MODELING NIGERIAN STOCK PRICE VOLATILITY USING EGARCH-X MODEL WITH DIFFERENT INNOVATIONS

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Abstract

This study explores the modelling performance of EGARCH-X using the skewed student's t, normal, and student's t innovations. The aim of the study was to determine the innovation that best captures the asymmetry and kurtosis exhibited by the returns on financial data. The descriptive statistics revealed that the distributions of returns on the stock prices were skewed and leptokurtic. The unit root test was carried out using the Augmented Dickey-Fuller (ADF) test. The result of the unit root test reveals that the returns on the series were stationary. The ARCH LM-test detected the presence of ARCH effects. The mean equation was estimated, and the EGARCH-X (1,1) model was fitted to the data, incorporating three exogenous variables (daily opening price, daily low price, and daily high price). The goodness of fit of the models was tested using Akaike Information Criterion, Bayesian Information Criterion, and Log-Likelihood. The models' performance, based on Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Log-Likelihood, revealed that EGARCH-X (1,1) with skewed student's t innovation performs better than EGARCH-X (1,1) with normal and student's t innovations. The findings of the study further revealed that volatility persists longer in the models with Student's t innovations, suggesting a slower mean-reverting process pertinent for an appropriate forecast of the financial market.

Keywords: EGARCH, EGARCH-X, Stock Price, Volatility, Exogenous Variables.

1. Introduction

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In the past few years, research on modelling and forecasting the volatility of financial time series that is, asset returns, has grown significantly. Within the field of finance, it has garnered significant interest from scholars and other professionals. This is because the concept of volatility is a crucial in a wide range of financial and economic applications, including risk management, asset pricing, and portfolio optimization (Marobhe, 2020). A key idea in finance is stock price volatility, which is the amount of variance in a stock's price over time. Derivative pricing, portfolio optimization, and risk management all depend on an understanding and modeling of this volatility (Yeasin et. al, 2020). For instance, time series models are especially helpful since they examine past pricing data to spot trends and project future price changes (Khan and Abdullah, 2019). The dispersion around the average return of an asset is measured by the volatility of stock return. Creating investment strategies that lower risk and increase stock returns requires careful consideration of volatility modelling. In the pricing of securities and options, its significance extends beyond just investors and other market players in the economy as a whole (Zakaria et.al, 2012).

For stock market participants, understanding stock return behaviors is essential because deviations from expectations can result in significant gains or losses. For this reason, developing dependable and effective models for precise stock market price prediction is one of the most fascinating challenges (Emenyonu et.al, 2023). Different economic sectors exhibit different patterns of stock price volatility. Additionally, these industries vary from one another in terms of seasonal characteristics, time series unpredictability, and trend patterns (Fateye and Ajayi, 2022).

Although the volatility of the stock market has been the central premise of the hypothesis of an efficient market, studies attempting to validate or invalidate the theory have looked at a number of crucial characteristics of different stocks and come to a variety of results. Investors can anticipate future price changes and efficiently manage investment risks by looking for trends in historical data (Mohammed et.al, 2022).

It is well recognized that the well-known GARCH models are used to model stock price volatility. Several versions of the model have emerged to address various aspects of stock price volatility. One of the key factors taken into account when modeling stock price volatility is: whether the effects are symmetric or not (Gallo and Pacini, 1998). The models selected to model

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the specific stock price data are greatly influenced by this factor. Therefore, the objective of the study is to determine the innovative model that best captures the asymmetry and kurtosis features exhibited by Nigerian stock returns using EGARCH-X model.

2. Literature Review

Nugroho et. al (2023) studied the exponential transformation to the exogenous variables in the GARCH-X(1,1) model. The model assumes that the returns error is normally and Student-t distributed. Empirical analysis was carried out based on stock price index data FTSE100 and SP500 daily period from January 2000 to December 2021. The Akaike Information Criterion value indicates that the proposed model outperforms the basic GARCH-X(1,1) model, where the best fit model is given by the Student-t distributed model. Emenyonu et.al (2023) estimated both the symmetric and asymmetric volatility models. The ARMA-GARCH, ARMA-EGARCH models were employed with the error distributions such as normal distribution, student t-distribution and skewed student-t distribution. The ARMA (2,1)-EGARCH (1,1) with student t-distribution was seen to be the most appropriate model.

