COMPARATIVE ANALYSES OF DISTRIBUTIONS IN ASSYMETRY GARCH MODELLING: A STUDY OF NGN/USD EXCHANGE RATE

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Abstract

In an era of globalization characterized by flexible exchange rate systems, including Nigeria, the examination of foreign exchange rate volatility has become critically significant in recent decades, attracting the interest of both scholars and policymakers. Examining the dynamic variability of exchange rate series with distributional assumptions is highly significant. This paper examines the volatility of the Naira/US dollar exchange rate utilizing the EGARCH (1, 1) model from the GARCH family, with particular attention to generalized t, skewed student t, and skewed normal distributions. Data on the secondary Naira/Dollar exchange rate was obtained from the Central Bank of Nigeria's website, covering the period from January 2003 to April 2023. Monthly exchange rate returns were utilized to estimate the GARCH parameters employing the previously described distributions. The findings demonstrated that the skew student t (ST) distribution had superior predictive capability for N/$ exchange rate volatility, as evidenced by its elevated log-likelihood, reduced AIC, and diminished BIC within the chosen EGARCH (1, 1) model family. Furthermore, the results from the forecast evaluation revealed the existence of generated conditional variance, suggesting that the variance reverts to a long-term mean. The EGARCH model provided significant insights into volatility dynamics; however, it is concluded that the selection of distribution is crucial for improving its performance, with the skew Student's t distribution, due to its flexibility and adaptability to varying market conditions, identified as the most effective estimator in this study

Keywords: *EGARCH, Exchange Rate, Generalized t, Skew Student t, Skew Normal, Volatility*

1. Introduction

The significance of the exchange rate between the NGN and the USD is of utmost importance in the fields of international finance, economics, and the socio-political context of Nigeria. An essential indicator of the stability and well-being of a country's economy is its exchange rate. It has a big impact on trade dynamics, economic policy, and living standards in general (Bala

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and Asemota, 2013; Adedotun et al, 2022). Since Nigeria gained independence, the country has experienced periods of stable and volatile money. Frequent bouts of naira depreciation characterized the 1970s and 1980s, while volatility in oil prices coupled with pervasive economic problems. Different types of exchange rate regimes have been used during successive decades to stabilize the currency: from a controlled float regime to a fixed system and back to a managed float system.

This exchange rate has historically been influenced by many factors. Recently, the Naira has been in great distress from the pressure of increased prices of oil, inflation, falling foreign exchange reserves, and fiscal imbalances (Adenekan et al., 2019). Therefore, the Central Bank of Nigeria, in its policy to mitigate some of these impacts, implemented policies like capital controls, among others, with some exchange rate regimes. The immediate and long-term effects of policy changes put in place in 2022 will affected the rate of the currency.

Least squares is undoubtedly the foundation of useful econometrics. This is not surprising given that econometricians are usually faced with the task of being able to predict the magnitude of the response of a variable as another variable changes. Nonetheless, the landscape is changing, and econometricians are in high demand to foresee and assess the level of model flaws. In the aforementioned conditions, the focus is on volatility inquiries, and the dominant approaches that have gained importance are the ARCH/GARCH models (Aako and Alabi, 2019).

The measurement of exchange rate volatility is an important indicator that quantifies the degree of variation observed in the average return of an investment or index (Peng et al., 2023). The measure in question has garnered considerable interest within the field of financial analysis, especially with the introduction of Engle's (1982) ARCH model, which was enhanced by Bollerslev (1986). As a result, the development of different Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models has significantly advanced our understanding of volatility.

Bordalo et al., (2024) provide insight into an additional crucial element of equity volatility, namely, volatility clustering. This phenomena suggests that the current shocks in volatility have a lasting impact on the anticipated levels of volatility in the future. Furthermore, it is observed that equity volatility has an asymmetric characteristic in terms of volatility innovation. The aforementioned asymmetry becomes apparent when a decline in returns leads to a subsequent rise in volatility that surpasses what would generally be expected during an increase in returns.

ISSN NUMBER: 1116-249X

The complex aspects of stock return volatility highlight its crucial significance in financial analysis and market dynamics, particularly in the area of the Nigerian stock market, where the relationship with the naira/dollar exchange rate is unavoidable.

Different versions of GARCH model, the EGARCH and TGARCH models, have been utilised in order to reflect asymmetries in the volatility of exchange rates (Nelson, 1991; Zakoian, 1994). These models recognise the differential effects of adverse shocks on volatility in comparison to direct shocks.

In the financial literature, considerable effort has been devoted to volatility analysis, with several important style facts established. Among these, one of the more important findings is that large changes in an asset price tend to be followed by further large changes-a phenomenon usually referred to as volatility clustering. The two seminal figures who first indicated this were Mandelbrot (1963) and Fama (1965). The second is, the empirical evidence generally has shown that the asset price distribution is thicker tailed than the normal distribution, which provides a better chance of extreme value asset return realization.

The GARCH model has proved helpful over the years in empirical finance, though it has had its drawbacks. One marked significant inadequacy involves the conditions of non-negativity constraints imposed on estimated parameters. Many variations to this main version of GARCH have resulted on the basis of these lacks identified by scholars. Another symmetric GARCH family class includes the TGARCH, first proposed by Zakoian (1994). Still another is the EGARCH model proposed by Nelson (1991). Both of these models consider that the positive and negative shocks could have different effects on the conditional variance. Some specific omissions of the GARCH model that EGARCH addresses are that it measures persistence in strongly stationary series, captures asymmetric responses in volatility to shocks, and removes parameter limits to ensure that conditional variance stays positive.

