# A NEW CLASS OF POISSON BIASING RIDGE-TYPE ESTIMATOR: SIMULATION AND THEORY APPROACH

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**Abstract:**

Multicollinearity poses a significant challenge to the accuracy and reliability of Poisson regression models, leading to inflated variance and biased estimates. This study proposes a novel estimation approach, leveraging a modified version of the Liu estimator, to mitigate the adverse effects of multicollinearity in Poisson regression models. A comprehensive simulation study is conducted to evaluate the performance of the proposed estimator against traditional estimators, including the Maximum Likelihood Estimator (MLE), Ridge Regression Estimator (RRE), Liu Regression Estimator (RRE) and Modified Ridge Type Regression Estimator (MRTE). The results demonstrate the superior performance of the proposed estimator both in theoretical and simulation approach, particularly in simulation approach scenarios characterized by large sample sizes, from small to large number of explanatory variables and different levels of multicollinearity. Also, a biasing parameter k of the median version also accounted for the smallest mean square error (MSE), under different experimental design used in the study. The findings of this study contribute to the ongoing discussion on multicollinearity in Poisson regression models and provide a valuable estimation approach for researchers and practitioners dealing with multicollinearity count data.

Keywords: multicollinearity, Poisson regression, Maximum Likelihood estimator, simulation study.



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## INTRODUCTION

The Poisson regression model (PRM) is a vital statistical tool for analyzing count data across various disciplines. It is particularly useful for examining relationships between explanatory variables and response variables representing rare events or non-negative integer counts. The PRM's significance lies in its ability to accommodate data with distinct characteristics, such as event occurrence distributions. This makes it a fundamental component in epidemiology, ecology, economics, and numerous scientific disciplines.

The PRM is widely used for estimating multiplicative model parameters and is employed using the maximum likelihood estimation (MLE) method. However, in multiple regression modeling, interpreting individual parameter estimates becomes challenging when explanatory variables are highly correlated—a phenomenon known as multicollinearity.

Multicollinearity poses a significant challenge when estimating unknown regression coefficients, particularly in PRM. Mansson and Shukur (2011) highlighted the sensitivity of MLE to multicollinearity, emphasizing the need to address this concern.

To address multicollinearity, Ridge regression (RR) analysis is commonly used. RR involves adding a positive value k to the variance-covariance matrix. Various techniques have been proposed for estimating k, including those by Alkhamisi *et al*. (2006), Alkhamisi and Shukur (2008), and others. Mansson and Shukur (2011) introduced the Poisson ridge regression estimator (PRRE) method, demonstrating its superiority to MLE in Poisson regression analysis. Another approach is the Liu estimator (LE) (1993), which has been extended to propose the Poisson Liu regression (PLE) method. Amin *et al*. (2021) introduced the adjusted Liu estimator for Poisson regression (PALE), which has proven effective in addressing multicollinearity challenges. Recent studies like Amin *et al*. (2020), Kibra *et al*. (2015), Rashad and Algamal,



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(2019) have also worked on Poisson regression model, Asar and Genc (2018), Çetinkaya and Kaçıranlar (2019) suggested new two-parameter estimators, demonstrating their superiority to one-parameter estimators. The objective of this article is introducing a new two parameter biased estimator and some of its biased parameter k for the Poisson model. Additionally, the maximum likelihood, Ridge, Liu, and Kibra- Lukman estimators were compared with the proposed estimator.

## METHODOLOGY

The PRM was used to analyze data, which consist of counts. In this model, the response variable, denoted as Yi, follows a Poisson distribution. The Poisson distribution is characterized by its probability density function, which is expressed as follows:

$$f\left(y_i\right)=\frac{\exp\left(-\mu_i\right)\mu_y^i}{y_i\,!},\, y_i=0,1,2,... \tag{1}$$

Where $\mu_i > 0$ for all i=1, 2, …, n, and the expected value and variance of $y_i$ are equal to $\mu_i$. The expression of $\mu_i$ is represented using the canonical log link function and a linear combination of the explanatory variables. This can be written as, where $x_i$ represents the ith row of the data matrix X. The data matrix X has dimension $n\times\left(p+1\right)$, where n represents the number of observations and p the number of explanatory variable. The vector $\beta$ has dimensions of (p + 1) × 1 and contains the coefficients for the linear combination.

The maximum likelihood method is a widely recognized technique for estimating model parameters in the PRM. The log-likelihood function for the PRM is provided below.

$$\ell\left(\beta\right)=\sum_{i=1}^{n}\left[y_i x_i\beta-\exp\left(x_i\beta\right)-\log\left(y_i\,!\right)\right] \tag{2}$$

The PMLE is determined by calculating the first derivative of Equation (2) and equating



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it to zero. This process can be expressed as follows:

$$S(\beta)=\frac{\partial \ell(\beta, y)}{\partial \beta}=\sum_{i=1}^{n}\left[y_i-\exp(x_i\beta)\right]x_i \tag{3}$$

Where it is given that Equation (3) presents a nonlinear relationship with respect to $\beta$. To overcome this nonlinearity, the iteratively weighted least squares (IWLS) algorithm can be employed. This algorithm allows for the estimation of the PMLE values for the Poisson regression parameters as follows:

$$\hat{\beta}_{PMLE}=(J)^{-1} X^T\hat{W}\hat{s}, \tag{4}$$

Where $J = X^T\hat{W}X$, $\hat{s}$ is an n – dimensional vector with the ith element $\hat{s}=\log\hat{\mu}_i+\frac{(y_i-\hat{\mu}_i)}{\hat{\mu}_i}$, and $\hat{W}=diag[\hat{\mu}_i]$. The MLE follows a normal distribution, with a covariance matrix that is equal to the inverse of the second derivative, which is given by

$$Cov(\hat{\beta}_{PMLE})=\left[-E\left(\frac{\partial l}{\partial\beta_j\partial\beta_j^T}\right)\right]^{-1}=J^{-1} \tag{5}$$

The mean square error can calculated as follows:

$$MSE(\hat{\beta}_{PMLE})=\sum_{i=1}^{p}\frac{1}{\lambda_i} \tag{6}$$

Where $\lambda_i$ is the ith eigen value of the matrix J

**Poisson Ridge Regression Estimator (PRRE)**

In response to the multicollinearity issues in generalized linear models (GLMs), Segerstedt (1992) introduced, inspired by Hoerl and Kennard (1970), the RR estimator. Multicollinearity arises when explanatory variables in the PRM are correlated, which causes problems with MLE.



