

FORECASTING OF NAIRA-DOLLAR EXCHANGE RATES FROM GDP AND EXPORTS SERIES: EVIDENCE FROM VECM-VAR MODEL

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Abstract

In practice, forecasting of multivariate time series variables cannot be achieved directly from the Vector Error Correction Model (VECM). As a result, it is achieved through the help of the Vector Error Correction Model-Vector Autoregressive (VECM-VAR) model, which is also a VAR model derived from the VECM model. To demonstrate this, the forecasting dynamics of the Exchange Rates (EXR) of Nigerian Naira (NGN) to United States Dollar (USD) was studied using Gross Domestic Product (GDP) and Exports of Goods and Services (EGS) as predictors through Vector Error Correction Model-Vector Autoregressive (VECM-VAR), which is the short-run component of VECM. Pre-tests supported the VECM-VAR (2) model, indicating one cointegrating equation among the natural logarithms of EXR, GDP, and EGS ($\ln EXR$, $\ln GDP$, and $\ln EGS$). The Error Correction Term (ECT) confirmed equilibrium adjustments in the $\ln EXR$ equation. The study's findings revealed a continuous depreciation trend for the Naira against the United States Dollar.

Keywords: *Naira-Dollar Exchange Rates; Forecasts; Vector Error Correction Model-Vector Autoregressive; Error Correction Term.*

1.0 Introduction

Predicting exchange rates is an intricate task that has garnered significant attention from researchers, and it is relevant to portfolio managers and administrators up to date (Morales-Arias & Moura, 2013). Hence, exchange rate forecasting enables investors to decide how and when to trade. Exchange rates refer to the relationship where one country's currency is exchanged for that of another country (James et al, 2012).

Mostly, exchange rates measure economic status on trade, investment, and monetary fronts (Mhamada & Tursoyb, 2021; Akintomide, 2021; Ebiowei & Umobong, 2023). Specifically, in Nigeria, exchange rates (EXR) have been significant in determining international trading activities due to the fact that the United States Dollar (USD) remains the major driver of the major and minor foreign currencies in the foreign exchange markets (see Garba et al, 2021). The recent works of Ogunnusi et al. (2024) and Akintunde & Ampitan (2024) also highlighted the importance of stabilizing the Nigerian Naira against the United States Dollar. According to Agwuegbo et al (2017), interest in exchange rate volatility stems from the empirical challenge of predicting future exchange rate values.

In the time series econometrics literature, several models can capture the forecasting dynamics of the exchange rates depending on the objective of a study. These models are classified as univariate and multivariate time series, respectively. The univariate model remains the most popular and user-friendly model for forecasting a time series such as the EXR. However, the choice of univariate time series to be employed for forecasting relies upon measures' frequency of the series. For instance, if a time series is measured at high frequency (daily or intraday), then the univariate volatility models are desirable (see Engle, 1982; Bollerslev, 1986). Nonetheless, if a time series is measured at low frequency, say monthly, quarterly, biannually, or multiyear, then the classical univariate time series models are desirable (Box and Jenkins, 1976). For the monthly, quarterly, and bi-annual data, there may seem to be potential for seasonality, thus, the seasonal univariate might suffice for monthly, quarterly, and bi-annual series.

Conversely, the multivariate time series models utilize more than a single time series variable for their analysis. The two major time series models here are the Vector Error Correction Model (VECM) proposed by Engle and Granger (1987) and the Vector Autoregressive (VAR) models proposed by Sims (1980). In most time series econometrics studies of empirical settings, VECM and VAR models are mainly employed for hypothesis testing rather than forecasting (see Alexiadis, 2017; Akanni et al, 2020; Akanni et al, 2021). The VECM is appropriate for capturing the forecasting and causal pattern of multivariate time series, provided cointegration is evident within that system of equations in a study. If there is evidence of cointegration in a system, then the cointegration property will work in the prediction of macroeconomic time series from other related macroeconomic time series (Alexiadis, 2017). The VECM-VAR model will likely outperform the standard VAR model with respect to parameter estimation as well as forecasting

accuracy for multivariate time series when cointegration exists variables. On the other hand, the standard VAR model can outperform the VECM-VAR under weak cointegration or zero cointegration in the systems (see Ongore et al, 2013).

Like other exchange rate series, the Naira-Dollar Exchange Rate (EXR) creates large problems for Nigeria's trade balance, economic stability, and policy formulation (see Chude & Chude, 2023). Despite the availability of economic indicators such as Gross Domestic Product (GDP) and Exports of Goods and Services (EGS), there remains a gap in understanding their long-run dynamics with the exchange rate. Unlike the VAR, which requires all variables to be stationary and may lose long-run information when differencing non-stationary data, the VECM retains this information, leading to more efficient and reliable forecasts for cointegrated systems.

This study addresses this gap by employing the short-run component {VECM-VAR (p)} of the Vector Error Correction Model (VECM) to examine the forecasting dynamics of the EXR, by capturing the immediate effects of GDP and EGS fluctuations under a cointegration scenario.