Bollerslev, (2023) stressed the point that a GARCH model with the assumption of normal distribution error fails to capture kurtosis as well as persistent autocorrelation in return series. Nelson, 1991 recommended incorporating generalized error distribution within the EGARCH model in order to handle such problems. On the other hand, Majose, (2010) advanced the argument that stationarity of TGARCH depends on the choice of distribution for the error term, generally assumed to be either Gaussian or Student-t. Also, as the level of tail heaviness in the error distribution increases, the magnitude of the leverage effect captured by the TGARCH model decreases and becomes less flexible.

ISSN NUMBER: 1116-249X

2. Review on related works

The study by Adenekan, et al. (2019), sought to understand how the volatility of the exchange rate between the NGN and the USD affects the return on the Naira exchange rate. In their research approach, the authors considered the daily percentage exchange rate returns of the Naira against the US dollar. The asymmetric characteristic in the pattern of volatility for the exchange rate is convincingly evident according to the findings of the researchers. Indeed, the study noticed that the negative shocks, translating to the declines in the return on exchange rate, tended to have a rather bigger force of dragging volatility downwards, as compared to the case with positive shocks of a similar magnitude.

Nnamani and David (2012) uses symmetric and assymetric volatility models in analyzing the variation of the Naira weekly exchange rate and eight other currencies. The researchers explicitly designated the residual distribution as normal and made the observation that volatility shown a significant level of persistence in seven of the examined currency pairs, but it exhibited explosiveness in one pair. It is noteworthy that the asymmetric model failed to detect a leverage effect for any of the analyzed currencies.

Emenogu, Adenomon, and Nweze (2020) studied the volatility of daily stock returns for Total Nigeria Plc from January 2001 to May 2017, using nine GARCH variant models to estimate and backtest Value-at-Risk (VaR). Most types of GARCH models were found to be consistently persistent in the capture of volatility, but out of these, some cases exhibited instability for both the iGARCH and eGARCH models, whereas the sGARCH and gjrGARCH models resulted in minor convergence issues. It was in the findings that across the models, mean reversion of returns ensued. The challenges and restrictions of GARCH models will be highlighted by using the forecasted volatility in Stock-specific risk management.

Okon et al. (2020) thus assessed the trend in the exchange rate between the Nigerian Naira and the US Dollar during the COVID-19 pandemic. In their study, the authors adopt a longitudinal design for a monthly dataset of exchange rate series for their ARIMA model-based forecasts. Results of diagnostic indicated an autoregressive and moving average pattern with a best fit to an ARIMA, that is ARIMA 2, 1, and 3. The residual analysis confirmed the strength of the model. Based on the evidence, the ARIMA model has been performing well in terms of out-of-sample forecasting of the dynamic nature of exchange rates. Besides, it has given immense

ISSN NUMBER: 1116-249X

insight into the fluctuations within the pandemic period, thus forming a base for future economic planning.

For that purpose, Spulbar et al. (2023) used the FTSE MIB Index, representative of 40 major equities on Italy's Borsa Italiana, with a daily closing price versus the WIG20 Index in Poland from December 2008 to December 2022. Both are classified under the category "developed markets" by the FTSE Equity Country Classification Report 2022, hence the indication of advanced financial systems. In fact, some evidence on the volatility with respect to the market behaviors is given through the volatility analyses of applied GARCH (1,1) and EGARCH (1,1) models in the study; thus, such indices are of a great importance to investors and policymakers.

Almansour et al., (2023) utilize daily data from 2012 to 2021 to analyze dynamic return volatility connections between S&P and Dow Jones sustainability indices with their conventional counterparts. Strong connectedness of sustainability and conventional indices for all periods was found, including pre- and during COVID-19. Conventional indices like the S&P500 and DJWI emerged as primary transmitters of the shocks while sustainability indices emerged as the key receivers. These findings have significant implications for investors, portfolio managers, and policymakers in terms of risk management and the understanding of volatility spillovers between indices, especially in periods of uncertainty.

Kaplan (2023) analyzed asymmetric GARCH models for the volatility of the exchange rate between MIST countries and the US dollar. Their study showed the presence of leverage effects and asymmetric spillovers in these exchange rates, reflecting the complex relationships between these countries and the US.

Kuang [2024] proposes one high-frequency-based long-memory realized volatility model in univariate form which enables the portfolio Value-at-Risk forecast to be improved. This paper presents a model incorporating realized variance and covariance measures without including any parametric covariance matrix, which can increase forecasting accuracy but at reduced computational complexity. The value of this approach over traditional approaches, such as GARCH type of models, is shown mainly around times of high volatility or during periods of structural instability. It outlines the transformative role of high-frequency data in risk management: providing a hands-on and efficient tool to better assess portfolio risk.

A fractionally integrated mechanism was added to APARCH models in a later study to simulate the behaviour of the yen against the US dollar. In contrast with the stock market, shocks to the

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value of the yen, either positive or negative, were observed to have an equivalent impact on future volatility levels. Drawing from this, the authors present the finding that no apparent differences seem to exist between the estimations from both stable and fractionally integrated models, which again provides further proof of the complex dynamics characterizing exchange rate volatility.