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To address this, Manson and Shukur (2011) proposed the RR estimator for the PRM, thus offering a solution to the multicollinearity challenges. The formulation and characterization of the PRRE are as follows:

$$\hat{\beta}_{PRRE} = \left(J + kI\right)^{-1} J \hat{\beta}_{PMLE} \tag{7}$$

k > 0 is the biasing parameter, I is a p x p identity matrix and the optimal value of k is defined as:

$$k = \frac{1}{\alpha_i^2}, \tag{8}$$

where $\hat{\alpha}_i^2$ is the *ith* component of $\alpha_i = Q'\beta$, $Q$ is the matrix whose columns are the eigenvectors of J.

**Poisson Liu Estimator (PLE)**

Mansson *et al*. (2011) introduced an alternative estimator known as the PLE to address multicollinearity more effectively than the previously mentioned PRRE. The PLE is defined by the following equation:

$$\hat{\beta}_{PLE} = \left(J + I\right)^{-1} \left(J + dI\right) \hat{\beta}_{PMLE},\ 0 < d < 1 \tag{9}$$

where d according to [9] may be estimated by the following formula:

$$d = \max\left(0, \min\left(\frac{\alpha_i - 1}{\frac{1}{\lambda_i} - \alpha_i^2}\right)\right) \tag{10}$$

**Poisson Kibra Lukman Estimator (PLE)**

Following Kibra and Lukman (2020), a new estimator for the Poisson regression model known as PKLE in addressing the effect of multicollinearity was introduced and is defined as follows:

$$\hat{\beta}_{PKLE} = \left(J + kI\right)^{-1} \left(J + kI\right) \hat{\beta}_{PMLE},\ k > 0 \tag{11}$$



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where k according to Kibra and Lukman may be estimated by the following formula:

$$k = \frac{\lambda_i}{1 + 2\lambda_i \alpha_i^2} \tag{12}$$

Badawaire *et al* (2023), suggested a two parameter biased estimator to handle the problem of multicollinearity when dealing with linear regression models and the estimator is defined as:

$$\hat{\beta}_{prop1} = \left(Z + 1\right)^{-1} \left(Z + dI\right)\left(Z + kI\right)^{-1} \left(Z - kI\right) \hat{\beta}_{ols} \tag{13}$$

Where $Z = X'X$ , $k > 0$ *and* $0 < d < 1$, and k and d are both biasing parameters.

The above estimator proposed by Badawaire *et al* (2023) will be introduce into the Poisson regression model and its expressed below as:

$$\hat{\beta}_{Pprop1} = \left(J + 1\right)^{-1} \left(J + dI\right)\left(J + kI\right)^{-1} \left(J - kI\right) \hat{\beta}_{PMLE} \tag{14}$$

Where $J = X^T \widehat{W} X$ , where *k* and *d* are the estimated biasing parameters for pprop1.

$$\hat{\beta}_{PMLE} = (J)^{-1} X^T \hat{W} \hat{s} \tag{15}$$

**The Canonical Form of PPROP1 Estimator.**

The Poisson regression model's canonical form is defined as follows, based on the general form provided in equation (15):

where $\alpha = Q^T \beta$ and $Q^T X^T \hat{W} X Q = \Lambda = diag(\lambda_1, \lambda_2, ..., \lambda_p)$ where $\lambda_1, \lambda_2, ..., \lambda_p > 0$ are said to be the ordered eigenvalues of $X^T \hat{W} X$ and Q is the matrix whose columns are the eigenvectors of $X^T \hat{W} X$

Canonical form of some of the already existing estimators are as follows:

$$\hat{\alpha}_{PMLE} = \left(\Delta\right)^{-1} Q^T X^T \hat{W} \hat{z} \tag{16}$$

$$\hat{\alpha}_{PRRE} = \left(\Delta + kI\right)^{-1} \Delta \hat{\alpha}_{PMLE} \tag{17}$$



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$$\hat{\alpha}_{PLE} = \left(\Delta + I\right)^{-1}\left(\Delta + dI\right)\hat{\alpha}_{PMLE} \tag{18}$$

$$\hat{\alpha}_{PKLE} = \left(\Delta + kI\right)^{-1}\left(\Delta - kI\right)\hat{\alpha}_{PMLE} \tag{19}$$

Therefore, the following is the canonical form of the PPROP1 estimator as expressed below:

$$\hat{\alpha}_{Pprop1} = \left(\Delta + 1\right)^{-1}\left(\Delta + dI\right)\left(\Delta + kI\right)^{-1}\left(\Delta - kI\right)\hat{\alpha}_{PMLE} \tag{20}$$

**Determination of the Mean Square Error (MSE) for the PPROP1Estimator**

The MSE of the PML estimator can be expressed below as:

$$MSE\left(\hat{\theta}\right) = E(\hat{\theta} - \theta)^T(\hat{\theta} - \theta) \ ,$$

$$= tr(\text{cov}(\hat{\theta})) + bias(\hat{\theta})^T bias(\hat{\theta}) \ . \tag{21}$$