Chaovanapoonphol et.al (2023) proposed an alternative model to analyze the major factors in terms of internal and external factors, which are expected to affect price volatility simultaneously when multiple exogenous Bayesian GARCH-X models are applied. The empirical results of the comparison between the multiple exogenous Bayesian GARCH-X models and the standard Bayesian GARCH-X model show that the standard error of the first model is the smallest compared to the others.

Vaz et.al (2017) empirically showed that EGARCH volatility forecasts may be improved by inserting an appropriate exogenous variable in the volatility equation. Several realized measures were tested as regressors and the robust to microstructure effects revealed that natural disasters have adverse effects on the Hong Kong Stock Market return and volatility with increasing magnitude.

Nguyen and Nguyen (2019) measured stock price volatility on Ho Chi Minh stock exchange (HSX) by applying symmetric models (GARCH, GARCH-M) and asymmetry (EGARCH and TGARCH) to measure stock price volatility on HSX. The results showed that GARCH (1,1) and

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EGARCH (1,1) models are the most suitable models to measure both symmetry and asymmetry volatility level of VN-Index.

Jimoh and Benjamin (2020) examined the nexus between the two key economic and financial variables (exchange rate and stock market price) and the most traded crypto currency in Nigeria. Using GARCH(1,1), EGARCH(1,1), and Granger causality technique. They estimated the reaction of the volatility of exchange rates and stock market prices to volatility in crypto currency prices.

Mohammed et.al (2022) studied the volatility of equity returns for two beverages traded on the Nigerian stock exchange. The ARCH effect test demonstrated that the two beverages negated the claim that there is no ARCH effect.

Pole and Cavusoglu (2021) investigated the effect of macroeconomic factors on stock return in the Nigerian stock market using Autoregressive Distributed Lag (ARDL) method of analysis. Findings revealed that money supply and aggregate industrial production positively and significantly affected stock return while exchange and inflation rates negatively affect stock return in the Nigerian stock exchange market. It was concluded that macroeconomic factors significantly affect stock return in the Nigerian stock market at short run and long run.

The method of EGARCH is limited to the use of Generalized Error Distribution (GED) innovation which follows the assumptions of normal distribution, whereas this assumption is sometime violated in the presence of exogenous variables (Korn and Erlwein-sayer, 2013). This model cannot capture the effect of accompanying exogenous variables which can give additional information in modelling and forecast of stock prices.

3. Methodology

Given the form and behavior of the generated data collected over time, financial time series offer a more comprehensive analytical approach to financial asset analysis, including stocks prices. The study employed secondary data covering daily returns of Nigerian stock prices from 30th January, 2012 to 3rd October, 2024 sourced from www.investing.com. The returns were computed using equation (1):

$$y_t = logp_t - logp_{t-1} \qquad (1)$$

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3.1 The GARCH Model

An extension of the ARCH model is the generalized ARCH (GARCH) model. While the ARCH model is straightforward, it frequently needs a large number of parameters to accurately capture the volatility process of an asset return (Ariyo Raheem et. al, 2023). The generalized ARCH (GARCH) model is formulated as follows:

Given equation $r_t = \mu_t + a_{t,}$ as the mean, then, the generalized ARCH (GARCH) volatility model is given as;

$$\sigma_t^2 = \propto_0 + \sum_{i=1}^{m} \propto_i \sigma_{t-1}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2, \qquad (2)$$

where;

$a_{t,} = \sigma_t \varepsilon_{t,}$, and $\{\varepsilon_{t,}\}$ is a sequence of independent and identically distributed (*iid*) random variables with mean 0 and variance 1.

$$\propto_0 > 0, \propto_1 \geq 1, \beta_j \geq 0 \text{ and } \sum_{i=1}^{\max(m,s)} (\propto_i + \beta_i) < 1,$$

where ε_t is often assumed to follow a standard normal or standardized Student-t distribution or generalized error distribution. The $\propto_i$ and β_j are referred to as ARCH and GARCH parameters, respectively. The GARCH (1,1) model is given as:

$$\sigma_t^2 = \propto_0 + \propto_1 \sigma_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3)$$

such that:

$$0 \leq \propto_1, \ \beta_j \leq 1, (\propto_1 + \beta_1) < 1$$

3.2 The Exponential GARCH-X Model

The EGARCHX-model notation was introduced by Nelson (1991) and used in vector form by Koutmos and Booth (1995):