Numerous published studies on the Naira exchange rate forecasting have concentrated on volatility techniques. For illustration, Adedotun et al. (2022) examined the annual Naira/Dollar exchange rates series using the GARCH, EGARCH, and TGARCH models. Their results revealed that the symmetric-GARCH (1,2) model best predicted the yearly Naira/Dollar exchange rates, whereas the EGARCH (1,4) was the most efficient model for capturing asymmetry effects. Akintunde et al. (2025) recent work highlighted the suitability of the Seasonal Autoregressive Integrated Moving Average (SARIMA) over the traditional Autoregressive Integrated Moving Average (ARIMA) in forecasting the monthly Naira/Dollar exchange rates under a seasonality scenario. Also, Olowe (2009) applied GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1), APARCH (1,1), IGARCH (1,1), and TS-GARCH (1,1) models to monthly Naira/Dollar exchange rates to show potential volatility in the series despite being measured monthly. Musa & Abubakar (2014) applied GARCH (1, 1), GJR-GARCH (1, 1), TGARCH (1, 1), and TS-GARCH (1, 1) models on daily Dollar/Naira exchange rates to establish that both the TGARCH (1, 1) and TS-GARCH (1, 1) models best estimated the parameters of the variance equation since the estimates are all statistically significant and have a lower AIC value. Through the use of linear and non-linear GARCH models on Nigeria Naira exchange rates, Yaya and Shittu (2014) discovered that the Nigeria Naira-United States Dollar (NGN-USD) exchange rate is the least volatile series based on unconditional volatility and diagnostic tests; however, the Nigeria Naira-Special Drawing Rights

(NGN-SDR) exchange rate displays the highest volatility. They also discovered that asymmetric nonlinearities are significant in the Naira-Danish Kroner (N-DKR) and Naira-Central and West African Francs (N-CFA) exchange rates. Furthermore, Oluwadare et al (2021) examined the Naira exchange rates using symmetric and asymmetric GARCH models. Their findings revealed that among asymmetric volatility models considered, the Exponential GARCH (EGARCH) and Asymmetric Power ARCH (APARCH) outperformed the symmetric forms in predicting the volatility of NGN exchange returns. Atoi and Nwambeke (2021) examined the dynamics of the international markets in Nigeria using dynamic conditional correlation GARCH (DCC-GARCH) as well as unrestricted bivariate BEKK-GARCH (1,1). They found that the EXR is crucial when we talk about moderating interest rate volatility in Nigeria. Etuk (2013) revealed that the Naira-Euro Exchange Rates (NEER) series exhibit seasonality and are adequately modelled using the ARIMA $(0, 1, 1) \times (1, 1, 1)_{12}$ model.

2.0 Research Methodology

2.1 Data Description

In the present study, low-frequency time series, specifically yearly multivariate time series datasets spanning 1960 to 2022, were obtained on Naira-Dollar Exchange Rates (EXR), Gross Domestic Product (GDP), and Exports of Goods and Services (EGS). For reference purposes, these datasets were collected from the World Bank Data (2022).

2.2 Model Specification

The Vector Error Correction Model (VECM) is desirable in the analysis as well as forecasting cointegrated processes of the first order only. However, including either level stationary series or second order integration in VECM will likely yield misleading results (see Engle and Granger, 1987). VECM extends the Vector Autoregressive (VAR) model and accounts for the dynamics within related multivariate time series settings (see Gujarati & Porter, 2009; Engle and Granger, 1987; Sims, 1980; Minitaeva, 2024).

Unlike the classical Simultaneous Equations model (SEM) which requires the tests for a priori assumptions of endogeneity or exogeneity, the VECM and VAR systems do not require these assumptions, because the independent variables in these multivariate time series models are sometimes purely considered as the lagged endogenous variables with or without exogenous variables (see Maysami & Koh, 2000; Gujarati & Porter, 2009). Mathematically, the VECM is specified as

$$\Delta Y_t = \sum_{j=1}^{k-1} \Gamma_j \Delta Y_{t-j} + \alpha \beta' Y_{t-k} + \mu + \varepsilon_t \quad (1)$$

Where $\sum_{j=1}^{k-1} \Gamma_j \Delta Y_{t-j}$ and $\alpha \beta' Y_{t-k}$ represent the VAR components in first differences and error correction components respectively in levels of equation (1), Y_t which is a $p \times 1$ vector of difference stationary time series variables of order one $\{I(1)s\}$, μ represents a $p \times 1$ vector of constants, k denotes the lag structure, ε_t is represents a $p \times 1$ vector of white noise error terms, Γ_j denotes a $p \times p$ matrix of short-term adjustments among variables across p equations at the j th lag, β' denotes a $p \times r$ matrix of cointegrating vectors, Δ denotes the integration of order one $\{I(1)s\}$, α denotes a $p \times r$ matrix of the speed of adjustment parameters denoting the speed of error correction mechanism. It should be noted that a larger α suggests a faster convergence toward long-run equilibrium in cases of short-run deviations from this equilibrium. However, the standard Vector Autoregressive $\{VAR(p)\}$ model is of the form;

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + \dots + A_p y_{t-p} + e_t \quad (2)$$

Where; $y_t = k$ vector of endogenous variables, A_1 to A_p and B are matrices of coefficients to be estimated, e_t is a vector of innovations that may be contemporaneously correlated but are uncorrelated with their immediate past or lagged values.