Bakkali et al. (2024) proposed two hybrid models, namely Vanilla-RGARCH and LSTM-RGARCH for volatility estimation in a financial framework. They combine the strong aspects of Realized GARCH and HAR models by soothing each of its failures in the representation of characteristics that capture features relative to asymmetry and long memory in financial time series. As shown, both the new models substantially outperform the usual accuracy increase in out-of-sample RGARCH and HAR volatility forecasts. The ability to learn even the most complicated trends has been outstanding in time series that reveal persistence or anti-persistence in volatility of financial time. It has been found that these models outperform the others in volatility prediction and, therefore, offer a better tool for risk management and investment decisions.

David et al., (2016) express the significance of naira against other major currency economies: US dollar, British pounds, euro, Japanese Yen. In this view point, GARCH (1, 1) technique was employed to analyze all uneven properties of volatility in unpredictable current rate of Nigerian currency. As a factor of fact, added more exogenous variable(s) just on how to improve better performance of employed methods whose predictive power are checked hereby.

The study of Abdullah et al. (2017) predicted volatility of the Bangladeshi currency rate using a GARCH model. By adopting daily data across four classes of GARCH models, for which the normal distribution competed with the Student's t-error distribution, it achieved better results of improved modeling performances on various diagnostic tests besides the accuracy of the modelled forecast.

The study by Naimy et al., (2021) examined the volatility pattern of six famous cryptocurrencies and looked into their relationship with a number of international currencies. The scope of their analysis involved examining various GARCH-type models and assessing the prediction precision of the Value at Risk metric. The duration of this study project extended over a period of four years, specifically from 2015 to 2019. Significantly, the results of their study highlighted the better predictive accuracy of the IGARCH model in forecasting global

ISSN NUMBER: 1116-249X

currencies, both when evaluated using the dataset used for model development (in-sample) and when applied to new data (out-of-sample).

Nugroho, Priyono, and Susanto (2021) employed GARCH (1,1) models with Skew Normal(SN) and Skew Student-t error distributions to evaluate the volatility of financial asset returns. The researchers utilised these models to analyse the daily returns of the FTSE100 and IBEX35 stock indices, covering the time period from 2000 to 2017 on monthly basis. The estimation of model parameters was conducted using Excel's Solver, employing the Generalised Reduced Gradient Non-Linear approach, as well as the Adaptive Random Walk Metropolis method adopted. The findings of the study reveal that Excel's Solver demonstrates effectiveness in accurately estimating parameters for GARCH(1,1) model with non-Normal distributions. The evaluation of model fitting performance was conducted by the log-likelihood ratio test. The results indicated that the Skew-ST distribution fitted with GARCH(1,1) exhibited the most optimal fit. Subsequently, the Student-t, Skew-Normal, and Normal distributions followed in terms of their fitting quality.

Several literatures have dwelt on fitting volatility models to Naira/Dollar exchange rate series at different time-intervals without taking into consideration, the distribution model that best fit in for the series due to its fatter tails, which can thereby eliminate fitting spurious variance model when this is taken into cognizance in predicting future volatility of the financial market series, hence, the motivation of this paper.

3. Methodology

This paper adopt the ex-post facto method. This method involves the utilization of secondary data for the analysis of research findings, as articulated by Huyler and McGill (2019). A sample of size of nineteen (19) years and four months data, which spans through a total of 232 months of the NGN/USD exchange rate data was conveniently extracted from the CBN statistical bulletin and allied websites that publish economic data. The data ranges from January 2003 to April 2023. The stated year was selected to capture a comprehensive period that includes both periods of stability and significant market events, such as the global financial crisis (2007-2009) and the COVID-19 pandemic, which had a profound impact on financial markets.

3.1 Data Transformation

The exchange rate series underwent a transformation, converting it into a sequence of returns. This transformation is a fundamental step in financial analysis and serves as the basis for

ISSN NUMBER: 1116-249X

various modelling and forecasting techniques. Empirical studies, as demonstrated by Christoffersen (2012), have shown that logarithmic returns tend to adhere more closely to the assumptions of normality, a fundamental concept in many statistical and financial models. This normality assumption is valuable because it underpins various statistical tools and techniques commonly employed in financial analysis. The monthly naira/dollar exchange rate price returns is defined as:

$$a_t = \ln\left(\frac{y_t}{y_{t-1}}\right) \tag{1}$$

Where: a_t represents return on exchange rate in period *t;* y_t and y_{t-1} represents exchange rate in period *t*; and $t-1$ respectively.

The analytical method used in this study consists of both descriptive and inferential. The descriptive method of data analysis consists of the measure of central tendency, location, and graphical illustration of the variables under study for movements of the NGN/USD exchange rate over time and its level of stationarity using the correlograms and partial autocorrelation functions plots while the inferential analytical method is the eGARCH model based on comparison with three error distributions such as the Generalized t, Skew normal and Skew Student's t distributions.

