DISCRETIZATION OF CLIMATIC CHARACTERISTICS AND MARKOV CHAIN MODELLING OF CROP GROWTH

ADAH, V*.; IKUGHUR, JA; NWAOSU, SC; UDOUMOH, EF

Department of Statistics, Joseph Sarwuan Tarka University, Makurdi, Nigeria

*Corresponding Author's Email: adahvictoria14@yahoo.com

Abstract

Climate change is impacted by multiple variables, and modeling the joint impact of climatic variables is of paramount interest; hence, this study presents a unique method that uses the logical operator to map bivariate data series to the univariate sequence. Each of the bivariate random variables can take only categorical values. The logical "AND" and "OR" were used for mapping these sequences and, subsequently, the Markov chain analysis. The method was applied to climatic variables (Rainfall and Temperature) to obtain favourable and unfavourable climate conditions for the growth of the yam crop. The Markov chain analysis indicates that the sequence of state for the yam crop is ergodic and thus, the favourable and unfavourable climatic conditions has a stable distribution. The logical "AND" has a low probability of favorability of 0.36 compared to the logical "OR", 0/75. The climate change impact (CCI) revealed that climate change adversely affects the growth of the yam crop. The mean recurrent time for favourable climate gave an insight into how to adapt to avoid losses. The study recommends that farmers invest more in the crop in question, considering climate change adaptation (CCA). This is because there is high climatic favorability during this period.

Keywords: Climatic condition, Indicator Function, Logical Operator, Markov chain, mixing time

INTRODUCTION

Statistical models are essential in evaluating climate change and giving insight into adapting, as these changes are widely considered as multipliers of existing threats to food security. Such models are typically used in analyzing the likelihood and severity of weather extremes (Wineman and Crawford, 2017).

Crop growth models are frequently utilized to examine how climate change affects agriculture and to aid in formulating adaptation strategies, despite the inherent complexity of both agricultural

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systems and climate change (Asseng et al., 2015; Carr et al., 2022). As the global climate undergoes rapid transformation, agricultural systems must adapt to ensure their sustainability. Assessments of the impacts of climate change are anticipated to be thorough, as they are crucial for determining the necessary adaptations (Falloon et al., 2014).

Many researchers are interested in quantifying, controlling, or adapting to climate change due to its rapidity. Nevertheless, most of these studies primarily focus on examining the effects of a single climatic variable, such as temperature or rainfall, on crop growth. Among them are Ayinde et al. (2011), on co-integration modeling, Aondoakaa (2012), Emaziye (2015), Mijinyawa and Akpenpuun (2015), Zakari et al. (2017), Olajire et al (2018), Uger (2018) and Ibrahim (2020) all on Multiple Regression model.

Markov chain models have been employed to analyze the variations in climatic parameters over time. Yoo et al. (2016) assessed the impact of climate change on rainfall using a two-state Markov chain model, while Raheem et al. (2015) employed a three-state Markov chain to investigate the patterns and distribution of daily rainfall in Uyo metropolis, Nigeria. The distribution pattern of rainfall was explored by Yusuf et al. (2016), Makokha et al. (2016), Reis et al. (2017), Nuga and Adekola (2018), and Agada et al. (2018), who applied the Kruskal-Wallis procedure, linear trend analysis, and the Markov chain model.

It is worthy of note that a multiplicity of climatic variables do impact climate change, therefore accurate modeling of multivariate climatic variables would allow for better decision-making that minimizes exposure to climate risk so as to take advantage of a favourable climate for cropping, among other human activities.

According to Cong and Brady, (2012) and Mesbahzadeh et al., (2019), the variables relevant to the study of climate are rainfall and temperature, as they significantly affect crop growth

The Markov chain model generally assesses how climate change influences the short-term and long-term patterns of each climate parameter based on the climatic needs of specific crops (Adah and Agada, 2019). As a result, it offers a modeling strategy that is appropriate for a univariate setup categorized into various states. Conversely, logical operators present a systematic approach to combine different states of each climate variable, leading to the realization of climatic conditions required for crop growth.