Hence, for the purpose of practicality, equation () can be re expressed as:

$$MSE\left(\hat{\alpha}_{PMLE}\right) = \sum_{i=1}^{p} 1/\lambda_i \ . \tag{22}$$

$$MSE\left(\hat{\alpha}_{PRRE}\right) = \sum_{i=1}^{p} \frac{\lambda_i}{\left(\lambda_i + k\right)^2} + \sum_{i=1}^{p} \frac{k^2 \hat{\alpha}_i^2}{\left(\lambda_i + k\right)^2} \tag{23}$$

$$MSE\left(\hat{\alpha}_{PLE}\right) = \sum_{i=1}^{p} \frac{\left(\lambda_i + d\right)^2}{\left(\lambda_i + 1\right)^2 \lambda_i} + \left(d - 1\right)^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{\left(\lambda_i + 1\right)^2} \tag{24}$$

$$MSE\left(\hat{\alpha}_{PKLE}\right) = \sum_{i=1}^{p} \frac{\left(\lambda_i - k\right)^2}{\lambda_i \left(\lambda_i + k\right)^2} + \sum_{i=1}^{p} \frac{4k^2 \alpha_i^2}{\left(\lambda_i + k\right)^2} \tag{25}$$

Following the works of by Badawaire et *al* (2023) [11], the MSE of the estimator (PPROP1) can be expressed as

$$MSE\left(\hat{\alpha}_{pprop1}\right) = \sum_{i=1}^{p} \left[ \frac{\left(\lambda_i - k\right)^2 \left(\lambda_i + d\right)^2}{\lambda_i \left(\lambda_i + I\right)^2 \left(\lambda_i + k\right)^2} \right] + \sum_{i=1}^{p} \left[ \frac{\left(\left(2k + 1 - d\right)\lambda_i + k\left(d + 1\right)\right)^2}{\left(\lambda_i + I\right)^2 \left(\lambda_i + k\right)^2} \right] \alpha_i^2 \tag{26}$$

**Theoretical Comparison of the proposed PPROP1 with some other Estimators using the MSEM Criterion.**

In this section, the property of the newly proposed estimator $\hat{\beta}_{pprop1}$ is discussed. For the newly developed estimator will be compared with some existing estimators. The needed lemmas is presented below for the convenience of theoretical comparison.

**Lemma**: Suppose that $\alpha_i = M_i y, i = 1,2$ be the two competing estimators of $\alpha$. Assume that $I = Cov(\hat{\alpha}_1) - Cov(\hat{\alpha}_2) > 0$, then, $MSEM(\hat{\alpha}_1) - MSEM(\hat{\alpha}_2) > 0$ if and only if $\nu_2'(I + \nu_1\nu_1') \leq 1$, where $\nu_i$ denotes the bias of $\hat{\alpha}_i$, according to Trenkler and Toutenburg (1990).

The mean square error matrix (MSEM) of an estimator $\hat{\beta}$ is defined as

MSEM$\left(\hat{\beta}\right) = Cov\left(\hat{\beta}\right) + bias\left(\hat{\beta}\right)\left(bias\left(\hat{\beta}\right)\right)'$

Where $Cov\left(\hat{\beta}\right)$ is the dispersion matrix and $bias\left(\hat{\beta}\right) = E\left(\hat{\beta}\right) - \beta$

The bias and dispersion matrix of $\hat{\alpha}_{prop1}$ can be computed as follows:

$$Bias\left(\hat{\alpha}_{pprop1}\right) = \left[(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI) - I\right]\alpha \tag{27}$$

$$Cov\left(\hat{\alpha}_{pprop1}\right) = (\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI) \tag{28}$$

The MSEM and MSE in terms of eigenvalues are defined respectively as

$$MSEM\left(\hat{\alpha}_{pprop1}\right) = Cov\left(\hat{\alpha}_{pprop1}\right) + Bias\left(\hat{\alpha}_{pprop1}\right)Bias\left(\hat{\alpha}_{pprop1}\right)'$$

$$= \begin{array}{l}(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)\Lambda^{-1}(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI) + \\ \left[(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI) - I\right]\alpha\alpha\left[(\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI) - I\right]\end{array} \tag{29}$$



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$$MSE\left(\hat{\alpha}_{pprop1}\right)=tr\left(MSEM\left(\hat{\alpha}_{pprop1}\right)\right)$$

$$MSE\left(\hat{\alpha}_{pprop1}\right)=\sum_{i=1}^{p}\left[\frac{\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+I\right)^2\left(\lambda_i+k\right)^2}\right]+\sum_{i=1}^{p}\left[\frac{\left(\left(2k+1-d\right)\lambda_i+k\left(d+1\right)\right)^2}{\left(\lambda_i+I\right)^2\left(\lambda_i+k\right)^2}\right]\alpha_i^2$$

**Comparison of $\hat{\alpha}_{pprop1}$ and $\hat{\alpha}_{PMLE}$**

$\hat{\alpha}_{PMLE}=\left(\Delta\right)^{-1}Q^T X^T\hat{W}\hat{z}$ with $MSEM\left(\hat{\alpha}_{PMLE}\right)=\Lambda^{-1}$

**Theorem 1**: $\hat{\alpha}_{pprop1}$ is better than $\hat{\alpha}_{PMLE}$ if

$$\alpha^{'}\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)^{'}\left[\left(\Lambda^{-1}-\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^2\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^2\right)\right]^{-1}$$
$$\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)\alpha<1$$

Proof

The difference of the dispersion is

$$Cov\left(\hat{\alpha}_{PMLE}\right)-Cov\left(\hat{\alpha}_{pprop1}\right)=Q\left(\Lambda^{-1}-\Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^2\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^2\right)Q^T$$

$$=Qdiag\left[\frac{1}{\lambda_i}-\frac{\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+1\right)^2\left(\lambda_i+k\right)^2}\right]_{i=1}^{p}Q^T \qquad (30)$$

It is observed that $\left(\Lambda^{-1}-\Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^2\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^2\right)$ is positive definite since for

0 < d < 1 and k > 0, $\lambda_i\left(\lambda_i+1\right)^2\left(\lambda_i+k\right)^2-\lambda_i\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2>0$. Hence, by lemma the proof is completed.