Specifically, the system of standard $\{VAR(p)\}$ as applied to the EXR, GDP, and EGS series is of the form

$$\ln EXR_t = \sum_{i=1}^p \alpha_{1i} \ln EXR_{t-i} + \sum_{i=1}^p \beta_{1i} \ln GDP_{t-i} + \sum_{i=1}^p \gamma_{1i} \ln EGS_{t-i} + \varepsilon_{1t} \quad (3)$$

$$\ln GDP_t = \sum_{i=1}^p \alpha_{2i} \ln EXR_{t-i} + \sum_{i=1}^p \beta_{2i} \ln GDP_{t-i} + \sum_{i=1}^p \gamma_{2i} \ln EGS_{t-i} + \varepsilon_{2t} \quad (4)$$

$$\ln EGS_t = \sum_{i=1}^p \alpha_{3i} \ln EXR_{t-i} + \sum_{i=1}^p \beta_{3i} \ln GDP_{t-i} + \sum_{i=1}^p \gamma_{3i} \ln EGS_{t-i} + \varepsilon_{3t} \quad (5)$$

Where the logarithm transformations of EXR, GDP, and EXR at current time t ($\ln EXR_t$, $\ln GDP_t$ and $\ln EGS_t$) denote the endogenous variables which are regressed on their respective lags ($i = 1 \dots p$), ε_{1t} , ε_{2t} and ε_{3t} represent the uncorrelated error terms in the system of equations.

Based on Johansen (1988), the three equations (2), (3), and (4) in the VAR (p) system can have a minimum of one cointegrating equation and a maximum of three cointegrating equations (see Engle and Granger, 1987). A cointegrating equation within a three-equation model implies an Error Correction Model (ECM). On the other hand, if there are two or three cointegrating equations in the system, then we have a Vector Error Correction Model (VECM).

Moreover, pre-tests are to ensure an appropriate model selection in multivariate time series settings, which include stationarity tests, optimal lag selection, cointegration tests, model estimation, Granger causality tests, diagnostic checking, and forecasting.

3.0 Results and Discussions

Here, we have the results of analysis done in predicting the Nigerian Naira-Dollar exchange rates (EXR) series from Gross Domestic Product (GDP) and Exports of Goods and Services (EGS) using Vector Error Correction Model (VECM), VECM-VAR, and VAR modelling techniques discussed in the research methodology section. Table 1 presents the descriptive analysis.

Table 1a: Descriptive Statistics for the EXR, GDP, and EGS series at level form

Vars	N	Mean	Sd	Min	Max	Skew	Kurtosis
EXR	63	7.76E+01	1.12E+02	0.55	415	1.51	1.46
GDP	63	1.93E+05	3.90E+05	66.4	1834770	2.79	7.87
EGS	63	2.61E+10	3.34E+10	3.88E+08	1.43695E+11	1.65	2.16

Each of the variables, as shown in Table 1a, has 63 observations. The EXR exhibits a positive skewness (1.51) and notable variability. The mean is 77.6, the standard deviation is 112, and the range is between 0.55 and 415. GDP has a standard deviation of 390,000, a mean of 193,000, and range of 66.4 to 1,834,770. It indicates extreme values because of its high kurtosis (7.87) and significant positive skewness (2.79). The EGS exhibits positive skewness (1.65), with a mean of 2.61×10^{10} and a standard deviation of 3.34×10^{10} . Its range is 3.88×10^8 to 1.437×10^{11} . According to the data, all factors show a tendency toward higher values and significant variability.

Table 1b: Descriptive Statistics for the log-transformed EXR, GDP, and EGS series at level form

Vars	N	Mean	Sd	Min	Max	Skew	Kurtosis
lnEXR	63	2.31	2.53	-0.6	6.03	0.09	-1.75
lnGDP	63	9.03	3.36	4.2	14.42	-0.01	-1.47
lnEGS	63	22.96	1.71	19.78	25.69	-0.4	-0.91

Similarly, Table 1b presents descriptive statistics for the log-transformed variables $\ln\text{EXR}$ (exchange rate), $\ln\text{GDP}$ (gross domestic product), and $\ln\text{EGS}$ (exports of goods and services), based on 63 observations each. All variables exhibit moderate variation, with $\ln\text{GDP}$ and $\ln\text{EGS}$ having higher means compared to $\ln\text{EXR}$. The skewness values for all three variables are close to zero, indicating relatively symmetric distributions. The kurtosis values are negative, suggesting flatter-than-normal distributions (platykurtic). Overall, the data appear reasonably well-behaved for time series analysis in their log-transformed forms.

Figure 1 shows time series plots. Until mid-1980s $\ln\text{EXR}$ remained almost stable and after that there was a steep rise and gradually it has been upward indicating consistent depreciation.

On the other hand, the $\ln\text{GDP}$ series showed a continuously rising deterministic trend throughout the period indicating steady economic growth. In the case of the $\ln\text{EGS}$, there was a sharp rise that was preceded by instability in the latter half of the 1980s. This is an indication that there is instability in spending over any particular period.