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Typically, the studies mentioned earlier have focused on the effects of climate change in univariate contexts. Cong and Brady (2012) and Mesbahzadeh et al. (2019) conducted a copular analysis to examine the relationship between rainfall and temperature. Nevertheless, their approach is appropriate for continuous random variables rather than for categorical random variables.

This study therefore, focuses on the impact of climate change on crop growth. This is to be achieved by using indicator function for discretization, then, the logical operators “AND” and “OR” in order to obtain the proposed climatic condition for the growth of crops.

MATERIALS AND METHODS

Discretization

Consider the indicator function, often referred to as the function that maps elements of the subset to “one”, 1 and all other elements to “zero”, 0 (Taboga, 2021).

Given the probability space (Ω,F,P), A $\in F$. The indicator function for the random variable, A, $1_A(\omega)$: Ω → R is defined in Equation (1):

$$I_A(\omega), \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases} \quad (1)$$

where, A is a set or a condition and ω is an element in the domain.
From the above definition, it can easily be seen that $I_A(\omega)$, is a discrete random variable with support

$$R_{I_A(\omega)}(\omega) = \{0,1\}$$

Connectivity of $I_A(\omega)$ and $I_B(\omega)$

Indicator functions can be combine using logical operators to provide new indicator functions

Theorem 1: Suppose $I_A(\omega)$ and $I_B(\omega)$ are two indicator functions, then there exist an indicator function $I_C(\omega)$, such that $I_C(\omega) = I_{A\cap B}(\omega) = I_A(\omega).I_B(\omega)$ and $I_C(\omega) = I_{A\cup B}(\omega) = \max(I_A(\omega), I_B(\omega))$.

Proof: The Boolean function has one or more input values and yields a result based on these input values: $[\{0,1\} * \{0,1\} \to \{0,1\}]$ (Cori and Lascar,2002, Conradie and Goranko, 2015).

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Model Specification

In this study. $I_A(\omega)$ and $I_B(\omega)$ are the indicator functions of rainfall and temperature respectively and $I_C(\omega)$, the climatic condition necessary for the growth of crops.

Thus, $I_C(\omega) = \mathrm{C_k}: [\mathrm{C_0}, \mathrm{C_1}, \mathrm{C_2}, \dots \mathrm{C_n}]$ is a sequence of climatic random variable for the growth of crops indexed by time, k.

Markov Chain Model Specified

Suppose the stochastic process $\{\mathrm{C_k}, \mathrm{k} \geq 0\}$ takes values in the state space S, then

(i) $P(C_0 = i) = \pi_i, \quad \forall i \in S,$ is called the initial distribution

(ii) $P(C_{k+1} = i_{k+1} \mid C_0 = i_0, C_1 = i_1, \dots, C_k = i_k) = P(C_{k+1} = j \mid C_k = i_k) \quad \forall k \geq 0$

and $\forall i_0, i_1, \dots, i_k, i_{k+1} \in S$. (2)

So the stochastic process $\{\mathrm{C_k}, \mathrm{k} \geq 0\}$ is a Markov Chain with initial distribution $\pi = \{\pi_i, \in \mathrm{S}\}$ and the transition probability matrix $P = (p_{ij}, i, j \in S)$

The Equation 2 is the Markov property, which means that the behavior of the chain in the next time increment depends only on the current state of the chain, which also shows that the probability of a future event will only depends on the probability of the previous event instead of the whole system evolution, an indication that Markov chains are memoryless process (Breen, 2018).