3.2 EGARCH Model Specification

According to Nelson (1991), the EGARCH (p, q) model integrates the asymmetrical influence of positive and negative shocks on volatility. In this model, it is acknowledged that negative shocks tend to induce higher levels of volatility, even when their magnitudes are identical to positive shocks. EGARCH is formulated in a logarithmic framework, which means that its parameters are not constrained and can assume negative values, while still guaranteeing a positive conditional variance. Furthermore, the conditional variance of EGARCH is expressed as a function of previous innovation, given its volatility dynamic with $p = q = 1$ as:

$$\ln(\sigma_t^2) = \omega + \beta\ln(\sigma_{t-1}^2) + \gamma\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{2}$$

In equation (2), β represents volatility persistence, while α captures the magnitude of the effect of past squared returns on present volatility. The asymmetry in volatility is captured by the

ISSN NUMBER: 1116-249X

leverage parameter γ. If γ is negative, it implies that negative returns posed greater influence on volatility than positive returns and vice versa. When γ is nonzero, the combined impact of a positive previous return on current volatility is given by (α + γ), while the impact of a negative previous return is (γ - α).

3.3 Estimation Techniques

The parameters in GARCH models are established by maximizing the likelihood function, which is formulated based on the presumed distribution of the residual term. Different distributions, including the Generalized t distribution, Skew Normal distribution, and Skewed Student's t-distribution, have been examined as potential options for modelling this innovation. The log-likelihood of parameter vector θ is given as:

$$L(\theta) = \sum_{i=1}^{T} I_t(\theta) = \sum_{i=1}^{T}\left(-\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma_t^2 - \frac{v_t^2}{2\sigma_t^2}\right) \tag{3}$$

The process of maximizing equation (3) entails establishing the initial values for the innovation (v_t^2) and the conditional variance (σ_t^2). In this paper, the conditional distribution of the innovation is defined as a Generalized Error Distribution. This choice allows for the incorporation of all the leptokurtosis present in the returns. The distribution in functional form is given as:

$$f_X(x', s) = \frac{Se^{-\frac{1}{2}\left|\frac{x}{\lambda}\right|^s}}{\lambda 2^{(s+1)/s} \bullet \Gamma(1/s)}, \text{ where } \lambda = \left[\frac{2^{-2/s}\Gamma(1/s)}{\Gamma(3/s)}\right]^{1/2} \tag{4}$$

The shape parameter s > 0 is constrained to be greater than 0. This distribution takes on the characteristics of a standard normal distribution when s equals 2, but it exhibits fat tails when "s" is less than 2. The log likelihood function for equation (4) is expressed as follows:

$$L_n(\theta) = \sum_{t=q+1}^{n}\left(\log\frac{s}{\lambda} - \frac{1}{2}\left|\frac{v_t}{\sigma_i\lambda}\right|^s - (1+s^{-1})\log 2 - \log[\Gamma(1/s)] - \frac{1}{2}\log(\sigma_t^2)\right) \tag{5}$$

3.3.1 Generalized *t* distribution

The Generalized t (GT) distribution is a generalization of the Student's t distribution that incorporates additional shape parameters to control its tails and shape (Wang and Romagnoli, 2005). It is defined by its pdf and cdf. The pdf and cdf of this distribution can be specified in

ISSN NUMBER: 1116-249X

terms of the multivariate Student's t distribution. The GT is denoted as $GT(\mu,\Sigma, v, \beta)$, where μ is the mean vector, Σ is the covariance matrix, v the degrees of freedom and β is the shape parameter. The pdf is expressed as:

$$f(x\,|\,\mu,\Sigma,v,\beta)=\frac{\Gamma\left(\frac{v+p}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\pi^{p/2}\,|\Sigma|^{1/2}\left(\frac{v}{v+\beta(x-\mu)^{T}-\Sigma^{-1}(x-\mu)}\right)^{\frac{v+p}{2}}} \tag{6}$$

In this case, p is the dimensionality of the distribution i.e number of variables, $\Gamma(\cdot)$ is the gamma function, $|\cdot|$ denote the determinant of the covariance matrix.

The cdf of the GT distribution does not have a closed-form expression and typically requires numerical methods for evaluation. However, it can be expressed in terms of the multivariate Student's t distribution.

3.3.2 Skew Normal Distribution

Azzalini (2011) posits that given a random variable *X* following Normal PDF *f(x)* and Normal CDF, the skew normal distribution for *X* can be expressed as:

$$g(x\,|\,\lambda) = 2f(x)F(\lambda x) \tag{7}$$

Where $\lambda \in \mathbb{R}$ is the skewness parameter.

If $X \sim N(0, \sigma^2)$, the p.d.f and c.d.f are respectively:

$$f(x)=\frac{1}{\sqrt{2\pi\sigma^{2}}}\ell^{-\frac{x^{2}}{2\sigma^{2}}} \tag{8}$$

$$F(x)=\frac{1}{2}\left[1+Erf\left(\frac{x}{\sqrt{2\sigma^{2}}}\right)\right] \tag{9}$$

Erf represents error function given by:

$$Erf(y)=\frac{2}{\sqrt{\pi}}\int_{0}^{y}e^{-z^{2}}dz \qquad 10)$$

Hence, the pdf for the skew normal (SN) distribution is expressed by

$$SN(\lambda) = g(x/\lambda) = \frac{1}{\sqrt{2\pi\sigma^2}} \ell^{\left(-\frac{x^2}{2\sigma^2}\right)} \left[1 + Erf\left(\frac{\lambda x}{\sqrt{2\sigma^2}}\right)\right] \quad (11)$$

Where $\sigma > 0$. The distribution lose its symmetrical property at $\lambda \neq 0$, will be left-skewed and exhibits negative skewness if $\lambda < 0$ and right-skewed with positive skewness exhibition if $\lambda > 0$.