(i) Conditional means equation:

$$y_t = \mu + \sum_{i=1}^{k} \beta_i x_{t,i} + \varepsilon_t \qquad (4)$$

where:

$$\varepsilon_t \sim N(0, \sigma_t^2)$$

y_t: is the dependent variable at time t

μ: is the constant mean (intercept term) for the mean equation

$x_{t,i}$: are the external regressors (exogeneous variables)

β_i: are the coefficients corresponding to each external regressor

ε_t: is the error for (residual) at time t

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(ii) Conditional variance equation:

The log-conditional variance equation of the EGARCH-X (p, q) augmented by the logarithm of the realized measure is given by

$$\ln(\sigma_{it}^2) = \omega + \sum_{i=1}^{p} \propto_j \frac{\varepsilon_{t-j}}{a_{t-j}} + \theta_1 \, lnX_{t-1} + \sum_{j=1}^{q} \beta_j \ln(\sigma_{t-j}^2)$$
$$+ \sum_{j=1}^{p} \gamma_i \left(\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}} - E\left[\frac{|\varepsilon_{t-j}|}{\sigma_{t-j}}\right]\right) + \sum_{m=1}^{r} \delta_m x_{t-m}, \quad (5)$$

where

δ_m: the coefficieint of the exogeneous variables

In equation (5), there are no restrictions on the parameter space so that the estimation procedure is simplified. For the EGARCH-X(1, 1) model, the persistence is equal to β_1 (Gallo and Pacini, 1998).

3.3 Error Distributions

The probability distribution of stock returns often exhibits fatter tails than the standard normal distribution. The existence of heavy-tailed property of stock returns is probably due to a volatility clustering in stock markets. Another source for heavy-tailed feature of stock returns seems to be the sudden changes in the financial time series. In practice, the returns are negatively skewed (Nugroho et.al, 2023). In order to capture this phenomenon, the normal distribution, student-t, and skewed student-t distributions are investigated.

For the GARCH models characterized by GED, student-t, and skewed student-t distributions are not adequately captured by Normal error distribution. The GED is estimated by maximizing the likelihood function below:

$$L(\theta_t) = -\frac{1}{2}\sum_{t-1}^{T}(ln2\pi + ln\sigma_t^2 + \frac{a_t^2}{\sigma_t^2}) \quad (6)$$

In the case of t-innovation, the volatility models considered are estimated to maximize the likelihood function of a Student's t distribution as follow:

$$L_{(std)}(\theta_t) = -\frac{1}{2}\ln\left(\frac{\pi(d)\Gamma(d/_{2^2})}{\Gamma[(d+1/_2)]^2}\right) - \frac{1}{2}\ln S_t^2 - \frac{d+1}{2}\left(1 + \frac{(r_t - X_{t,\theta})^2}{S_t^2 \, (d-2)}\right) \quad (7)$$

In the case of the skewed student's t-innovation, the volatility models is to maximize the likelihood function of the skewed student's t distribution. The probability density function of the skewed student's t distribution is

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$$f(x) = \begin{cases} \frac{bc}{w}(1+\frac{1}{v}(\frac{b(x-\varepsilon)}{w})^2)^{-\frac{v+1}{2}}, if\ x\ \geq \varepsilon \\ \\ \frac{bc}{w}(1+\frac{1}{v}(\frac{b(x-\varepsilon)}{w})^2)^{-\frac{v+1}{2}}, if\ x\ < \varepsilon \end{cases} \quad (8)$$

The log of the likelihood function is given as follow:

For $x\ \geq\ 0$:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}t\left(\frac{x}{v}\right)T\left(\lambda.\frac{x}{v+1},v+1\right)\right)$$

$$= log\left(\frac{2}{v+1}\right) + log\left(t\left(\frac{x}{v}\right)\right) + log\left(T\left(\lambda.\frac{x}{v+1},v+1\right)\right) \quad (9)$$

Log-likelihood for $x\ <\ 0$:

$$logL(x;\ v,\lambda) = log\left(\frac{2}{v+1}.t\left(\frac{-x}{v}\right)[T-1\left(\lambda\,.\frac{x}{v+1},v+1\right)]\right)$$

$$= log\left(\frac{2}{v+1}\right) + \log\left(t\left(\frac{-x}{v}\right)\right) + log[T-1\left(\lambda\,.\frac{x}{v+1},v+1\right)]\Big) \quad (10)$$

3.5 Model Selection

Model selection is done using information criteria, and the model with the least information criteria value across the error distributions is adjudged the best fitted. If the number of parameters in the model is denoted as p, then the AIC is defined by:

$$AIC(p) = -2In(Ml) + 2p, \quad (11)$$

Where Ml is the maximum likelihood estimate.
The BIC) is given as:

$$BIC(p) = -2In(Ml) + pIn(N), \quad (12)$$

where N is the number of observations.

