118*, 105895.

Giacometti, R., Torri, G., Rujirarangsan, K., & Cameletti, M. (2023). Spatial Multivariate GARCH Models and Financial Spillovers. *Journal of Risk and Financial Management*, *16*(9), 397.

ISSN NUMBER: 1116-249X

Islam, M. A. (2017). An Empirical Evaluation of Hedging Effectiveness of Crude Palm Oil Futures Market in Malaysia. *International Journal of Economics and Financial Research*, *3*(11), 303-314.

Ledoit, O., Santa-Clara, P., & Wolf, M. (2003). Flexible multivariate GARCH modeling with an application to international stock markets. *Review of Economics and Statistics*, *85*(3), 735-747.

Livingston Jr, G. C., & Nur, D. (2023). Bayesian inference of multivariate-GARCH-BEKK models. *Statistical Papers*, *64*(5), 1749-1774.

Maharana, N., Panigrahi, A. K., & Chaudhury, S. K. (2024). Volatility Persistence and Spillover Effects of Indian Market in the Global Economy: A Pre-and Post-Pandemic Analysis Using VAR-BEKK-GARCH Model. *Journal of Risk and Financial Management*, *17*(7), 294.

Metsileng, L. D., Moroke, N. D., & Tsoku, J. T. (2020). The application of the multivariate GARCH models on the BRICS exchange rates. *Academic Journal of Interdisciplinary Studies*, *9*(4), 23-23.

Minovic, J. (2007). *Multivariate GARCH models: theoretical survey and model application*. MSc. Thesis, Faculty of Economics, University of Belgrade. Serbia.

Neifar, M. (2020). Multivariate GARCH Approaches: case of major sectorial Tunisian stock markets.

Shittu, O. I., Marcus, N. S., & Acha, C. K. (2019). Multivariate GARCH modelling of volatility of Nigerian stock market and some economic indices. *Scientia Africana*, *18*(2), 183-192.

Stelzer, R. (2008). On the relation between the VEC and BEKK multivariate GARCH models. *Econometric Theory*, *24*(4), 1131-1136.

ISSN NUMBER: 1116-249X