3.3.3 Skew Student-t Distribution

Unlike the Skew Normal distribution, which solely captures skewness, the Skew Student-t (ST) distribution, introduced by Fernandez and Steel (1998), accommodates both skewness and kurtosis. A random variable X following an ST distribution with mean 0 and variance σ^2 has a pdf given by:

$$ST(\gamma, v) = f(\gamma, v) = \frac{2\Gamma\left(\frac{v+1}{2}\right)}{(\gamma + \gamma^{-1})\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma^2}} \left[1 + \frac{x^2}{(v-2)\sigma^2}(\gamma^2 1_{(x\leq 0)} + \gamma^{-2} 1_{(x\leq 0)})\right]^{\frac{v+1}{2}} \quad (12)$$

Where $\gamma > 0$ represent skewness parameter and $v > 2$ is the degrees of freedom controlling the distribution tail thickness. The distribution lose its symmetry when $\gamma \neq 0$ but when $\gamma > 0$, then the distribution is left-skewed. Importantly, as γ approaches zero and ν tends towards infinity, the Skew Student-t distribution converges to the standard normal distribution (Bollerslev, 1986).

3.4 Model selection based on the identified distributions

Commonly employed model selection criteria is the Akaike Information Criterion (AIC). This metric evaluate model fit by balancing the log-likelihood against model complexity. A penalty term, typically twice the number of parameters, is incorporated to mitigate overfitting. This study focuses on the AIC, given as:

$$AIC = 2K + \ln\left(\frac{RSS}{n}\right) \quad (13)$$

"*k*" signifies the number of parameters fitted in the model, "RSS" is the Residuals Sum of Squares and "n" is the number of observations

ISSN NUMBER: 1116-249X

.4. Results and Discussion

Table 1: Descriptive Statistics of ₦/$ Exchange Rate

Estimates	₦/$ Series	₦/$ Returns
Minimum	117	-1.275
Maximum	460.5	0.349
Mean	201.9	0.000339
1st Quarter	131.8	-0.00314
Median	155.1	0.000
3rd Quarter	304.5	0.00502
Standard Deviation	97.693	0.07897
Skewness	1.208	-13.4572
Kurtosis	0.035	221.671
JB test Statistics	Chi-sq =149.19 (p-value 0.000)	Chi-sq =1273686 (p-value 0.000)

Source: *Extracted from R-Outputs*

Descriptive statistics of exchange rate series and log returns are presented in Table 1. result indicated that from the original series that the minimum, maximum, mean and standard deviation of the Naira/Dollar exchange rates recorded between January 2003 to April, 2023 were found to be N117, N460.50k, N201.90k and 97.693 respectively. It can be seen that the variability of the original series to the average is large, which implies higher rates of fluctuation in the series, which depicts an evidence of volatility. However, skewness and kurtosis of the original exchange rate series were found to be 1.208 (not equal zero) and 0.035 (less than 3). This implies that the series are not from a normal distribution, an indication of rejecting the null hypothesis of normality. This is as well confirmed from the Jarque Bera test statistics of 149.19 with associated $p < 0.05$ level of significance.

Taking the log returns of the exchange rate into cognizance, result indicated that exchange rate has a quite small positive average return (about 0.034%), with standard deviation of 7.9%. More so, normality of the exchange rate returns was as well tested by using the Jarque-Bera test under null hypothesis that the return series is normally distributed as result of the test statistic value 1273686 confirms of rejecting null hypothesis. Also, the skewness and kurtosis of the returns also indicated the non-normality of the series as it shows an evidence of fatter tails.

ISSN NUMBER: 1116-249X

4.1 Stationarity Check, Volatility Check and determination of the ARMA order of GARCH Model

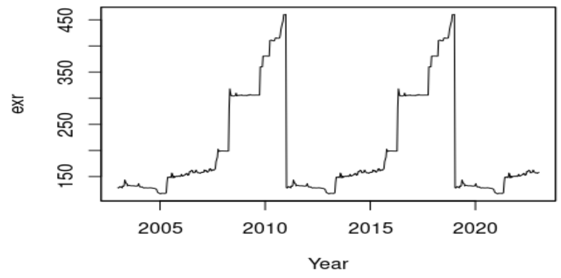

Fig. 1 *Time Series Plot of Nigeria Exchange Rate Prices between January 2003- April, 2023*

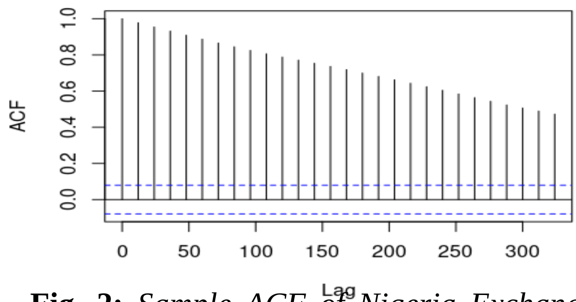

Fig. 2: *Sample ACF of Nigeria Exchange Rate Prices*

It can be evidenced from fig. 1, depicting the time-plot of the original exchange rate price series that this series was found to be irregular from time to time which also depicts its element of unit root.