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4 Results and Discussion

Table 1: Descriptive Statistics of the Daily Return Series of Nigeria Stock Prices.

	RHigh	RLow	ROpen	RPrice
Mean	0.000188	0.000187	0.000188	0.000188
Median	3.50E-05	0.000120	2.81E-05	2.58E-05
Maximum	0.040390	0.036480	0.038935	0.040650
Minimum	-0.036942	-0.030692	-0.032700	-0.027163
Std. Dev.	0.004250	0.004248	0.004409	0.004409
Skewness	0.545567	0.063953	0.463010	0.515040
Kurtosis	13.05801	10.80650	10.68303	10.70170
Jarque-Bera	13378.53	7967.690	7827.655	7891.813
Probability	0.000000	0.000000	0.000000	0.000000
Sum	0.590615	0.587400	0.590715	0.588888
Sum Sq. Dev.	0.056638	0.056589	0.060974	0.060975
Observations	3137	3137	3137	3137

The descriptive statistics for daily returns of Nigeria stock prices in table 1 show small positive averages for all variables (mean: 0.000188), indicating minor gains. The returns display low volatility (standard deviation: 0.0044) and a slight positive skewness (0.06-0.55), suggesting a tendency for higher positive returns. High kurtosis values (10.7-13.1) and the significant Jarque-Bera test (p-value: 0.000) indicate non-normal distribution with heavy tails, suggesting that extreme price changes occur more frequently than in a normal distribution.

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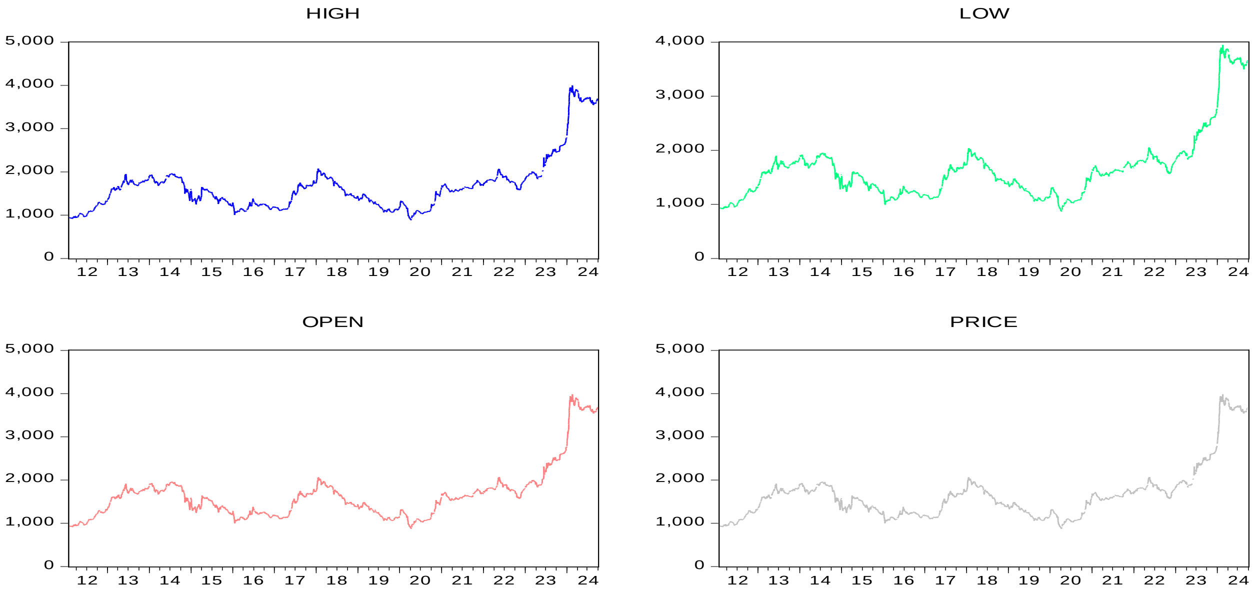