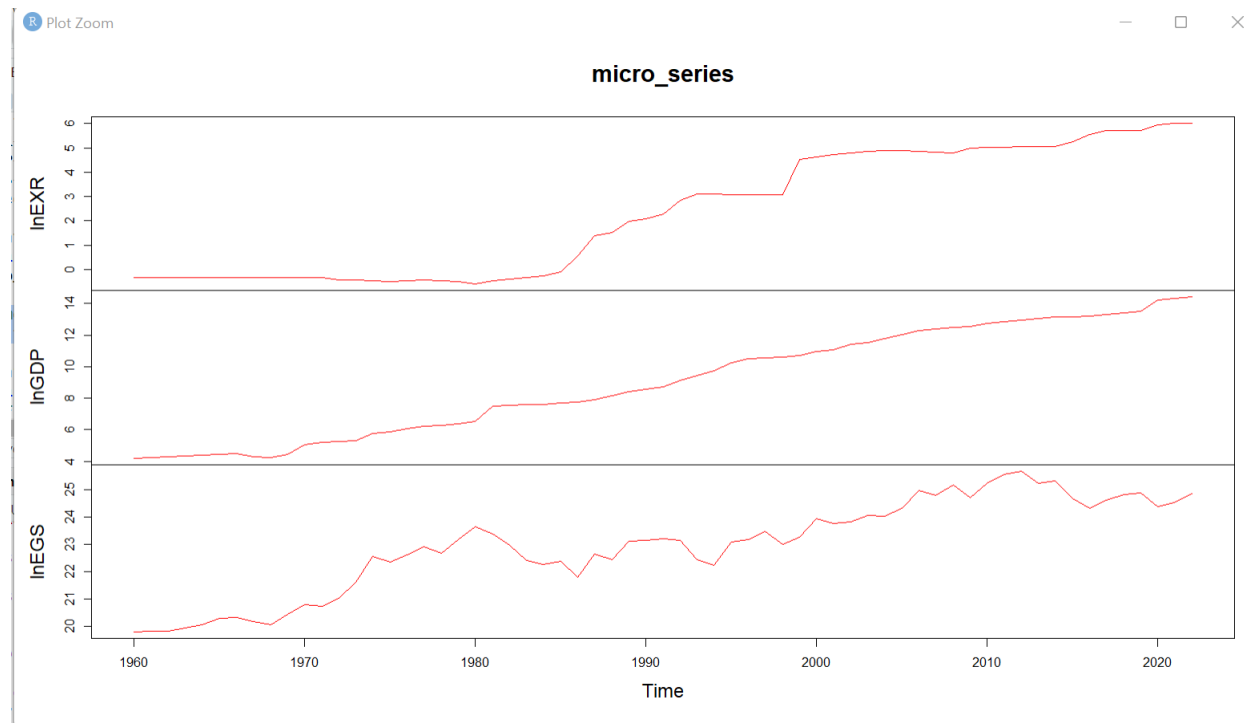


Figure 1: Time series plots for $\ln\text{EXR}$, $\ln\text{GDP}$, and $\ln\text{EGS}$ series at the level form

The $\ln\text{EXR}$, $\ln\text{GDP}$, and $\ln\text{EGS}$ series exhibit non-zero mean and varying variance (non-constant variance). In other words, these raw series are non-stationary, which need differencing once or twice for stationarity. Results of the stationarity test are presented in Table 2. Based on these results, the series are non-level stationary time series.

Table 2: Stationarity test results for the series

Variable	At the level form			After the first difference		
	ADF-statistics	5% CV	p-values	ADF-statistics	5% CV	p-values
lnEXR	1.5109	-1.95	0.1361	-3.7565	-1.95	0.000402 ***
lnGDP	0.1759	-2.89	0.861	-3.0138	-1.95	0.00382 **
lnEGS	-1.5699	-2.89	0.1219	-5.1806	-1.95	2.91e-06 ***

Note: 5% CV denotes 5% Critical Value

However, estimates of the first differences of these series later confirmed that the series are difference stationary processes of order one $\{I(1)s\}$ which make them examinable by multivariate time series models.

Also, these plots of the first differenced lnEXR, lnGDP, and lnEGS series in Figure 2 established zero means and constant variances of the system throughout the period.