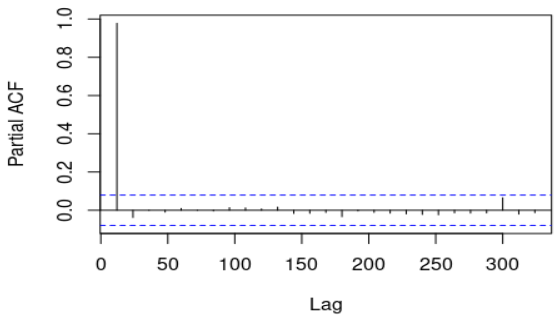

Fig. 3: *Sample PACF of Nigeria Exchange Rate Prices*

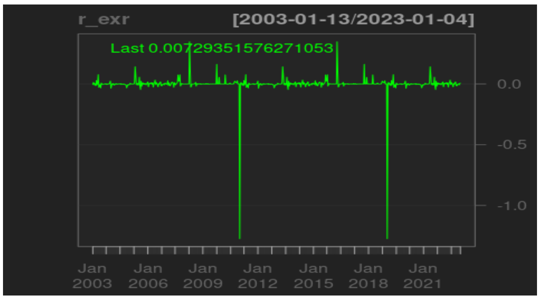

Fig. 4: Volatility Clustering of Monthly ₦/$ Exchange Rate Returns

The ACF and PACF plots of fig. 2 and fig. 3 indicated from the ACF that the positive spikes implies that the large current values of exchange rate at each lag correspondents at the next lag, as it was found to be statistically significant. However, the ACF was found to tail off to zero at other lags as it indicated decrement of autocorrelations from lag to lag. It can as well be evidenced that the sample PACF is within the upper and lower bound, implying a positive AR(p) to be calibrated to the family of GARCH models.

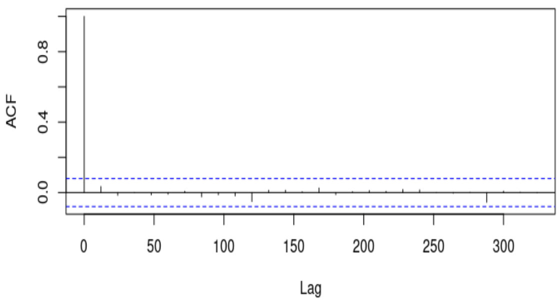

***Fig. 5**: ACF **of Monthly** ₦/$ Exchange Rate Returns*

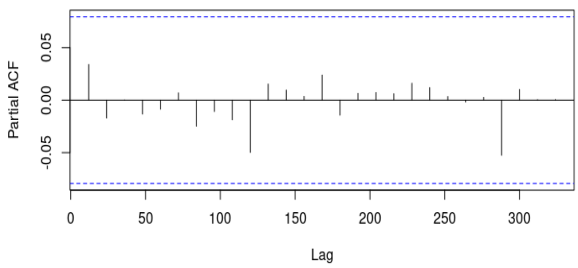

***Fig. 6:** PACF of Monthly ₦/$ Exchange Rate Returns*

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The volatility clustering of fig. 4 shows volatility (high fluctuations) at different point in time. The ACF of the returns tailed off to zero at some lags, an indication that the series can be modelled using GARCH family of models, taking into consideration different distributions that best suit the series.

Table 2: ADF Stationarity Test

Series	Test statistic	Lag order	p-value	Remarks
₦/$ Series	-2.4744	8	0.3775	Presence of unit root
₦/$ returns	-8.5459	8	0.0100	Absence of unit root

Source: *Extracted from R-Studio Output*

The N/$ exchange rate series is non-stationary, showing that its levels fluctuate over time and are influenced by underlying trends. However, the log N/$ returns are stationary, indicating that taking differences has removed the unit root, allowing for consistent statistical analysis over time. This finding is crucial in time series modelling, as many models (such as the GARCH models) require stationary data for reliable forecasts and volatility analysis.

Table 3: ARCH Test for Presence of Volatility

Series	Test statistic	df	p-value	Remarks
₦/$ Series	566.51	12	0.0000	Presence of ARCH Effects

Source: *Extracted from R-Studio Output*

Table 3 provides the test of ARCH effect in the data. The low p-value < 0.05 indicates a statistically significant result, indicating that the null hypothesis of no ARCH effects can be rejected. Consequently, this test reveals the presence of ARCH effects in the N/$ exchange rate series. The presence of ARCH effects suggests that the N/$ exchange rate series exhibits volatility clustering. This pattern is common in financial time series and indicates that the volatility of the exchange rate is not constant over time. Since volatility is significant in this series, it is appropriate to apply volatility modelling approach to capture these changing variance patterns, as they are essential for accurate forecasting and risk management in exchange rate analysis.

ISSN NUMBER: 1116-249X

4.2 Mean and Variance Equations Parameter Estimates

Table 4: Parameter estimates of EGARCH (1, 1) Mean Equation with Competing Distributions

	Generalized t			**Skew Normal**			**Skew Student *t***		
Parameter	**Estimates**	**t-value**	**p-value**	**Estimates**	**t-value**	**p-value**	**Estimates**	**t-value**	**p-value**
ξ	-0.576	-1370.60	0.000***	-0.359	-8017	0.000***	-0.872	-297.43	0.000***
η	0.580	1370.598	0.000***	0.366	18079	0.000***	0.864	287.07	0.000***
ω	-6.233	-9.624	0.000***	-0.699	-14430	0.000***	-3.145	-8.350	0.000***

, **, and * represents significant @10%, 5% and 1% levels respectively*

Source: Extracted from RStudio Output

Table 4 presents the parameter estimates of an EGARCH(1,1) model applied to the Naira/Dollar exchange rate series, analyzed using three competing distributions Taking the mean equation (ξ) into consideration, the p-values indicate that ξ is statistically significant at the 1% level across all distributions. The negative ξ estimtes across distributions implies that the exchange rate has a downward trend in the mean equation, implying that the returns may be trending negatively over time. More so, the positive and significant η values across distributions indicate asymmetry in the volatility, confirming that the EGARCH model appropriately captures asymmetric responses in volatility to positive and negative shocks in the exchange rate. In addition, the negative intercept (ω) shows that the model assumes a lower baseline level of log-variance, capturing the fact that volatility changes over time rather than remaining constant.