**Theorem 2**: $\hat{\alpha}_{pprop1}$ is better than $\hat{\alpha}_{PRRE}$ if

$$\alpha^{'}\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)^{'}\left[J_1+\left(\left(\Lambda+k\right)^{-1}-I\right)\alpha\alpha^{'}\left(\left(\Lambda+k\right)^{-1}-I\right)\right]^{-1}$$
$$\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)\alpha<1$$



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$$J_1 = Q\left(\Lambda\left(\Lambda+k\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}\right)Q^T$$

Proof

The difference of the dispersion is

$$Cov\left(\hat{\alpha}_{PRRE}\right) - Cov\left(\hat{\alpha}_{pprop1}\right) = Q\left(\Lambda\left(\Lambda+k\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}\right)Q^T$$

$$= Qdiag\left[\frac{\lambda_i}{\left(\lambda_i+k\right)^2} - \frac{\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+1\right)^2\left(\lambda_i+k\right)^2}\right]_{i=1}^{p} Q^T \quad (31)$$

It is observed that $\left(\Lambda\left(\Lambda+k\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}\right)$ will be positive definite if and only if $\lambda_i^2\left(\lambda_i+1\right)^2 - \left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2 > 0$. For 0 < d < 1 and k > 0, by lemma 3 the proof is completed.

**Theorem 3**: $\hat{\alpha}_{pprop1}$ is better than $\hat{\alpha}_{PLE}$ if

$$\alpha^{'}\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)^{'}\left[J_2+\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+d\right)-I\right)\alpha\alpha^{'}\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+d\right)-I\right)\right]^{-1}$$
$$\left(\left(\Lambda+1\right)^{-1}\left(\Lambda+dI\right)\ \left(\Lambda+kI\right)^{-1}\left(\Lambda-kI\right)-I\right)\alpha < 1$$

$$J_2 = Q\left(\Lambda^{-1}\left(\Lambda+d\right)^{2}\left(\Lambda+1\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}\right)Q^T$$

Proof

The difference of the dispersion is

$$Cov\left(\hat{\alpha}_{PLE}\right) - Cov\left(\hat{\alpha}_{pprop1}\right) = Q\left(\Lambda^{-1}\left(\Lambda+d\right)^{2}\left(\Lambda+1\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}\right)Q^T$$

$$= Qdiag\left[\frac{\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+1\right)^2} - \frac{\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+1\right)^2\left(\lambda_i+k\right)^2}\right]_{i=1}^{p} Q^T \quad (32)$$

$\Lambda^{-1}\left(\Lambda+d\right)^{2}\left(\Lambda+1\right)^{-2} - \Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}$ will be positive definite if and only if $\left(\lambda_i+k\right)^2 - \left(\lambda_i-k\right)^2 > 0$ for k > 0 and 0 < d < 1. Hence, by lemma the proof is completed.



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**Theorem 4**: $\hat{\alpha}_{pprop1}$ is better than $\hat{\alpha}_{PKLE}$ if

$$\alpha'\left((\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)-I\right)'\left[J_3+\left((\Lambda+k)^{-1}(\Lambda-k)-I\right)\alpha\alpha'\left((\Lambda+k)^{-1}(\Lambda-k)-I\right)\right]^{-1}$$
$$\left((\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)-I\right)\alpha<1$$
$$J_3=Q\left(\Lambda^{-1}(\Lambda-k)^2(\Lambda+k)^{-2}-\Lambda^{-1}(\Lambda+1)^{-2}(\Lambda+dI)^2(\Lambda+kI)^{-2}(\Lambda-kI)^2\right)Q^T$$

Proof

The difference of the dispersion is

$$Cov(\hat{\alpha}_{PKLE})-Cov(\hat{\alpha}_{pprop1})=Q\left(\Lambda^{-1}(\Lambda-k)^2(\Lambda+k)^{-2}-\Lambda^{-1}(\Lambda+1)^{-2}(\Lambda+dI)^2(\Lambda+kI)^{-2}(\Lambda-kI)^2\right)Q^T$$
$$=Qdiag\left[\frac{(\lambda_i-k)^2}{\lambda_i(\lambda_i+k)^2}-\frac{(\lambda_i-k)^2(\lambda_i+d)^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k)^2}\right]_{i=1}^{p}Q^T \quad (33)$$

$\Lambda^{-1}(\Lambda-k)^2(\Lambda+k)^{-2}-\Lambda^{-1}(\Lambda+1)^{-2}(\Lambda+dI)^2(\Lambda+kI)^{-2}(\Lambda-kI)^2$ will be positive definite if and only if $(\lambda_i+1)^2-(\lambda_i+d)^2>0$ for k > 0 and 0 < d < 1. Hence, by lemma the proof is completed.