A homogenous Markov chain (HMC) is such that, $\forall k \geq 0$, $\forall i, j \in S$, the probability

$$P(C_{k+1} = j \mid C_k = i) = p_{ij} \tag{3}$$

is independent of k. Thus, the probability in each case depends on the time difference and not on the points in time. The one step time homogenous Markov chain is given by:

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & \cdots & p_{1s} \\ \vdots & \vdots & \vdots \\ p_{s1} & \cdots & p_{ss} \end{bmatrix}$$

The matrix is stochastic (that is, nonnegative and all rows sum to one) since for all i:

$$\sum_{j=1}^{n} P_{ij} = \sum_{j=1}^{n} P\left(C_1 = s_j \middle| C_0 = s_i\right) \tag{4}$$

$$P(C_1 \in S | C_0 = s_i) = s_i) = 1; \quad P_{ij} \geq 0$$

The two state transition probability matrix is:

$$P_{ij} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

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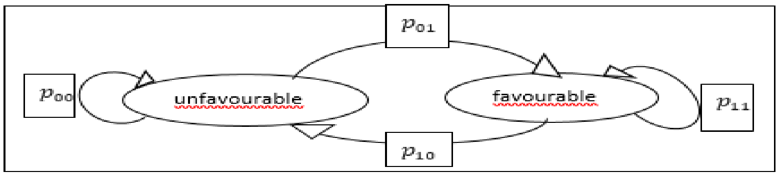

Figure 1: Transition Probability Diagram of the Climatic States

The transition probability diagram in figure 1 has all states connected in both directions, this makes the chain irreducible.

where P_{ij} is the probability of transiting from state $i \; to \; j, i = 0{,}1 \; and \; j = 0{,}1$. Here, 0 represent unfavourable state while 1 favourable state. The maximum likelihood estimators of $\hat{P}_{ij}$ are given by:

$$\hat{P}_{ij} = \frac{f_{ij}}{\sum_{j=0,1} f_{ij}} = \frac{f_{ij}}{f_{i.}} \tag{5}$$

f_{ij} represent the number of j period preceded by i period. Hence estimates of the probability that the climatic state is unfavourable (0) and favourable (1) can be obtained respectively as

$$\widehat{P_0} = \frac{f_0}{\sum_{i=0,1} f_i} \; and \; \hat{P_1} = \frac{f_1}{\sum_{i=0,1} f_i} \tag{6}$$

Dependency of Transitions

The pattern of states, 'unfavourable' and 'favourable' conditions for crop growth can be analyzed statistically to determine whether the sequence of observations adheres to a chain dependent process. The subsequent events may exhibit characteristics of Markov chain models if they are depended, as noted by Mohamad et al. (2017) and Samsuddin and Ismail (2019). In cases where consecutive events are independent, the statistic α is defined by

$$\alpha = 2 \sum_{i,j}^{S} f_{ij} \ln\left(\frac{p_{ij}}{p_j}\right), \; \alpha \sim \chi^2_{(r-1)(c-1)} \tag{7}$$

Markov chain Stationary distribution

The stationary distribution of a Markov chain illustrates how the chain behaves as time approaches infinity, representing the long-term likelihood of being in each state and offering insights into the system's steady-state condition.

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Definition 1: The distribution π is referred to as stationary or invariant distribution, if $\forall j \in S$,

$$\sum_{i \in S} \pi(i)\, P_{ij} = \pi(j) \tag{8}$$

which can be written in a compact form using matrix notation as

$$\pi = \pi \mathrm{P} \tag{9}$$

The vector π, consisting of non-negative components, serves as a stationary distribution for a Markov chain characterized by transition matrix P. Once a row in a Markov chain attains a stationary distribution, it will consistently retain this distribution in all subsequent iterations of the process (Neamat, 2023).

Suppose the Markov chain $\{C_n, n \geq 0\}$ has a limiting distribution δ, then for an arbitrary initial distribution π on S, $\lim_{n\to\infty} \pi P^n = \delta$. Conversely,

$$\lim_{n\to\infty} \pi P^n = \left(\pi \lim_{n\to\infty} P^{n-1}\right) P = \delta P \tag{10}$$

Thus $\delta = \delta P$, so δ is a unique stationary distribution. This distribution is nonzero everywhere on S (Valenzuela,2022)

Theorem 1: (Fundamental Limit Theorem). Consider an irreducible and aperiodic Markov chain represented by $\mathrm{C}_0, \mathrm{C}_1, \mathrm{C}_2, \ldots \mathrm{C}_n$, which has a stationary distribution denoted as $\pi(\cdot)$. Assume that C_0 follows the distribution π_0, an arbitrary initial distribution. Then, we have $\lim_{n\to\infty} \pi_n(i) = \pi(i)$. for every state i.