Figure 1: Time plot of the Nigeria Daily Stock Prices: Opening, High, Low and Closing Prices from 2012-2024

Table 2: Unit Root Test for the Daily Returns of Nigerian Stock Prices

Null Hypothesis: Price has a unit root					
		Price	High	Low	Open
Augmented Dickey-Fuller test statistic		0.921764	0.670221	0.714746	1.140905
Test critical values:	1% level	-3.432246	-3.432248	-3.432247	-3.432246
	5% level	-2.862263	-2.862264	-2.862264	-2.862263
	10% level	-2.567199	-2.567200	-2.567199	-2.567199
	p-value	0.9958	0.9915	0.9925	0.9979

The Unit Root Test results in table 2 shows that the Nigeria daily stock price variables (Price, High, Low, and Open) are non-stationary. The Augmented Dickey-Fuller (ADF) test statistics for all variables are positive and higher than the critical values at the 1%, 5%, and 10% significance levels. Additionally, the p-values are very high (ranging from 0.9915 to 0.9979), providing no statistical evidence to reject the null hypothesis of a unit root. This suggests that the stock prices follow a random walk and do not exhibit mean reversion, implying non-stationarity in the data. Table 3 shows a unit root of the return series.

Table 3: Unit Root Test for the Return on Daily Stock Price after Differencing

Null Hypothesis: PRICE has a unit root					
		RPrice	RHigh	RLow	ROpen
Augmented Dickey-Fuller test statistic		-40.83293	-38.23835	-40.47863	-41.48840
Test critical values:	1% level	-3.432246	-3.432246	-3.432246	-3.432246
	5% level	-2.862263	-2.862263	-2.862263	-2.862263
	10% level	-2.567199	-2.567199	-2.567199	-2.567199
	p-value	0.0000	0.0000	0.0000	0.0000

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The Unit Root Test results for the return on daily stock prices (RPrice, RHigh, RLow, and ROpen) in table 3 indicate that the returns are stationary. The Augmented Dickey-Fuller (ADF) test statistics for all variables are highly negative, significantly lower than the critical values at the 1%, 5%, and 10% significance levels. With p-values of 0.0000, the null hypothesis of a unit root is strongly rejected. This suggests that the stock price returns exhibit mean reversion and stability over time, rather than following a random walk, indicating stationarity in the data.