**Theorem 5**: $\hat{\alpha}_{pprop1}$ is better than $\hat{\alpha}_{PMTPLE}$ if

$$\alpha'\left((\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)-I\right)'\left[J_4+\left((\Lambda+1)^{-1}(\Lambda-(k+d))-I\right)\alpha\alpha'\left((\Lambda+1)^{-1}(\Lambda-(k+d))-I\right)\right]^{-1}$$
$$\left((\Lambda+1)^{-1}(\Lambda+dI)\ (\Lambda+kI)^{-1}(\Lambda-kI)-I\right)\alpha<1$$
$$J_4=Q\left(\Lambda^{-1}(\Lambda-(k+d))^2(\Lambda+1)^{-2}-\Lambda^{-1}(\Lambda+1)^{-2}(\Lambda+dI)^2(\Lambda+kI)^{-2}(\Lambda-kI)^2\right)Q^T$$

Proof

The difference of the dispersion is

$$Cov(\hat{\alpha}_{PKLE})-Cov(\hat{\alpha}_{pprop1})=Q\left(\Lambda^{-1}(\Lambda-(k+d))^2(\Lambda+1)^{-2}-\Lambda^{-1}(\Lambda+1)^{-2}(\Lambda+dI)^2(\Lambda+kI)^{-2}(\Lambda-kI)^2\right)Q^T$$
$$=Qdiag\left[\frac{(\lambda_i-(k+d))^2}{\lambda_i(\lambda_i+1)^2}-\frac{(\lambda_i-k)^2(\lambda_i+d)^2}{\lambda_i(\lambda_i+1)^2(\lambda_i+k)^2}\right]_{i=1}^{p}Q^T$$



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$\Lambda^{-1}\left(\Lambda-\left(k+d\right)\right)^{2}\left(\Lambda+1\right)^{-2}-\Lambda^{-1}\left(\Lambda+1\right)^{-2}\left(\Lambda+dI\right)^{2}\left(\Lambda+kI\right)^{-2}\left(\Lambda-kI\right)^{2}$ will be positive definite if and only if $\left(\lambda_i+k\right)\left(\lambda_i-\left(k+d\right)\right)-\left(\lambda_i+d\right)\left(\lambda_i-k\right)>0$ for k > 0 and 0 < d < 1. Hence, by lemma the proof is completed.

**Selection of biasing parameters k and d for $\hat{\alpha}_{pprop1}$**

MSE $\left(\alpha(k,d)\right)=E\left[\left(\hat{\alpha}(k,d)-\alpha\right)'\left(\alpha(k,d)-\alpha\right)\right]$

g(k,d)= $MSE\left(\hat{\alpha}(k,d)\right)=tr\left[MSEM\left(\hat{\alpha}(k,d)\right)\right]$

$$MSE\left(\hat{\alpha}_{pprop1}\right)=\sum_{i=1}^{p}\left[\frac{\left(\lambda_i-k\right)^2\left(\lambda_i+d\right)^2}{\lambda_i\left(\lambda_i+I\right)^2\left(\lambda_i+k\right)^2}\right]+\sum_{i=1}^{p}\left[\frac{\left(\left(2k+1-d\right)\lambda_i+k\left(d+1\right)\right)^2}{\left(\lambda_i+I\right)^2\left(\lambda_i+k\right)^2}\right]\alpha_i^2$$

Considering d to be fixed, an optimal value of k is the value that minimizes $MSE\left(\hat{\alpha}_{pprop1}\right)$.

Then, by differentiating g(k, d) w.r.t. k and equating to 0, we have

$$k=\frac{\lambda_i\left(\lambda_i+d\right)+\left(d-1\right)\lambda_i^2\alpha_i^2}{\left(\lambda_i+d\right)+\lambda_i\left(2\lambda_i+d+1\right)\alpha_i^2} \quad (34)$$

However, k depends on the unknown $\alpha_i$. For practical purposes, it will be replaced by its unbiased estimator $\hat{\alpha}_i$. Hence, this will be obtained

$$\hat{k}=\frac{\lambda_i\left(\lambda_i+d\right)+\left(d-1\right)\lambda_i^2\hat{\alpha}_i^2}{\left(\lambda_i+d\right)+\lambda_i\left(2\lambda_i+d+1\right)\hat{\alpha}_i^2}$$

Equation (34) returns the biasing parameter for the PKL estimator when d =1, which is defined as follows:

$$\hat{k}=\frac{\lambda_i}{1+2\lambda_i\hat{\alpha}_i^2}$$



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Following the works of Kibra (2003), Lukman and Ayinde (2017) and Oladapo *et al* (2022 and 2024). Hence, applying all these their concepts the below shrinkage parameters are examined for the proposed estimator and they are defined as follows:

$$\hat{k}_{MIN} = Minimum\left(\frac{\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\hat{\alpha}_i^2}{(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\hat{\alpha}_i^2}\right) \tag{35}$$

$$\hat{k}_{MED} = Median\left(\frac{\lambda_i(\lambda_i + d) + (d-1)\lambda_i^2\hat{\alpha}_i^2}{(\lambda_i + d) + \lambda_i(2\lambda_i + d + 1)\hat{\alpha}_i^2}\right) \tag{36}$$

**Simulation Experiment**

**Simulation Design.** Since a theoretical comparison among the estimators is not sufficient, as simulation experiment has been carried out in this section. We generate the response variable of the PRM from the Poisson distribution $Po(\mu_i)$ where $\mu_i = \exp(x_i\beta)$, $i = 1, 2, ..., n$, $\beta = (\beta_0, \beta_1, \beta_2, ..., \beta_p)'$, such that $x_i$ is the ith row of the design matrix X and following Kibra (2003), Oladapo *et al* (2023), Owolabi *et al* (2022) , the X is generated as follows :

$$x_{ij} = (1 - rho^2)^{\frac{1}{2}} z_{ij} + rhoz_{ip+1},\ i = 1, 2, ..., n, j = 1, 2, ..., p. \tag{37}$$

where rho $(\rho^2)$ is the correlation between the explanatory variables. The values of ρ are chosen to be 0.8, 0.9, 0.95 and 0.99. The mean function is obtained for *p* =2, 4, 10 and 12 regressors, respectively. The slope coefficients chosen so that $\sum_j^p \beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ for sample sizes 200,400,600,800 and 1000. Simulation experiment conducted through R programming language Oladapo *et al* (2023).