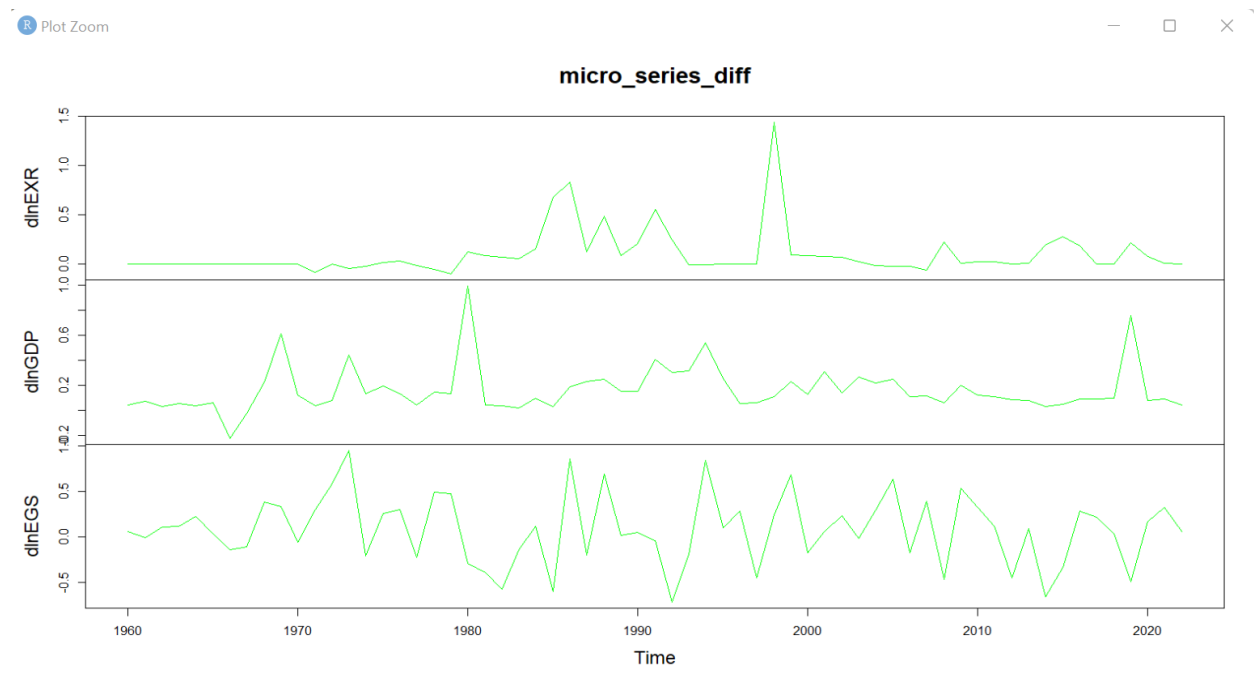


Figure 2: Time series plots for lnEXR, Ln GDP, and lnEGS series after differencing once. This indicates that these macroeconomic time series are stationary and all $I(1)s$. Once the stationarity status has been determined, the optimal lag for the multivariate time series is the next to be determined.

Table 3: Optimal lag selection for the series

	1	2	3	4	5
AIC(n)	-7.85***	-7.729	-7.553	-7.378	-7.214
HQ(n)	-7.725***	-7.48	-7.179	-6.88	-6.591
SC(n)	-7.53***	-7.089	-6.594	-6.099	-5.615
FPE(n)	0.0004***	0.0004***	0.001	0.001	0.001

The estimates of the Akaike Information Criterion (AIC), Hann-Quinn (HQ), Schwarz Criterion (SC), and Final Prediction Error (FPE) for five (5) lags (lag1, lag2, lag3, lag4, and lag5) are reported in Table 3. Of all these lags, only lag 1 reported the lowest values of all the selection criteria, indicating that one is the optimal lag in the study. The multivariate time series model can be a cointegration or non-cointegration model depending on the presence or absence of cointegration in the system. The Vector Error Correction Model (VECM) is desirable if cointegration is present in the system, otherwise, the Vector Autoregressive (VAR) model is desirable (see Garba et al, 2020). Results of cointegration test are shown in Table 4.

Table 4: Johansen cointegration test results

Trace statistic estimates				
Hypothesized order of cointegration	test statistic (TS)	10% CV	5% CV	1% CV
H(2) (cointegration of order 2)	5.69	7.52	9.24	12.97
H(1) (cointegration of order 1)	14.73	17.85	19.96	24.6
H(0) (no cointegration)	37.12	32	34.91	41.07
Eigenvalue statistics				
Hypothesized order of cointegration	test statistic (TS)	10% CV	5% CV	1% CV
H(2) (cointegration of order 2)	5.69	7.52	9.24	12.97
H(1) (at least cointegration of order 1)	9.03	13.75	15.67	20.2
H(0) (no cointegration)	22.39	19.77	22	26.81

For the Trace statistics, one cointegrating relationship exists among the EXR, GDP, and EGS series at the 5% chosen significance level as $r = 0$ (TS = 37.12 > 5% Critical Value = 34.91). Moreover, we reject the null hypothesis of at least one cointegration of order one ($r \leq 1$) (Test statistic = 14.73 > 5% Critical Value = 19.96) AND the null hypothesis of two or more cointegrations of

order one ($r \leq 2$) (Test statistic = 5.69 > 5% Critical Value = 9.24). These mean there is no evidence of more than one cointegrating relationship in the system.

Similarly, the Eigenvalue statistics reported the same results as the Trace statistics. In essence, only one cointegrating relationship exists among the EXR, GDP, and EGS series at the 5% chosen significance level under both the trace and eigenvalue statistics.

The long-run estimates for the $\ln EXR_t$, $\ln GDP_t$, and $\ln EGS_t$ equations are reported in Table 5. As usual, the Error Correction Term (ECT) measures the speed of adjustment after a shock. In the $\ln EXR$ equation, ECT (= -0.1186) is negative and statistically significant (p-value < 0.05), indicating the existence of cointegration in this equation. Moreover, the ECT (= -0.1186) indicates that about 11.86% of deviations from equilibrium are corrected each period. In the $\ln GDP$ equation, ECT (0.06196) is insignificant, suggesting no cointegration in this equation.

Table 5: Long-run Estimates

	ECT	Intercept	$\ln EXR_{t-1}$	$\ln GDP_{t-1}$	$\ln EGS_{t-1}$
Equation $\ln EXR_t$	-0.1186	1.76323	0.20006	-0.1344	-0.0784
Equation $\ln GDP_t$	0.06196	-0.7174	0.03753	0.10185	0.01249
Equation $\ln EGS_t$	-0.0244	0.44855	0.1485	-0.2354	-0.027
	Estimate	Std. Error	t value	Pr(> t)	
$\ln EXR:ECT$	-0.1186	0.05271	-2.2503	0.02838***	
$\ln GDP:ECT$	0.06196	0.04413	1.40387	0.16588	
$\ln EGS:ECT$	-0.0244	0.09243	-0.2643	0.79249	

Also, in the $\ln EGS$ equation, ECT (-0.0244) is insignificant, showing no cointegration in this equation.

As discussed in the methodology section, the estimates of the VECM-VAR (2) model in Table 6 were automatically obtained from the ECM estimates in Table 5 via the `vec2var` function in the R package.