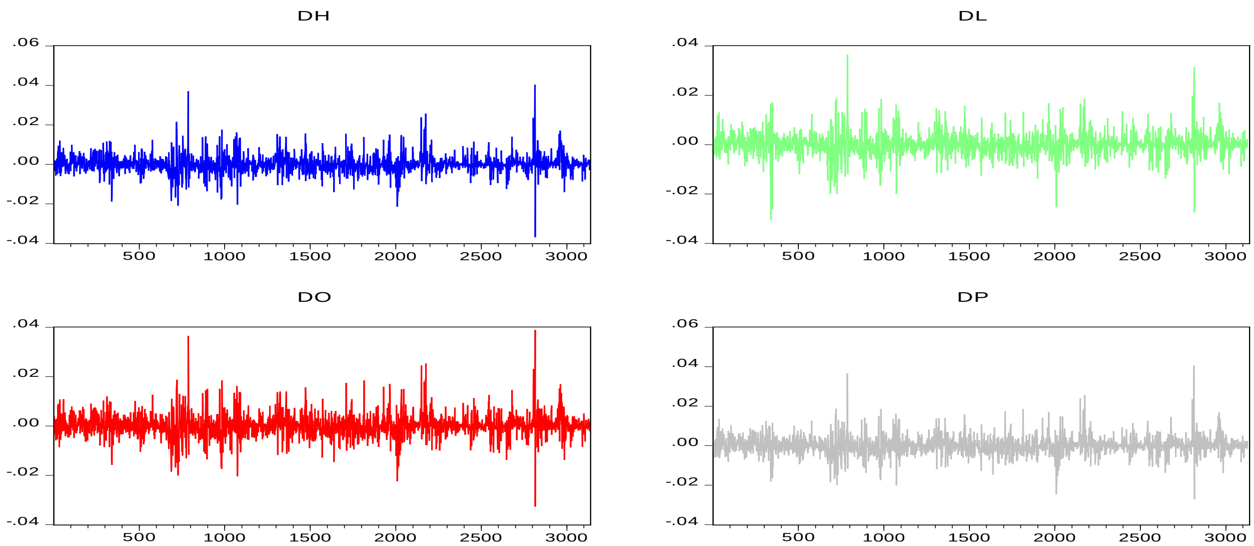

Figure 2: Time plot of the Return Series for Nigeria daily Stock Prices

DO, DP, DH and DL is the return for the daily opening price, closing price, highest price and lowest price respectively.

Table 4: ARCH LM-test: Null hypothesis: no ARCH effects

Chi-squared	370.93
Df	12
p-value	2.2e-16

Table 3 displays the results of the ARCH LM-test, which tests the null hypothesis of no ARCH effects. The test statistic is a Chi-squared value of 370.93 with 12 degrees of freedom. The p-value is extremely low at 2.2e-16, indicating strong evidence against the null hypothesis. This suggests the presence of ARCH effects in the data. With these outcomes, fundamental conditions for applying GARCH family models have been fulfilled.

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Table 5: E-GARCH-X (1,1) with Three Exogenous Variables and Normal Innovation

Parameters	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000027	0.000014	1.95613	0.05045
δ_1	0.883833	0.011961	73.89293	0.00000
δ_2	0.736774	0.009034	81.55272	0.00000
δ_3	-0.713465	0.007772	-91.80184	0.00000
ω	-2.626394	0.317775	-8.26496	0.00000
α_1	0.015154	0.020108	0.75362	0.45107
β_1	0.794371	0.024614	32.27301	0.00000
γ_1	0.575459	0.039454	14.58547	0.00000
Persistence	0.794371			
Half-life	3.011003			
Robust Standard Errors:				
μ	0.000027	0.000013	2.05407	0.039969
δ_1	0.883833	0.031137	28.38509	0.000000
δ_2	0.736774	0.019227	38.31876	0.000000
δ_3	-0.713465	0.009836	-72.53601	0.000000
ω	-2.626394	0.799105	-3.28667	0.001014
α_1	0.015154	0.031427	0.48219	0.629669
β_1	0.794371	0.062138	12.78389	0.000000
γ_1	0.575459	0.088278	6.51868	0.000000

The E-GARCH-X (1,1) model results in table 4 indicate significant dynamics in the volatility of the series. The mean return (μ) is very small (0.000027) and marginally significant with a p-value of 0.05045, suggesting that the average daily return is close to zero. The leverage effect (α_1) is strongly negative (-0.454869), showing that negative shocks increase volatility more than positive shocks, and this effect is highly significant ($p = 0.0000$). The intercept (ω) in the volatility equation is also negative and significant (-2.626394, $p = 0.0000$), indicating a low baseline level of volatility. Additionally, the three exogenous variables ($\delta_1 = 0.883833$, $\delta_2 = 0.736774$, $\delta_3 = -0.713465$) significantly affect volatility, with δ_1 and δ_2 positively contributing to volatility and δ_3 having a negative effect, all with p-values of 0.0000.

Table 6: EGARCH-X(1,1) with Three Exogenous Variables with Student's t Innovation

Parameters	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000024	0.000008	3.05381	0.00226
δ_1	0.922247	0.010843	85.05690	0.00000
δ_2	0.806656	0.016550	48.74009	0.00000
δ_3	-0.784333	0.015956	-49.15583	0.00000
ω	-2.095834	0.327843	-6.39280	0.00000
α_1	0.021651	0.027553	0.78579	0.43199
β_1	0.837575	0.025415	32.95607	0.00000
γ_1	0.616114	0.053832	11.44518	0.00000
Persistence	0.8375751			
Half-life	3.910688			

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Robust Standard Errors				
μ	0.000024	0.000003	7.48784	0.000000
δ_1	0.922247	0.018318	50.34536	0.000000
δ_2	0.806656	0.044248	18.23041	0.000000
δ_3	-0.784333	0.039192	-20.01277	0.000000
ω	-2.095834	0.480998	-4.35726	0.000013
α_1	0.021651	0.028779	0.75233	0.451855
β_1	0.837575	0.037408	22.39039	0.000000
γ_1	0.616114	0.065508	9.40514	0.000000
shape	3.683532	0.329435	11.18137	0.000000