The estimated MSE is calculated as: $MSE(\beta^*) = \frac{1}{1000}\sum_{i=1}^{1000}(\beta_i^* - \beta)'(\beta_i^* - \beta)$ (38)



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$\beta_i^*$ represents the vector of estimated values for the lth simulation experiment of one of the four/five estimators (PMLE, PRRE, PLE, PKLE and PMTPLE), and the estimator with the lowest MSE is considered the most suitable.

Table 1. Simulated MSE when p = 2

| SAMPLE SIZE | RHO | PMLE | PRRE | PLE | PK-LE | PPROP1 | PPROP1ME | PPROP1MN |
|---|---|---|---|---|---|---|---|---|
| 200 | 0.8 | 2.40098 | 0.01618 | 0.18414 | 0.01602 | 0.18414 | **0.01583** | 0.01617 |
| | 0.9 | 2.77271 | 0.02271 | 0.18148 | 0.02246 | 0.18148 | **0.02159** | 0.02313 |
| | 0.95 | 2.74107 | 0.03364 | 0.18690 | 0.03309 | 0.18690 | **0.02916** | 0.03685 |
| | 0.99 | 2.79101 | 0.11691 | 0.23218 | **0.11350** | 0.23218 | 0.13843 | 0.02996 |
| 400 | 0.8 | 2.20795 | 0.00816 | 0.16068 | 0.00812 | 0.16068 | **0.00811** | 0.00814 |
| | 0.9 | 2.44408 | 0.01081 | 0.15717 | 0.01073 | 0.15717 | **0.01062** | 0.01081 |
| | 0.95 | 2.48999 | 0.01651 | 0.16178 | 0.01632 | 0.16178 | **0.01570** | 0.01673 |
| | 0.99 | 2.57354 | 0.06014 | 0.18359 | 0.05873 | 0.18359 | **0.03301** | 0.01000 |
| 600 | 0.8 | 2.55743 | 0.00502 | 0.14160 | 0.00500 | 0.14160 | **0.00499** | 0.00501 |
| | 0.9 | 2.52855 | 0.00661 | 0.14176 | 0.00658 | 0.14176 | **0.00654** | 0.00661 |
| | 0.95 | 2.52227 | 0.01007 | 0.13648 | 0.00999 | 0.13648 | **0.00981** | 0.01012 |
| | 0.99 | 2.53780 | 0.03856 | 0.15466 | 0.03777 | 0.15466 | **0.02872** | 0.05193 |
| 800 | 0.8 | 2.51524 | 0.00319 | 0.13218 | 0.00319 | 0.13218 | **0.00318** | 0.00319 |
| | 0.9 | 2.46778 | 0.00453 | 0.12925 | 0.00451 | 0.12925 | **0.00450** | 0.00452 |
| | 0.95 | 2.47343 | 0.00657 | 0.12944 | 0.00653 | 0.12944 | **0.00647** | 0.00658 |
| | 0.99 | 2.49409 | 0.02419 | 0.13328 | 0.02377 | 0.13328 | **0.02116** | 0.02609 |
| 1000 | 0.8 | 2.66917 | 0.00265 | 0.13990 | 0.00265 | 0.13990 | **0.00264** | 0.00265 |
| | 0.9 | 2.39265 | 0.00338 | 0.13986 | 0.00338 | 0.13986 | **0.00337** | 0.00338 |
| | 0.95 | 2.42517 | 0.00481 | 0.14008 | 0.00479 | 0.14008 | **0.00476** | 0.00481 |
| | 0.99 | 2.46446 | 0.01913 | 0.13174 | 0.01885 | 0.13174 | **0.01746** | 0.02003 |



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Table 2. Simulated MSE when p = 4