In comparing the distributions, each distribution (Generalized t, Skew Normal, and Skew ST) yields highly significant parameter estimates, with differences in their magnitude, reflecting each distribution's fit to the EGARCH model for the exchange rate. The Skew ST distribution, with larger absolute values for ξ, η, and ω, may imply a better fit for capturing more pronounced asymmetry and heavier tails in the exchange rate volatility compared to the other distributions.

Overall, the EGARCH (1,1) model with competing distributions confirms the presence of significant asymmetric volatility in the Naira/Dollar exchange rate. This model was selected

ISSN NUMBER: 1116-249X

due to its suitability for modelling volatility that account for asymmetry in the volatility response to positive and negative shocks.

Each distribution taking the mean equation into consideration provides robust parameter estimates, but the Skew ST distribution's higher parameter magnitudes may indicate a superior ability to model both skewness and heavy tails in the data.

Table 5: Parameter estimates of EGARCH Volatility Equation with Competing Distributions

	Generalized t			**Skew Normal**			**Skew Student *t***		
Parameter	**Estimate**	**t-value**	**p-value**	**Estimate**	**t-value**	**p-value**	**Estimate**	**t-value**	**p-value**
μ	$2.46x10^{-4}$	202.593	0.000***	0.003	11453	0.000***	0.001	3.063	0.002***
α	0.656	4.611	0.000***	-0.127	-12451	0.000***	1.202	5.824	0.000***
β	0.164	2.217	0.027**	0.913	15221	0.000***	0.400	5.529	0.000***
γ	0.353	3.619	0.000***	-0.401	-13164	0.000***	1.360	6.622	0.000***

, **, and * represents significant @10%, 5% and 1% levels respectively*

Source: Extracted from R-Studio Output

Table 5 depicts the parameter estimates of the EGARCH (1, 1) volatility equation with competing distributions. In this table, result shows that the positive and significant values for μ across all distributions indicate a positive long-term average variance, which represents a baseline level of volatility in the exchange rate series. While the actual values differ by distribution, the significance suggests that the baseline volatility is statistically robust across model specifications.

Parameter α captures the impact of past shocks on current volatility. The positive values for the Generalized t and Skew ST distributions suggest that positive shocks (or "good news") increase volatility. In contrast, the negative value for the Skew Normal distribution (-0.127) indicated an opposite effect, indicating that positive shocks reduce volatility under this specification.

ISSN NUMBER: 1116-249X

Parameter β (Persistence of Volatility) is statistically significant for all distributions, with significance at the 5% level for Generalized t ($p = 0.027$) and 1% level for Skew Normal and Skew ST. This parameter reflects the persistence of volatility over time. The higher values for the Skew Normal (0.913) and Skew ST (0.400) distributions suggest that the volatility tends to be more persistent under these models, with shocks having a lasting effect. A lower β for Generalized t (0.164) implies a quicker decay in the impact of shocks, leading to less persistent volatility.

More so, The γ parameter indicates the presence of leverage effect in volatility responses. Positive values for Generalized t and Skew Student t distributions indicate that negative shocks ("bad news") increase volatility more than positive shocks do. The negative γ for the Skew Normal distribution implies an inverse leverage effect, where positive shocks lead to increased volatility, suggesting this model may capture a different type of asymmetric response.

Each distribution model exhibits statistically significant parameter estimates, yet the Skew Student t distribution yields larger parameter magnitudes for α, β, and γ. This imply that the Skew Student t distribution better captures extreme values and asymmetry in the volatility pattern of the exchange rate series, potentially offering a more refined fit for series exhibiting heavy tails and volatility clustering.

4.2 Model Selection Criteria and Diagnostic Check

Table 6: EGARCH Model and its Distribution Selection

Generalized t			**Skew Normal**			**Skew Student *t***		
LL	**AIC**	**BIC**	**LL**	**AIC**	**BIC**	**LL**	**AIC**	**BIC**
1836.072	-6.004	-5.946	916.385	-2.983	-2.984	1844.176	-6.027	-5.962

LL, AIC and BIC represents Log-likelihood, Akaike Information Criterion and Bayesian Information Criterion respectively.
Source: Extracted from R-Studio Output

In choosing the best distribution that fits in for EGARCH model, table 6 depicts the log-likelihood, AIC and BIC of each distribution. Results showed from the asymmetry EGARCH model indicated that the generalized student *t* distribution possess the highest log-likelihood (1844.176), lowest AIC (-6.027) and lowest BIC (-5.962), followed by the Generalized t

ISSN NUMBER: 1116-249X

distribution. The Skew Normal distribution, with much higher AIC(-6.027) and BIC (-5.962) values, is the least suitable for capturing the dynamics of the Naira/Dollar exchange rate series.