For any irreducible ergodic Markov chain, the limit $\lim_{n\to\infty} \mathrm{P}_{ij}^{(n)}$ exists and does not depend on i.

Furthermore,

$$\lim_{n\to\infty} \mathrm{P}_{ij}^{(n)} = \pi_j > 0 \tag{11}$$

where the π_j uniquely satisfy the following steady-state equations

$$\pi_j = \sum_{i=1}^{M} \pi_i \mathrm{P}_{ij} \quad for\ j = 0,1,2, \ldots, N \tag{12}$$

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$$\sum_{j=1}^{N} \pi_j = 1$$

$$\pi_j = (\pi_0, \pi_1, \pi_2, .., \pi_N)$$

The π_j is referred to as the Markov chain's steady-state probabilities. For a Markov chain with a two state, state space, S= {0,1},

$$P = \begin{pmatrix} 1 - p_{01} & p_{01} \\ p_{10} & 1 - p_{10} \end{pmatrix}$$

p_{01} and p_{10} are real numbers in the interval [0,1]

The n-step transition probabilities, P^n is:

$$P^n = \frac{1}{p_{01} + p_{10}} \begin{pmatrix} p_{10} + p_{01}(1 - p_{01} - p_{10})^n & p_{01} - p_{01}(1 - p_{01} - p_{10})^n \\ p_{10} - p_{10}(1 - p_{10} - p_{01})^n & p_{01} + p_{10}(1 - p_{10} - p_{01})^n \end{pmatrix}$$

$$\lim_{n \to \infty} P^n = \frac{1}{p_{01} + p_{10}} \begin{pmatrix} 1 - p_{01} & p_{01} \\ p_{10} & 1 - p_{10} \end{pmatrix} \quad (13)$$

We observe that both rows are identical, which means that the final state is independent of the initial state. Hence, the equilibrium distribution is represented by the vector:

$$(\pi_0, \pi_1) = \left(\frac{p_{01}}{p_{01} + p_{10}}, \quad \frac{p_{10}}{p_{01} + p_{10}} \right) \quad (14)$$

where π_0 and π_1 are the long run or steady state chance of climate unfavourability and favourability respectively.

Irreducibility

For certain Markov chains, it is crucial that no matter which state the chain begins in, there is a non-zero probability that it will reach every other state within a finite amount of time.

Let $\{C_k, \mathrm{k} \geq 0\}$ represent a Markov chain with a discrete set of states S. We say that i leads to j (denoted as $i \to j$) if there exists some k ≥ 0 such that ${P^n}_{ij} > 0$. We say that i communicates with j (denoted as $i \to j$ if both $i \to j$ and $j \to i$ hold true.

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Suppose Let $\{C_k, \mathrm{k} \geq 0\}$ represent a Markov chain with a discrete state space. The chain $\{C_k, \mathrm{k} \geq 0\}$ is considered irreducible if for every pair of states i and j in S, it holds that i is reachable from j and vice versa, implying that any state can be accessed from any other state.

Aperiodicity

An irreducible, positive recurrent Markov chain might fail to converge because of its periodic properties, which can lead to the chain oscillating among different subsets instead of distributing evenly across the entire state space over time.

Suppose $\{C_k, \mathrm{k} \geq 0\}$ has a state space S and $j \in S$ and is a Markov chain. Then,

(i) The period $\mathrm{d(j)}$ *of state j is defined as* $\mathrm{d(j)} = g.c.d\{\mathrm{n} \geq 1: P^{n}{}_{jj} > 0\}$ Here g.c.d is the greatest common divisor.