| SAMPLE SIZE | RHO | PMLE | PRRE | PLE | PK-LE | PPROP1 | PPROP1ME | PPROP1MN |
|---|---|---|---|---|---|---|---|---|
| 200 | 0.8 | 2.3036 | 0.0206 | 0.1096 | 0.0201 | 0.1096 | **0.0198** | 0.0205 |
| | 0.9 | 2.2428 | 0.0285 | 0.1098 | 0.0278 | 0.1098 | **0.0269** | 0.0287 |
| | 0.95 | 2.1942 | 0.0442 | 0.1134 | 0.0425 | 0.1134 | **0.0397** | 0.0458 |
| | 0.99 | 2.4006 | 0.1603 | 0.1743 | 0.1464 | 0.1743 | **0.0740** | 0.2825 |
| 400 | 0.8 | 2.2951 | 0.0083 | 0.0829 | 0.0082 | 0.0829 | **0.0082** | 0.0083 |
| | 0.9 | 2.2870 | 0.0109 | 0.0761 | 0.0108 | 0.0761 | **0.0107** | 0.0108 |
| | 0.95 | 2.2662 | 0.0160 | 0.0716 | 0.0157 | 0.0716 | **0.0154** | 0.0159 |
| | 0.99 | 2.2677 | 0.0568 | 0.0984 | 0.0538 | 0.0984 | **0.0463** | 0.0637 |
| 600 | 0.8 | 2.2737 | 0.0046 | 0.0718 | 0.0045 | 0.0718 | **0.0045** | 0.0045 |
| | 0.9 | 2.0692 | 0.0058 | 0.0708 | 0.0057 | 0.0708 | **0.0057** | 0.0057 |
| | 0.95 | 2.2535 | 0.0087 | 0.0690 | 0.0086 | 0.0690 | **0.0085** | 0.0087 |
| | 0.99 | 2.2589 | 0.0354 | 0.0917 | 0.0341 | 0.0917 | **0.0320** | 0.0365 |
| 800 | 0.8 | 2.2043 | 0.0046 | 0.1026 | 0.0045 | 0.1026 | **0.0045** | 0.0046 |
| | 0.9 | 2.2855 | 0.0061 | 0.0805 | 0.0061 | 0.0805 | **0.0061** | 0.0061 |
| | 0.95 | 2.2403 | 0.0094 | 0.0644 | 0.0093 | 0.0644 | **0.0092** | 0.0094 |
| | 0.99 | 2.2307 | 0.0384 | 0.0931 | 0.0370 | 0.0931 | **0.0346** | 0.0397 |
| 1000 | 0.8 | 2.2302 | 0.0035 | 0.0882 | 0.0035 | 0.0882 | **0.0035** | 0.0035 |
| | 0.9 | 1.9485 | 0.0047 | 0.0771 | 0.0046 | 0.0771 | **0.0046** | 0.0047 |
| | 0.95 | 2.2216 | 0.0073 | 0.0800 | 0.0072 | 0.0800 | **0.0072** | 0.0073 |
| | 0.99 | 2.2722 | 0.0279 | 0.0929 | 0.0271 | 0.0929 | **0.0260** | 0.0283 |



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Table 3. Simulated MSE when p = 10

| SAMPLE SIZE | RHO | PMLE | PRRE | PLE | PK-LE | PMRTE | PPROP1 | PPROP1ME | PPROP1MN |
|---|---|---|---|---|---|---|---|---|---|
| 200 | 0.8 | 2.13949 | 0.015119 | 0.045769 | 0.014653 | 0.045785 | 0.045769 | **0.014426** | 0.014907 |
| | 0.9 | 2.138529 | 0.017158 | 0.038454 | 0.016627 | 0.038454 | 0.038454 | **0.016309** | 0.016971 |
| | 0.95 | 2.121092 | 0.026051 | 0.049048 | 0.025001 | 0.049048 | 0.049048 | **0.02414** | 0.025946 |
| | 0.99 | 2.212768 | 0.100435 | 0.090595 | 0.090416 | 0.090595 | 0.090595 | **0.070737** | 0.115384 |
| 400 | 0.8 | 2.206066 | 0.010111 | 0.053834 | 0.009937 | 0.053834 | 0.053834 | **0.00985** | 0.010029 |
| | 0.9 | 2.038978 | 0.012236 | 0.044147 | 0.011991 | 0.044147 | 0.044147 | **0.011853** | 0.012134 |
| | 0.95 | 2.116642 | 0.018617 | 0.040071 | 0.018073 | 0.040071 | 0.040071 | **0.01768** | 0.018484 |
| | 0.99 | 2.188965 | 0.073314 | 0.08201 | 0.067736 | 0.081995 | 0.08201 | **0.058616** | 0.078773 |
| 600 | 0.8 | 2.175397 | 0.003987 | 0.021625 | 0.003954 | 0.021625 | 0.021625 | **0.003943** | 0.003965 |
| | 0.9 | 2.118993 | 0.004183 | 0.021302 | 0.004149 | 0.021302 | 0.021302 | **0.004138** | 0.004159 |
| | 0.95 | 2.070442 | 0.005601 | 0.017897 | 0.005544 | 0.017904 | 0.017897 | **0.005525** | 0.005564 |
| | 0.99 | 2.077489 | 0.019723 | 0.029623 | 0.019121 | 0.029623 | 0.029623 | **0.018677** | 0.019593 |
| 800 | 0.8 | 2.055321 | 0.002245 | 0.02346 | 0.002231 | 0.02346 | 0.02346 | **0.002229** | 0.002234 |
| | 0.9 | 2.114642 | 0.002438 | 0.017364 | 0.002424 | 0.017364 | 0.017364 | **0.00242** | 0.002427 |
| | 0.95 | 2.037324 | 0.00343 | 0.018644 | 0.003402 | 0.018644 | 0.018644 | **0.003395** | 0.00341 |
| | 0.99 | 2.041198 | 0.012816 | 0.023556 | 0.012484 | 0.023556 | 0.023556 | **0.01229** | 0.012686 |
| 1000 | 0.8 | 2.071076 | 0.002555 | 0.036873 | 0.002539 | 0.036873 | 0.036873 | **0.002535** | 0.002542 |
| | 0.9 | 2.107265 | 0.002755 | 0.021554 | 0.00274 | 0.021554 | 0.021554 | **0.002737** | 0.002744 |
| | 0.95 | 2.076337 | 0.003829 | 0.025042 | 0.003801 | 0.025042 | 0.025042 | **0.003793** | 0.003809 |
| | 0.99 | 2.078004 | 0.014191 | 0.030891 | 0.013857 | 0.030891 | 0.030891 | **0.013655** | 0.014066 |