Table 6: Estimates of the VECM-VAR (2) Model

Var	$\ln EXR_{t-1}$	$\ln GDP_{t-1}$	$\ln EGS_{t-1}$	$\ln EXR_{t-2}$	$\ln GDP_{t-2}$	$\ln EGS_{t-2}$	Constant
$\ln EXR_t$	1.0814541	0.008307797	-0.1944191	-0.2000641	0.1343744	0.07836734	1.763227
$\ln GDP_t$	0.0994891	1.027318834	0.07311756	-0.0375297	-0.101853	-0.0124945	-0.71744
$\ln EGS_t$	0.1240696	-0.20600596	0.94910843	-0.1485027	0.2353978	0.02698546	0.4485463

As seen in Table 6, the VECM-VAR (2) has an optimal lag of 2 while the ECM that generated it has an optimal lag order of one {ECM (1)}. Expectedly, the VECM-VAR model estimates should have been estimated based on an optimal lag of one, but were estimated automatically using lag 2.

Table 9: Out-sample forecast of EXR series from the VECM-VAR (2) model

Year	$\ln EXR$	Lower	Upper	CI
2023	6.21268	5.78951	6.63584	0.42317
2024	6.42811	5.77627	7.07995	0.65184
2025	6.62231	5.80353	7.44108	0.81878
2026	6.79224	5.83481	7.74967	0.95743
2027	6.944	5.86231	8.02569	1.08169
2028	7.08321	5.88646	8.27996	1.19675
2029	7.21389	5.90919	8.51859	1.3047
2030	7.33879	5.9322	8.74539	1.4066
2031	7.45978	5.95664	8.96291	1.50314
2032	7.57811	5.98323	9.17299	1.59488

Forecast results in Table 9 reveal a constant upward trend within the forecasted period which indicates a continuous depreciation of the NGN against USD. Figures 4 and 5 display the forecast plots which display forecasts for the $\ln EXR$, $\ln GDP$, and $\ln EGS$ series.

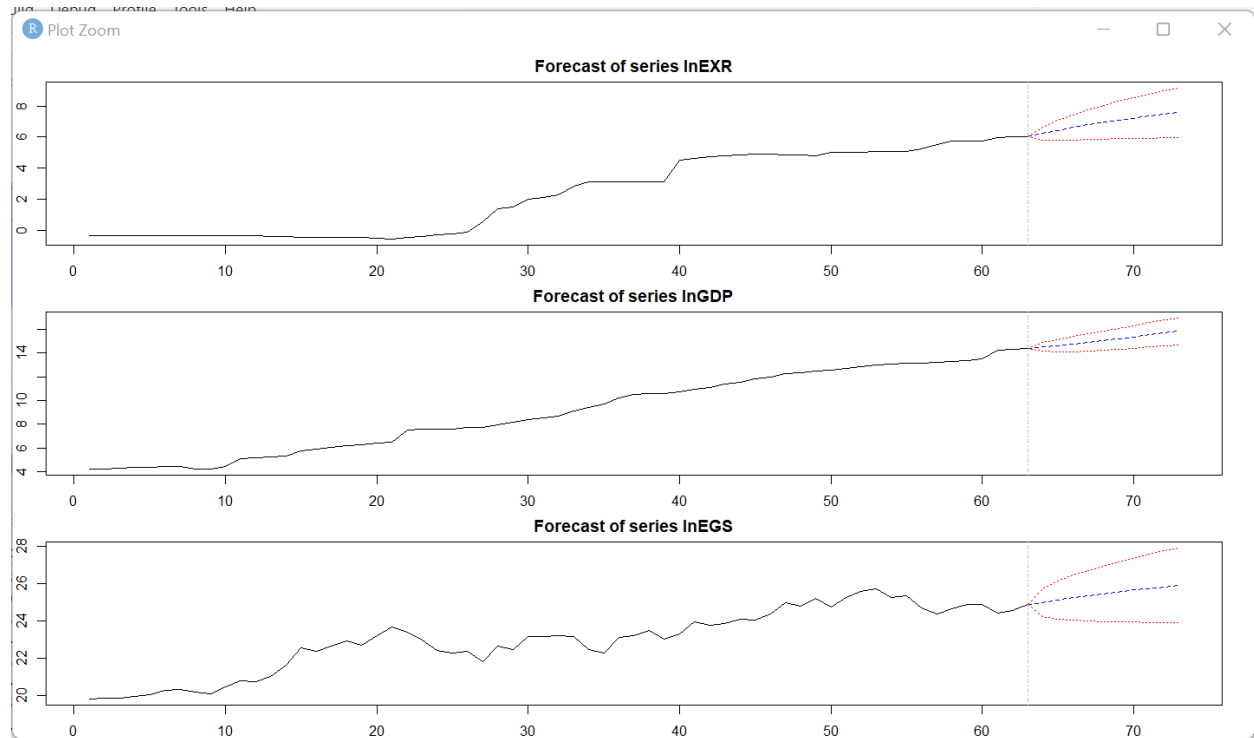


Figure 4: Forecast plots of EXR, GDP, and EGS under the VECM-VAR model

Based on the forecast plot in Figure 4, the three series reveal upward trends with increasing forecasted values, specifically noticeable for the $\ln\text{EXR}$ and $\ln\text{GDP}$ series. Also, the forecast confidence interval widens for all series, indicating greater uncertainty as the forecast horizon extends.