(ii) If $\mathrm{d(j)} = 1$, then j is an aperiodic state. A Markov chain is aperiodic if all the states are aperiodic.

Mean First Passage Times

The short time behavior of the Markov chain is analyzed through the concept of first passage times, which are also referred to as hitting times. When the chain begins in state i, it might go back to its initial state an infinite number of times with a probability of one, or it may return only a limited number of times, ultimately drifting away and never coming back.

Suppose $\{C_k, \mathrm{k} \geq 0\}$ has a discrete state space S and $j \in S$ and is a Markov chain. The first passage time to state j is defined by

$$T_j = \min\{\mathrm{k} \geq 1 \mid C_k = \mathrm{j}\} \qquad (15)$$

When $C_0 = \mathrm{j}$, the time T_j is referred to as the first return time (Zhang, 2020), which is the count of transitions taken by the process to move from state i to state j for the first occasion. This is referred to as the first passage time when transitioning from state i to state j. When j is equal to i, this first passage time represents the number of transitions required for the process to return to its original state i. In this situation, the first passage time is known as the recurrence (return) time for state i.

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Let the probability from state i to j for the first time be represented by f_{ij}:

$f_{ij} = \mathrm{P}(T_J < \infty \mid C_0 = i)$, then

(i) state j is recurrent if $f_{jj} = 1$,

(ii) state j is transient if $f_{jj} < 1$,

if all states are recurrent, the Markov chain too is recurrent. The expected time of return for state j is

$$u_j = \mathrm{E}(T_J \mid C_0 = \mathrm{j})$$

Suppose the chain is initially in state i and conditioned on the state of the Markov chain after one time-step, then the mean first passage times, m_{ij} in terms of the transition matrix P for a Markov chain is given by

Consider the scenario where the chain starts in state i, and given the state of the Markov chain after one time-step, the mean first passage times also known as the expected time of return denoted as m_{ij}, can be expressed in terms of the transition matrix P for a Markov chain.

$$m_{ij} = p_{ij} + \sum_{k \neq j} p_{ik}\left(m_{kj} + 1\right)$$

$$m_{ij} = \textstyle\sum_{k \neq j} p_{ik} m_{kj} + 1 \qquad (16)$$

Let the matrix of the mean first passage times be represented by M, Z represent the fundamental matrix, Z_{dg} denote the diagonal matrix of Z and zero elsewhere. Let E be an s × s matrix of all ones. D_{dg} is diagonal matrix with the jth entry $\frac{1}{a_j}$(Collins,1975).Then Equation 16 can be rewritten so that M is the sole solution to the matrix equation

$$M = \left(I - Z + EZ_{dg}\right)D \qquad (17)$$

$where\ Z = (I - (P - A))^{-1}$

I is an identity matrix

P is the regular matrix

A is the limiting matrix of P.

Assume state j is recurrent, then state j is:

(i) positive recurrent if $u_j < \infty$

(ii) null recurrent if $u_j = \infty$

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A Markov chain is considered positive recurrent if and only if each individual state exhibits positive recurrence. A key condition for the convergence of a Markov chain is that the state is positive recurrent.

Mixing Time of a Markov Chain

The mixing time t_{mix} of the Markov chain is the number of time steps required for the chain to be within a fixed threshold, ε of its stationary distribution, within a total variation distance(d_{TV}). When the chain is ergodic, its mixing time is the number of steps required to converge to its stationary distribution within a constant precision.

$$t_{mix}(\varepsilon) \coloneqq \min\{\in \mathcal{N}: \max_{x \subseteq S} d_{TV} \, |\sigma_x P^n - \pi| < \varepsilon\}, \quad \text{for all } x \in S \tag{18}$$

It is often common to define the standard mixing times as

$t_{mix} \coloneqq t_{mix}(\varepsilon)$ (Hsu *et al.*, 2019)