Hence, the Skew Student t distribution is the optimal choice for modeling the volatility of the exchange rate, as it better captures the asymmetric heavy tails often present in financial time series. This model selection can help improve volatility forecasts and provide insights for risk management in currency markets. The findings corroborate with that of Abdullah et al. (2017) where there study found that the Student's t-distribution outperformed the normal distribution in terms of diagnostic tests and forecasting accuracy using BDT/USD. The AR (2)–GARCH (1,1) model, using the student's t-distribution, provided the best out-of-sample volatility forecasts.

Table 7: Ljung-Box Dependence test of EGARCH model based on Skew Student t-distribution

Chi-squared	df	p-value
0.9993	1	0.3175

Source: Extracted from R-Studio Output

The Ljung-Box statistics examines the H_0 of independence in the residuals of the return exchange rate series. It can be seen that the chi-squared statistic and associated p-values > 0.05 level of significance suggested the failure in rejecting H_0 that the autocorrelation at the two lags are zero in the fitted EGARCH model. This implies that the model has captured dependence in the series.

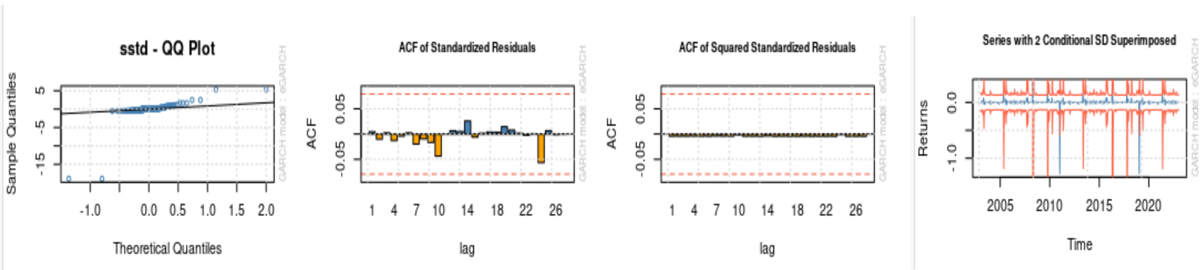

Fig. 7: *Residual plots with 2 conditional SD Superimposed EGARCH model from the best identified Skew-Student t Distribution*

Confirmatory analysis of the parsimonious EGARCH model as best identified with the skew student t distribution can be evidenced in fig. 4.8. From the QQ plot, the theoretical quantiles were found to be in line with the sample quantiles except in few points. More so, the ACF of standardized residuals and squared residuals were within the upper and lower bounds as there are no significant spikes, an indication that the models have captured goodness of fit.

ISSN NUMBER: 1116-249X

4.3 Forecasting Exchange Rate Volatility EGARCH Models from the Best Identified Distribution

Lead forecast of maximum of twelve months was done to confirm the behaviour of volatility in a short time (May, 2023-April, 2024).

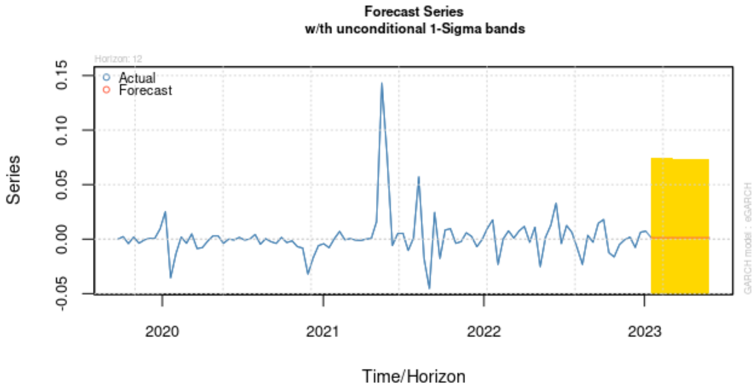

Fig. 8: EGARCH (1, 1) Forecast from the best Skew Normal Distribution

It can be seen from the forecast graphs of the identified model that volatility in the US$/Naira exchange rate remain constant as time passes. This might be as a result of the produced conditional variance, implying a mean reverting. Hence, the variance reverts towards a long term mean.

Conclusion

This study involved the application of the prominent EGARCH-type model in conjunction with three distributional models: the generalized Student's t, skew normal, and skew Student's *t* distributions. The objective was to assess their efficiency in modeling and estimating volatility within the naira/dollar exchange rates series spanning between January 2003 to April, 2023. After careful examination, the skew Student's t distribution clearly outperforms the other distributional models in terms of efficiency and effectiveness. The distribution has not only provided a sound framework for modeling volatility, but it has performed excellently in capturing the intricacies of the naira/US dollar exchange rate series, particularly with regard to skewness and thick tails. The choice of distribution has impacted on the performance of the model quite greatly, the EGARCH model has given an insight into the information on volatility dynamics. This, therefore, concluded that the skew Student's t distribution outperformed the other distributions since it can adapt to changing market conditions. The Skew Student's t Distribution is ideal for risk management and exchange rate-related decision making since it is precise in the estimation of risks. Exact risk assessments are necessary to improve investment and hedging strategies. Future research should also discuss the application of the Skewed

ISSN NUMBER: 1116-249X

Student's t distribution in various financial contexts, including but not limited to its suitability for other currency pairs and financial instruments.

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