The EGARCH-X (1,1) model results in table 5 indicate significant dynamics in volatility, utilizing Student's t-distribution for innovations. The estimated mean return (μ) is 0.000024, which is statistically significant with a t-value of 3.05381 and a p-value of 0.00226, suggesting a positive average daily return. The leverage parameter (α_1) is -0.462503, indicating that negative shocks to returns have a more substantial impact on future volatility than positive shocks, with a highly significant t-value of -27.00734 ($p = 0.00000$). The intercept in the volatility equation (ω) is -2.095834, showing a low baseline level of volatility when no shocks are present, and it is also significant with a t-value of -6.39280 ($p = 0.00000$).

The three exogenous variables significantly influence volatility: δ_1 (0.922247, $p = 0.00000$) and δ_2 (0.806656, $p = 0.00000$) positively contribute to volatility, while δ_3 (-0.784333, $p = 0.00000$) negatively affects it. The persistence of volatility is indicated by β_1, which is 0.837575 with a t-value of 32.95607 ($p = 0.00000$), reflecting a strong tendency for volatility to remain high after an increase. The leverage effect (γ_1) is 0.616114 ($p = 0.00000$), suggesting that negative returns lead to increased future volatility.

Table 7: EGARCH-X(1,1) with Three Exogenous Variables and Skewed Student's t Innovation

Parameters	Estimate	Std. Error	t value	Pr(>|t|)
μ	0.000035	0.000017	2.0792	0.037603
δ_1	0.918587	0.009196	99.8845	0.000000
δ_2	0.815799	0.010841	75.2506	0.000000
δ_3	-0.789495	0.009497	-83.1281	0.000000
ω	-2.110177	0.327454	-6.4442	0.000000
α_1	0.045245	0.035991	1.2571	0.208719
β_1	0.836344	0.025397	32.9308	0.000000
γ_1	0.613563	0.053770	11.4108	0.000000
Persistence	0.8363443			
Half life	3.878508			
Robust Standard Errors				
μ	0.000035	0.000016	2.16778	0.030175
δ_1	0.918587	0.013409	68.50629	0.000000
δ_2	0.815799	0.017939	45.47659	0.000000
δ_3	-0.789495	0.012435	-63.48880	0.000000
ω	-2.110177	0.473571	-4.45588	0.000008
α_1	0.045245	0.047282	0.95691	0.338611
β_1	0.836344	0.036828	22.70971	0.000000
γ_1	0.613563	0.062605	9.80052	0.000000
Skewness	1.033565	0.040719	25.38314	0.000000
Shape	3.659886	0.286762	12.76281	0.000000

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The results for the EGARCH-X (1,1) model with three exogenous variables and innovations following a skewed Student's t-distribution in table 6 reveal significant volatility dynamics in the financial time series. The estimated mean return (μ) is 0.000035, which is statistically significant with a t-value of 2.0792 (p = 0.037603), indicating a positive average daily return. The leverage effect (α_1) is -0.462574, demonstrating that negative shocks have a more pronounced impact on future volatility, supported by a highly significant t-value of -27.6394 (p = 0.000000). The intercept in the volatility equation (ω) is -2.110177, reflecting a low baseline volatility when no shocks occur, and this value is significant with a t-value of -6.4442 (p = 0.000000).
The three exogenous variables significantly influence volatility, with δ_1 at 0.918587 (p = 0.000000) and δ_2 at 0.815799 (p = 0.000000) positively affecting volatility, while δ_3 at -0.789495 (p = 0.000000) negatively impacts it. The persistence of volatility is indicated by β_1 at 0.836344 (p = 0.000000), suggesting a strong tendency for volatility to remain high after increases. The leverage effect (γ_1) is 0.613563 (p = 0.000000), indicating that negative returns contribute to higher future volatility. Additionally, the skewness parameter is 1.033565 (p = 0.000000), reflecting a positive skew in the distribution of innovations, and the shape parameter is 3.659886 (p = 0.000000), confirming that the distribution has heavier tails than the normal distribution.