Climate Change Impact and Adaptation Model

Climate Change Impact (CCI) is modeled as the long run percentage unfavourability of climate for crop growth. Crop growth is impacted by climate negatively if CCI is above 50% (Agada et al., 2019). For a two-state Markov chain, the long run steady state of the chain is obtained by solving the equation;

$$\Pi = \Pi \mathrm{P} \tag{19}$$

where $\pi = \pi_0, \pi_1$ is the steady state probability vector and P is the matrix of transition probability. The computational formula for Climate Change Impact (CCI) is given as:

$$\mathrm{CCI} = \pi_0 * 100\% \tag{20}$$

Climate Change Adaptation (CCA) is modeled as the mean return time (period) of favourable climate for crop growth. For a two-state Markov chain of order one, the mean return time (MRT) for each state is computed as the reciprocal of the steady state chances.

Mathematically we have

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$$\text{MRT} = \left(\frac{1}{\pi_0}, \frac{1}{\pi_1}\right) \quad (21)$$

where $\frac{1}{\pi_0}$ is the mean return time for climate unfavourability and $\frac{1}{\pi_1}$ is the mean return time for climate favourability.

Hence Climate Change Adaptation (CCA) can be inferred from $\frac{1}{\pi_1}$ as the number of time period after which farmers can invest more in the crop in question. This is because there is high climatic favourability during this period and less in $\frac{1}{\pi_0}$(time period), a state with a low MRT corresponds to a common, frequently visited event or condition and a high MRT indicates a rare event that takes longer to recur.

Case Study: Application

Daily data on rainfall (mm) and temperature (℃) for Makurdi, Nigeria were obtained from National Aeronautics and Space Administration (NASA) (Modern Era Retrospective Analysis Version 2 (MERRA-2)) for the period of thirty-seven (37) years (1984-2022).This study considered the yam crop, hence the annual rainfall and temperature were obtained from the daily data. The annual rainfall and temperature requirements for the growth of yam is 1035mm-1500mm and 25℃ - 30℃ respectively (Agada *et al*, 2019). The annual rainfall and temperate amount were discretized using Equation 1.

RESULTS AND DISCUSSION

Table 1: Dependency Test of Sequence of Chains

Logical	α	P-value
AND	5.7328	0.0167
OR	5.8552	0.0155

The Markov chain property test to check whether or not the sequence of successive states of climatic condition for the growth of crops are independent of each other can be seen on table 1. The values of the test statistic for the growth of yam using the logical "AND" and logical "OR" is 5.7328 and 5.8552 respectively. Since the associated p-value is less than 0.05, ($p<0.05$), the null hypothesis that the sequence of successive transitions is independent is rejected. In other words,

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the state of climate for the growth of crop is dependent on the previous state, and thus have the Markov chain property.

Matrix of Transition Probabilities for the logical "AND"

$$\begin{matrix} & F' & F \\ \begin{matrix} F' \\ F \end{matrix} & \begin{pmatrix} 0.625 & 0.375 \\ 0.66666667, & 0.33333333 \end{pmatrix} \end{matrix}$$

The matrix of transition probabilities of logical "AND" shows the transition probabilities estimates for the yam crop growth. For the logical "AND", the yam crop has a probability of 0.625(63%) of an unfavourable period succeeding unfavourable period, a favourable period succeeding unfavourable period with probability 0.375 (39%), an unfavourable period succeeding a favourable period with probability 0.666 (67%) and favourable period succeeding favourable period to be 0.8703 (87%). The likelihood of a favourable period succeeding a favourable period is not high and very low for the yam crop, indicating an adverse effect of climate change on the study crop. The findings corroborate Emaziye (2015) and Agada *et al* (2019) findings but contradict the conclusions of Ibrahim (2018), whose study shows an increase in rainfall and, hence, an increase in the yield of yam.