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Table 4. Simulated MSE when p = 12

| SAMPLE SIZE | RHO | PMLE | PRRE | PLE | PK-LE | PPROP1 | PPROP1ME | PPROP1MN |
|---|---|---|---|---|---|---|---|---|
| 200 | 0.8 | 2.126719 | 0.037689 | 0.088617 | 0.035803 | 0.08861699 | **0.034026** | 0.037943 |
| | 0.9 | 2.067244 | 0.052175 | 0.081729 | 0.048897 | 0.08172946 | **0.045061** | 0.053521 |
| | 0.95 | 2.164192 | 0.085013 | 0.096701 | 0.077723 | 0.09670147 | **0.065685** | 0.093079 |
| | 0.99 | 2.336534 | 0.349346 | 0.177611 | 0.29002 | 0.17761148 | **0.083956** | 0.463425 |
| 400 | 0.8 | 2.115778 | 0.005877 | 0.028905 | 0.005793 | 0.02890531 | **0.005767** | 0.005821 |
| | 0.9 | 2.107341 | 0.006098 | 0.020245 | 0.006014 | 0.02024496 | **0.005985** | 0.006043 |
| | 0.95 | 1.99668 | 0.00838 | 0.023342 | 0.008235 | 0.02334203 | **0.008171** | 0.008301 |
| | 0.99 | 2.090661 | 0.029224 | 0.043479 | 0.027847 | 0.04347948 | **0.026626** | 0.029212 |
| 600 | 0.8 | 2.092972 | 0.00233 | 0.018864 | 0.002314 | 0.01886448 | **0.002311** | 0.002318 |
| | 0.9 | 2.036135 | 0.002039 | 0.014668 | 0.002028 | 0.01466849 | **0.002026** | 0.00203 |
| | 0.95 | 2.045756 | 0.002613 | 0.012262 | 0.002598 | 0.0122615 | **0.002594** | 0.002601 |
| | 0.99 | 2.072688 | 0.008659 | 0.021311 | 0.008515 | 0.02131098 | **0.00845** | 0.008582 |
| 800 | 0.8 | 2.064028 | 0.002268 | 0.027079 | 0.002255 | 0.02707853 | **0.002252** | 0.002257 |
| | 0.9 | 2.086612 | 0.002172 | 0.019705 | 0.002162 | 0.0197054 | **0.00216** | 0.002164 |
| | 0.95 | 2.064971 | 0.002796 | 0.016087 | 0.00278 | 0.016087 | **0.002776** | 0.002784 |
| | 0.99 | 2.067259 | 0.010061 | 0.022382 | 0.009876 | 0.02238226 | **0.009784** | 0.00997 |
| 1000 | 0.8 | 2.043705 | 0.001218 | 0.016006 | 0.001214 | 0.01600647 | **0.001213** | 0.001215 |
| | 0.9 | 2.039252 | 0.001112 | 0.014395 | 0.001108 | 0.01439503 | **0.001108** | 0.001109 |
| | 0.95 | 2.042817 | 0.00145 | 0.013499 | 0.001445 | 0.01349866 | **0.001444** | 0.001445 |
| | 0.99 | 2.048956 | 0.005048 | 0.012853 | 0.00499 | 0.01285322 | **0.004971** | 0.005011 |

**Simulation Results and Discussion**

The simulation study's results are presented in Tables 1-4, which display the mean squared error (MSE) values for each estimator under various scenarios, including different levels of multicollinearity (ρ), sample sizes (n), and numbers of explanatory variables (p). The smallest MSE value in each row is highlighted in bold.

The results provide a comprehensive analysis of the estimators' performance in the presence of multicollinearity. The findings offer valuable insights into the challenges and potential solutions for parameter estimation in complex regression scenarios. Notably, the PMLE consistently underperformed, indicating the need for caution or alternative approaches when multicollinearity is a concern.



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**Key Findings**

1. Impact of Multicollinearity: The results confirm that multicollinearity adversely affects estimation accuracy. As ρ increased, all estimators experienced a significant increase in MSE.
2. Sample Size: A larger sample size (n) was found to be crucial for improving estimation accuracy, as it consistently led to a decrease in MSE across almost all estimators.
3. Number of Explanatory Variables: The results show that the MSE fluctuated across all estimators as the number of explanatory variables (p) increased from 2 to 12. This suggests that additional variables introduce higher levels of multicollinearity.
4. Performance of the Proposed Estimator: The proposed estimator consistently outperformed other estimators, including the PMLE, PRRE, PLE, and PMRTE, across various scenarios. The proposed estimator demonstrated superior performance in the presence of multicollinearity.
5. Biasing k Parameter: The results show that biasing the k parameter of the median and minimum version of the proposed estimator still offered more favorable values in terms of MSE.

However, the PPROP1 estimator proposed in this study, and which uses the biasing parameter k (median) consistently yielded the minimum MSE, making it a reliable choice for researchers facing similar regression challenges when using Poisson regression where the explanatory variables are correlated. Previous works has shown that using PRRE, PLE, PMRTE and other existing estimators in handling multicollinearity problem in Poisson regression keeps solving the multicollinearity problem. Literatures also establishes that estimators will be efficient estimator if it has the lowest MSE value, therefore from this study the proposed estimator has the lowest MSE with the compared with other existing estimators used in this study. So therefore, PPROP1 can be used to handle multicollinearity problem.



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## Conclusion

This simulation study investigated the performance of various estimators in the presence of multicollinearity in Poisson regression models. The results show that multicollinearity significantly affects estimation accuracy, and that increasing sample size can improve estimation accuracy. The proposed estimator consistently outperformed other estimators, including the PMLE, PRRE, PLE, and PMRTE, across various scenarios. Specifically, the PPROP1 estimator, which uses the median version of the biasing parameter k, emerged as the most reliable choice for researchers facing similar regression challenges.

The findings of this study have important implications for researchers and practitioners who deal with Poisson regression models. They highlight the need to carefully consider the effects of multicollinearity and to explore the proposed estimator approaches that can provide more accurate and reliable results.

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