FLEXIBLE ACCELERATED FAILURE TIME MODEL WITH SPATIAL DEPENDENCY: APPLICATION TO UNDER-FIVE MORTALITY RATE

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ABSTRACT

Under-five mortality (U5M) data often exhibit complex spatial dependencies and nonlinear temporal patterns that conventional survival models may fail to capture adequately. This study provides an extension to Bayesian Accelerated Failure Time (AFT) model that simultaneously accounts for nonlinear (NL) effects of continuous covariate, time-varying acceleration factors, and spatial heterogeneity in child survival analysis. Using data from the 2018 Nigeria Demographic and Health Survey (NDHS), the study extended the Weibull AFT model by incorporating B-splines for nonlinear effects of continuous covariate, random-walk time-varying coefficients, and intrinsic conditional autoregressive (ICAR) spatial random effects. Model performance was evaluated using Deviance Information Criterion (DIC) and Watanabe-Akaike Information Criterion (WAIC). Results demonstrate that the extended model (flexible spatial AFT model) significantly outperforms traditional parametric specifications. The findings revealed breastfeeding as the strongest protective factor (time ratio [TR]=5.95, 95% CI: 5.65-6.25), followed by complete antenatal care utilization (TR=2.14, 95% CI: 1.86-2.46) and longer birth intervals (TR=1.17, 95% CI: 1.12-1.23). Spatial analysis identified significant geographic clustering, with northern Nigerian states showing higher survival times than southern regions. The time-varying effects revealed that urban residence advantages diminish as children age while breastfeeding protection remains stable. This study provides a methodological advancement in survival analysis by simultaneously integrating NL effect of continuous covariates, non-constant acceleration factor and spatial effects within the AFT framework, offering policymakers a refined tool for targeted U5M interventions. The approach is broadly applicable to clustered survival data in global health and demographic research.

***Keywords**:* Accelerated Failure Time model, under-five mortality, spatial survival analysis, Bayesian inference, nonlinear effects, time-varying coefficients

1.0 Introduction

Under-five mortality (U5M) data from Demographic and Health Surveys (DHS) often exhibit spatial dependencies due to regional disparities in healthcare access, socioeconomic conditions, and environmental factors. Traditional survival models, such as the Accelerated Failure Time (AFT) model, frequently overlook these spatial effects, leading to biased estimates and obscured area-level determinants of child survival (Egbon, 2022). The classical AFT model relies on restrictive assumptions, including constant acceleration factors and linear covariate effects on log

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survival time (Cox & Oakes, 1984). However, in multivariable analyses, these assumptions may not hold for certain covariates, limiting the model's applicability. Similarly, flexible extensions of the Cox proportional hazards (PH) model often encounter violations of key assumptions, such as non-constant hazard ratios (violating PH) or nonlinear covariate effects on log hazard sometimes both for the same continuous covariate.

While flexible extensions of PH models have been extensively developed to address non-proportionality and nonlinear effects (Grambsch & Therneau, 1994; Huang & Liu, 2006; Sauerbrei et al., 2007), similar advancements in AFT models remain limited. Recent methodological progress has introduced semiparametric AFT models (Zhang et al., 2018; Lin et al., 2021), demonstrating superior performance in handling time-varying and nonlinear covariate effects, particularly in oncology, genomics, and clinical trials (Wu et al., 2021; Kang et al., 2020). However, their application to clustered survival data requiring spatial dependency adjustments remains underexplored, highlighting a critical gap in the literature.

Empirical studies on U5M have predominantly employed Cox PH, parametric survival, and spatial survival models. Spatial analyses in Kenya (Daniel, 2021) and Nigeria (Fagbamigbe & Nnanatu, 2022; Egbon et al., 2022) reveal significant geographic clustering of mortality risks, with socioeconomic, maternal, and healthcare access factors as key determinants. Studies across Sub-Saharan Africa (Fenta et al., 2025; Yalew et al., 2022) highlight the role of birth spacing, maternal education, and household wealth in child survival. In Nigeria, Cox PH models (Wegbom et al., 2019; Okoli et al., 2022) and Bayesian spatial approaches (Ghilagaber et al., 2013) consistently identify regional disparities, with higher mortality risks in northern states. Similar patterns are observed in Bangladesh (Fatima-Tuz-Zahura et al., 2017) and India (Singh & Singh, 2023), where multilevel survival models underscore the influence of birth order, maternal age at birth, and sanitation.