Matrix of Transition Probabilities for the logical "OR"

$$\begin{matrix} & F' & F \\ \begin{matrix} F' \\ F \end{matrix} & \begin{pmatrix} 0.3 & 0.7 \\ 0.23076923 & 0.76923077 \end{pmatrix} \end{matrix}$$

The sequence of climatic conditions for the growth of the yam crop for the logical "OR" has a 30% chance of unfavourable period following unfavourable period, 70% chance of an unfavourable to favourable period, 23% chance of a favourable to unfavourable period and a 77% of chance of favourable to favourable period. This indicates the climatic condition for the growth of the crop is more "sticky" in state 1 than state 0, as P(1→1)>P(0→0).

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Table 2: Stationary Distributions

Logical	$\pi_{F'}$	π_F
AND	0.64	0.36
OR	0.24793388	0.75206612

$F'$is unfavourable and F is favourable

The long-term stationary distribution of climatic conditions for yam crop growth is presented in Table 2. In the long run, the climatic conditions necessary for yam growth in Makurdi will consist of 64% unfavourable and 36% favourable conditions. This steady-state transition probability matrix for yam crop growth in Makurdi indicates the following: regardless of whether the initial state is favourable or unfavourable, the likelihood that the climate will be favourable for yam growth in the near future is 0.36 (36%), while there is a 0.64 (64%) probability that the climate will be unfavourable in the same time frame. It was noted that the complete transition matrix reflects a greater likelihood of unfavourable probabilities; this suggests that no matter what state the chain starts in, it is more probable that the next state will be unfavourable. This signals that, in the long term, the climatic conditions for the crops analyzed in this research are not likely to be conducive to growth. If no actions are taken, this could result in decreased production. Furthermore, it indicates that the yam crop is susceptible to climate change. These findings align with the results from Agada et al. (2019) and Konduri et al. (2020). The logical “OR” shows a 0.2479 probability of remaining in an unfavourable state and a 0.7521 probability of being in a favourable state in the long term. The “AND” decreases the probability of favorability, while the “OR” increases it due to its more inclusive nature.

Table 3: Mean First Passage Time(MFPT)

Logical	F' to F	F to F'
AND	2.67	1.50
OR	1.43	4.33

$F'$is unfavourable and F is favourable

The mean first passage time(MFPT) in Table 3 quantifies how long the climatic requirement for the growth of the yam crop tends to stay in one state before transitioning to the other. The expected

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number of years it takes the climate to be unfavourable before transitioning to a favourable state is 2.67 years, and it takes about one and a half years to transition from a favourable state to an unfavourable one. For the logical “OR”, the expected number of years for the climatic system to transition from the unfavourable to the favourable state is 1.43 years and about 4 years to transition from the favourable to the unfavourable state.

Table 4: Mean Return Time

Logical	F' to F'	F to F
AND	1.56	2.78
OR	4	1.33

$F'$is unfavourable and F is favourable

Table 4 illustrates the mean return time (MRT) associated with yam growth. It is evident that the anticipated time until the climatic conditions become favourable, assuming that the process commenced when conditions were already favourable, is approximately one and a half years. Conversely, the expected duration until the climatic conditions turn unfavourable, given that the process initiated under unfavourable conditions, is around 2.78 years. The climatic conditions required for yam growth, when considering the logical “OR,” indicate an average transition period of about 4 years from an unfavourable to another unfavourable state and around 1.33 years to revert to a favourable state, assuming the process began in a favourable state.

Table 5: Climate Change Impact/ Adaptation

Logical	CCI	CCA
AND	64%	2.78 years
OR	23%	1.33years

Table 5 shows how climate change has impacted the yam crop and how to adapt to avoid losses. For the logical “AND,” climate has impacted the yam crop by 64%; this indicates that climate has affected the crop negatively. This finding aligns with the research conducted by Agada et al. in 2019. Farmers have the opportunity to invest in yam cultivation every three years, as the average return period for favourable conditions is 2.78 years. The yam crop is affected by approximately

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23% due to the logical "OR." However, climate change has less than a 50% effect on the climatic conditions for yam growth; thus, farmers can still capitalize on the favourable period and make investments in the crop every one and a half years.