Figure 6 displays the Forecast Error Variance Decomposition (FEVD) results for the logarithms of Exchange Rate ($\ln\text{EXR}$), GDP ($\ln\text{GDP}$), and Exports of Goods and Services ($\ln\text{EGS}$) over a 10-period horizon. In the FEVD for $\ln\text{EXR}$, the variance in $\ln\text{EXR}$ is increasingly explained by its shocks over time, with contributions from $\ln\text{GDP}$ and $\ln\text{EGS}$ stabilizing at smaller proportions. $\ln\text{GDP}$'s FEVD indicates its variance is predominantly driven by its shocks throughout the horizon, with minimal influence from $\ln\text{EXR}$ and $\ln\text{EGS}$. Similarly, $\ln\text{EGS}$ variance is largely explained by its shocks, with negligible contributions from $\ln\text{EXR}$ and $\ln\text{GDP}$. Stability percentages were noticed across horizons, signifying consistent relationships among series over time. This suggests strong own-variable dominance in the dynamics, with limited cross-variable influence.

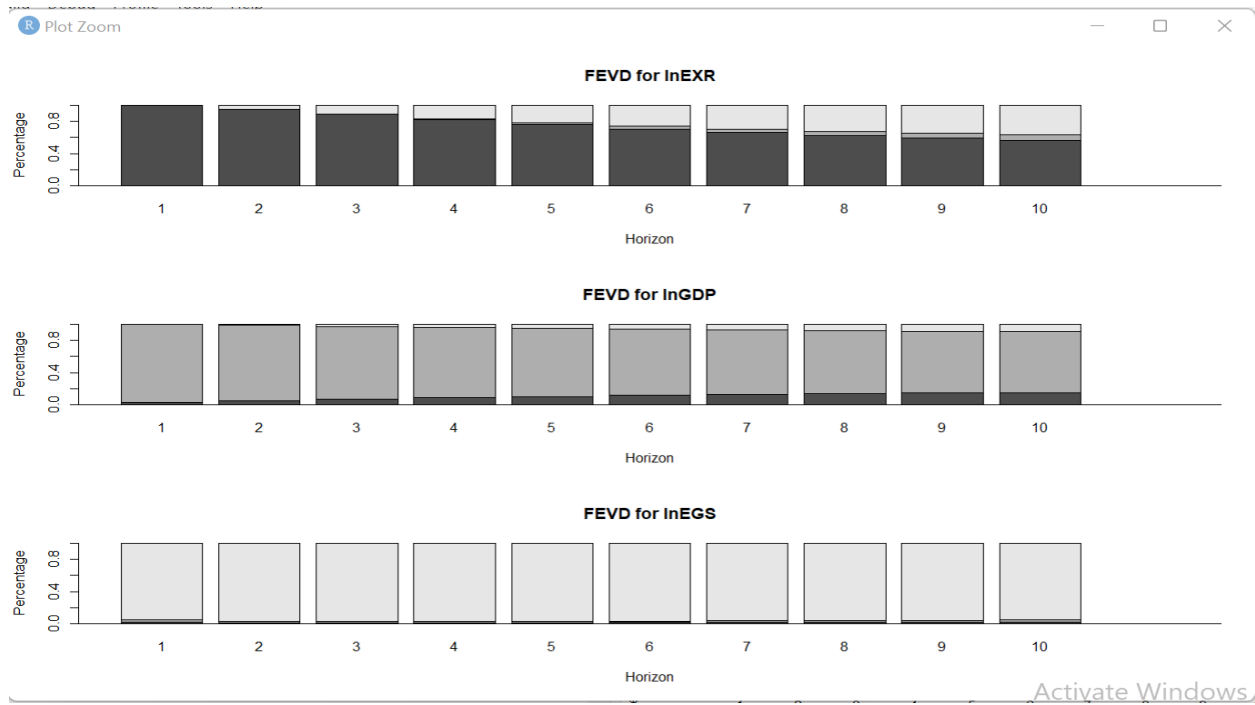


Figure 6: Forecast Error Variance Decomposition (FEVD) plots

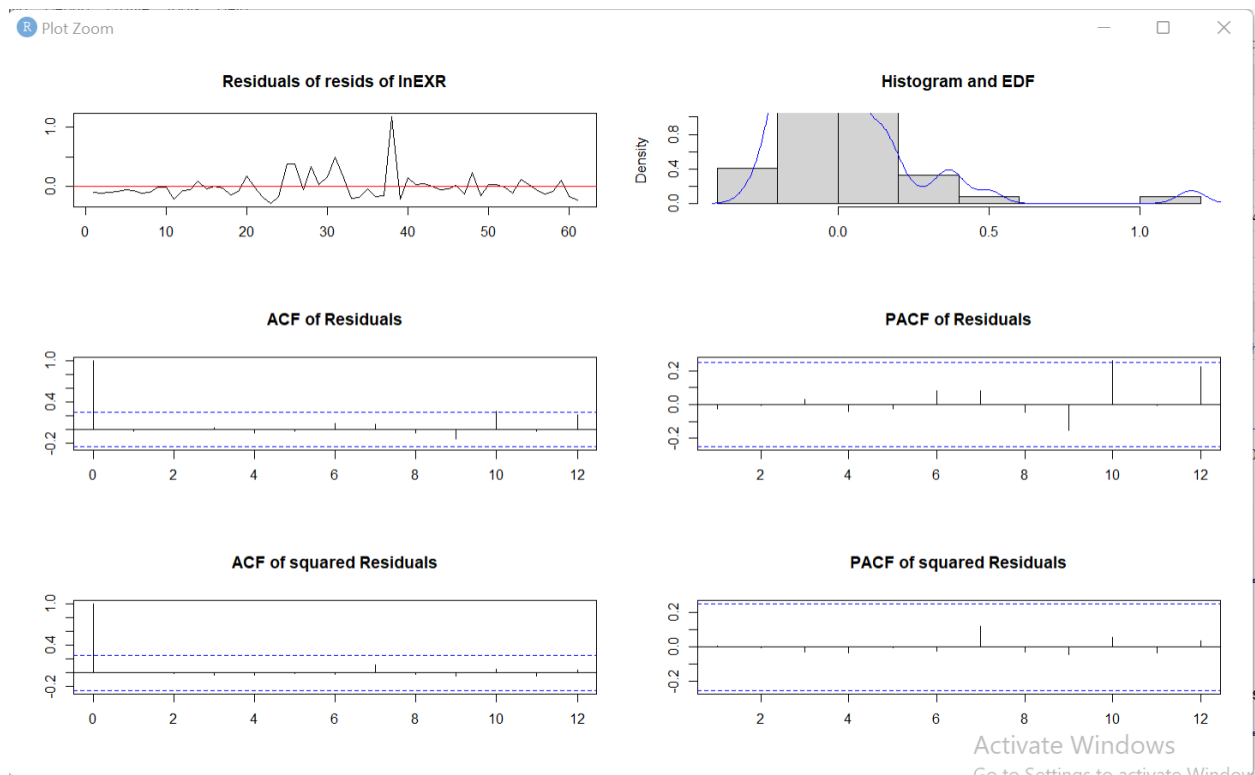


Figure 7: Diagnostic plots for the residuals from the VECM-VAR model

Diagnostic plots in Figure 7 above comprise residuals of $\ln EXR$, histogram, empirical density function (EDF), and correlograms of residuals and squared residuals, respectively.

In these plots, the residuals of $\ln EXR$ series are seen fluctuating around a mean of zero and constant variance, which reveals that the VECM-VAR {2} model is good for these series. These are evident from the insignificant spikes that are found within the 95% confidence bounds. Also, the histogram, as well as the empirical density function (EDF), indicates that the residuals are only a little skewed, departing from normality. The correlogram of residuals revealed no autocorrelation in the residuals of the fitted model (i.e., VECM-VAR {2}) since all the spikes of the autocorrelation (ACF) and partial autocorrelation (PACF) are within the 95% confidence bounds.

4.0 Summary of Findings

This work examines the forecast dynamics of Naira-Dollar Exchange Rates (EXR) from Gross Domestic Product (GDP) and Exports of Goods and Services (EGS) using the short-run component of the Vector Error Correction Model (VECM-VAR) model. The VECM-VAR model, as used in this context, is derived from VECM to VAR in levels through the use of the `vec2var` function in R. Pre-examination of the natural logarithmic transformed series ($\ln EXR$, $\ln GDP$, and $\ln EGS$) at level form revealed that these series exhibit various upward trends, albeit with fluctuations indicating non-stationarity in the series. As a result, the raw series were identified as non-stationary, requiring differencing to achieve stationarity. Through Augmented Dickey-Fuller (ADF) tests, the first differences confirmed that each