Table 8: Goodness of Fits of the Models with Different Innovations

Models	AIC	BIC	SIC	Hannan-Quinn	Log-Likelihood	Half-life
EGARCH-X (1,1) with Normal Innovation	-10.131	-10.114	-10.131	-10.125	15899.99	3.0110
EGARCH-X (1,1) with Student’s t Innovation	-10.268	-10.249	-10.268	-10.261	16115.21	3.9107
EGARCH-X (1,1) with Skewed Student’s t Innovation	-10.268	-10.249	-10.268	-10.261	16115.21	3.9107

The table presents the goodness of fit for various EGARCH-X (1,1) models utilizing different innovation distributions: Normal, Student's t, and Skewed Student's t. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) indicate that the student’s t model has the lowest AIC of -10.268 and BIC of -10.249, suggesting it provides a better fit than the Normal model, which has an AIC of -10.131 and BIC of -10.114. The Skewed Student's t model shows the same AIC and BIC values as the student’s t model, indicating comparable goodness of fit.
Additionally, the Log-Likelihood value is highest for the student's t and skewed student's t models at 16115.21, compared to the Normal innovation model at 15899.99, further confirming the superior performance of the former. The half-life, which measures the time it takes for volatility to decay by half, is 3.011003 for the Normal model, while both the student's t and skewed student's t models exhibit a longer half-life of 3.910688. This suggests that volatility persists longer in the models with Student's t innovations, indicating a slower mean-reverting process. Overall, the results highlight the enhanced capacity of the student’s t and skewed Student's t models to effectively capture the dynamics of the data compared to the Normal innovation model.

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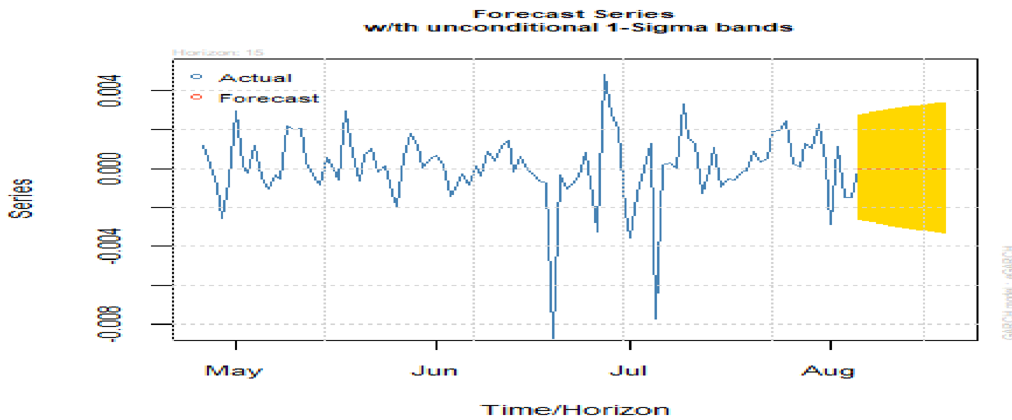

Figure 3: Forecast Plot with Exogenous Variables

Table 9: Forecast Horizon

Forecast	
RMSE	0.001932
MAE	0.000108
Theil Inequality Coefficient	0.221566
Bias proportion	0.002922
Variance Proportion	0.000108
Covariance Proportion	0.996971
Symmetric MAPE	76.6313

4. Conclusion

The results from the analysis of the EGARCH-X (1,1) models with various innovation give insights into the volatility dynamics of the financial time series being studied with respect to probability distributions. The goodness of fit tests and all the performance measures used show that EGARCH-X (1,1) with the student’s t and skewed Student's t innovations perform significantly better than the model with Normal innovations. Specifically, the skewed student’s t model got the lowest AIC (-10.268) and BIC (-10.249), underscoring its ability to provide a better fit for data on stock price. This finding is in consonance with the findings of Nugroho et. al (2023) who studied the exponential transformation of exogenous variables in the GARCH-X(1,1) model and concluded that the returns error is Student-t distributed. The half-life analysis adds another layer of understanding regarding volatility persistence. The longer half-life observed in the student’s t and Skewed Student's t models (3.910688) compared to the Normal model (3.011003) indicates that volatility takes longer to revert to its mean in these models. This result aligns with financial theory which posits that markets can experience prolonged periods of high volatility following shocks, reflecting investor behavior and market dynamics.

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