Table 6: Mixing Time

Logical	No of Steps
AND	3
OR	4

The mixing time for the yam crop is at the time, t=3 for the logical "AND", t=4 for the logical "OR". The sequence of favourable and unfavourable state is ergodic Markov chain since the chain is irreducible and aperiodic. This implies that the time it takes the sequence of climatic states for the growth of yam crop to reach its stationary distribution is two years and three years for the logical "”AND the logical "OR" respectively

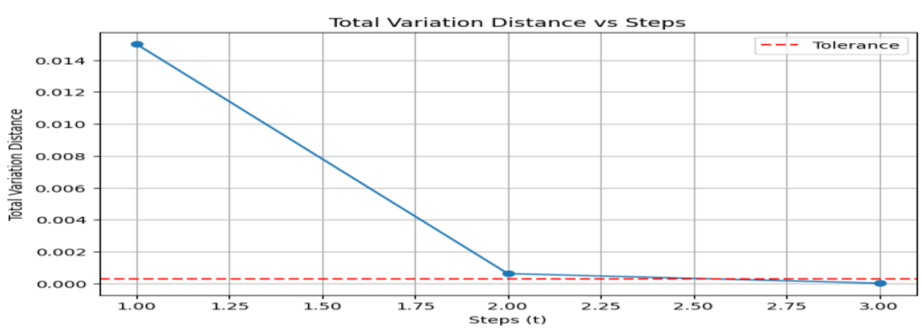

Figure 2: Mixing Time for the logical "AND"

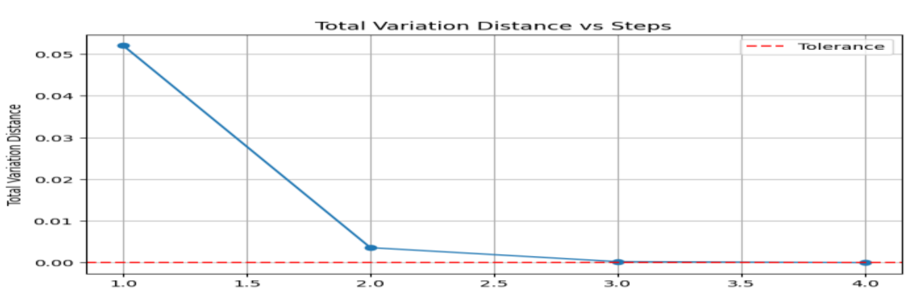

Figure 3: Mixing Time for the logical "OR"

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Figures 2 and 3 show the relationship between total variation distance and mixing time**.** The blue line in Figure 2 shows how the total variation distance decreases over time (t), indicating the convergence of the Markov chain's distribution to the stationary distribution. It starts at 0.015 (maximum difference) and approaches 0 as the chain mixes with a threshold of about 0.001; it can be observed that at time t=1 the chain is far from the stationary distribution and gets closer to the stationary distribution as the total variation distance decreases. Figure 3 shows the maximum difference is about 0.05 with a threshold of 0.001 at time, t= 1, and a mixing time of t=3. This implies that the time it takes the yam crop to be close to its threshold, of the stationary distribution is 3 years.

CONCLUSION

Two sequences of rainfall and temperature provide the necessary climatic conditions for crop growth. The climatic sequence exhibits the Markov Property, offering a dependable approach for analyzing and forecasting time series data that demonstrate Markov dependency (see table 1). All identified states are communicating, aperiodic, and ergodic, thus exhibiting limiting distributions. The analysis of the first-order Markov chain demonstrates that the climatic sequence related to yam growth tends to be more likely unfavourable, indicating the need to develop effective strategies to mitigate the impact of climate change and support food sustainability. This study indicates that Makurdi, Nigeria faces unfavourable yam yields annually, with favourable yields occurring approximately every three years. Based on these findings, it is advised that farmers in Makurdi should reduce their investment in yam during periods of unfavourable conditions and increase it during favourable times.

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