series was a difference stationary series of order one $I(1)$, with stationary time series characteristics (i.e., constant mean and constant variance). Also, the Akaike Information Criterion (AIC), Hannan-Quinn (HQ), Schwarz Criterion (SC), and Final Prediction Error (FPE) all selected an optimal lag of one for the multivariate time series. Based on the Johansen cointegration tests, one cointegrating equation exists in the three-equation model, which justifies using the Vector Error Correction Model (VECM) to examine the forecasting dynamics of these series. The negative and statistically significant Error Correction Term (ECT) ($p\text{-value} < 0.05$) of the long-run estimates indicated equilibrium adjustments in the $\ln EXR$ equation. The estimated VECM-VAR (2) model from the Error Correction Model {ECM (1)} utilized 2 lags for its estimation instead of the expected 1 lag as indicated in the ECM (1) estimates. Moreover, forecast plots demonstrated upward trends for $\ln EXR$, $\ln GDP$, and $\ln EGS$,

with increasing uncertainty over time. Forecast Error Variance Decomposition (FEVD) showed that variances in each series were primarily driven by their shocks, with minimal cross-variable influence. Diagnostic plots obtained from the residuals' tests confirmed that the VECM-VAR (2) is the optimal model, indicating no evidence of autocorrelation and almost constant variance.

5.0 Conclusion and Recommendations

In this work, we investigate the forecasting dynamics of the Naira-Dollar Exchange Rates (EXR) from Gross Domestic Product (GDP) and Exports of Goods and Services (EGS) using the Vector Error Correction Model (VECM-VAR). The VECM-VAR is also a VAR model derived from the VECM model, however, it is expected to perform better than the classical VAR model under a strong cointegration scenario. Our pre-tests justify the use of the VECM-VAR (2) model to examine the forecasting dynamics of the natural logarithm of EXR, GDP, and EGS series (lnEXR, lnGDP, and lnEGS) since there is one cointegrating equation in the three-equation model used in this work. Moreover, the Error Correction Term (ECT) indicated equilibrium adjustments in the lnEXR equation. Our findings show a continuous depreciation trend for the Naira against the United States Dollar, *which is in agreement with Akintunde et al (2025) findings.*

Our findings recommend that policymakers should concentrate on promoting exports because they directly impact the stability of the Naira exchange rate and Nigerian GDP growth.

Since the VECM-VAR model effectively captures long-run relationships and equilibrium adjustments, it is preferable for forecasting over traditional VAR models when cointegration exists. Cointegration models (s) are vital in ensuring reliable and accurate exchange rate predictions.

6.0 References

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