Despite these advances and numerous studies, few integrate flexible AFT models (model that relaxes the two key assumptions of classical AFT) with spatial dependencies, leaving a critical gap in modeling U5M's complex risk structures. This study bridges this gap by employing an advanced spatial AFT framework to enhance the precision and interpretability of U5M determinants in clustered DHS data. The specific objectives of the study include (i) to provide an extended Bayesian Accelerated Failure Time model that incorporates nonlinear covariate effects, time-varying coefficients, and spatial dependencies for analyzing U5M patterns in Nigeria; (ii) to

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compare the performance of the extended flexible AFT with spatial random effect model against conventional AFT models using Bayesian model selection criteria (DIC and WAIC) (iii) to identify the key determinants of under-five mortality and, (iii) to map spatial variations in child mortality risk across Nigerian states in order to identify significant geographic clusters of high and low mortality risk

2.0 Materials and Methods

The study utilized data from the 2018 Nigerian Demographic and Health Survey (NDHS), which were extracted, cleaned, and analyzed using STATA and the Integrated Nested Laplace Approximation (INLA) approach, implemented via the R-INLA package. The outcome variable was the time to death within five years of birth, with children who survived beyond this period treated as censored observations. An event (U5M) was recorded if a child died before reaching their fifth birthday.

2.1 Variables Classification

The careful selection of variables from the NDHS data is crucial for ensuring the study's relevance and validity. The variables chosen are those that are directly related to the research hypothesis and are supported by existing literature on the determinants of U5M. By focusing on a targeted subset of variables, researchers can effectively address the study's objectives and provide meaningful insights into the factors influencing U5M.

For the purpose of this study, the maternal age birth (Mage) was kept in its metrical forms while the other variables were categorical; breastfeeding status was coded “1” for children that were breastfed and “0” for those that were not breastfed (reference category); Preceding Birth Intervals (PBI) was coded 2 for More than 33 months, 1 for 24 – 33 months and 0 for less 24 months (reference category), Maternal Educational qualification (MEQ) was coded 3 for those with “Higher education”, 2 for those with “secondary education”, 1 for those with “primary education” and 0 for those with “No formal education” (Reference category). Wealth Index (WID) was coded 0, 1, 2, 3 and 4 for “poorest (reference category)”, “poorer”, “Middle”, “rich” and “richest” respectively. The Maternal Body Mass Index (BMI) was coded 0, 1 and 2 for “Underweight (reference category)”, “Normal” and “Overweight/obese” respectively. Antenatal Care Utilization (ACU) was coded 0 and 1for those with “incomplete (reference category)” and “complete” antenatal care utilization respectively. The sex of child was coded “0” for those male and “1” for female. Type of Place of Residence (TPR) was coded “0” for urban (reference category) and “1”

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for rural. Mosquito Net Use (MNU) was coded "0" for not usage and "1" for those that used it. Source of Drinking Water (SoDW) was coded "0" for improved and "1" for unimproved.

2.2 Accelerated Failure Time Model Formulation

In the conventional Accelerated Failure Time model, the natural logarithm of the event time, $Log(T)$, is modeled as a linear function of the covariate vector M (Wei, 1992, Kalbfleisch & Prentice, 2002):

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \beta_j M_j + \sigma\varepsilon \quad (1)$$

Taking the exponent of both side of equation (1) resulted to:

$$T = \exp(\beta_0 + \sigma\varepsilon) \exp\left(\sum_{j=1}^{p} \beta_j M_j\right) = T_0 \exp\left(\sum_{j=1}^{p} \beta_j M_j\right) \quad (2)$$

Where:

$M_j, j = 1,2, \dots, p$ are the covariates, $\beta_j, j = 0,1, \dots, p$ are the regression coefficient, $\sigma(> 0)$ is the scale parameter and ε is a random error which has a specified distribution. The term $\exp\left(\sum_{j=1}^{p} \beta_j M_j\right)$ is the acceleration factor. Thus, if $\beta_j > 0$ consequently, $\exp\left(\sum_{j=1}^{p} \beta_j M_j\right) > 0$, the covariates M_j decelerated the survival process and if $\beta_j < 0$ consequently, $\exp\left(\sum_{j=1}^{p} \beta_j M_j\right) < 0$, the covariates M_j accelerated the survival process.

In order to account for the spatial dependency, the Conventional AFT model (1) has been extended by incorporating spatial random effect (Egbon et al., 2022) as given as below:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \beta_j M_j + w_i + \sigma\varepsilon \quad (3)$$

$$w_i = n_i + v_i$$

Where the frailty term w_i incorporate the effect of both heterogeneity via the non-spatial frailty n_i and spatial dependency through the spatial frailty v_i. The non-spatial frailty n_i is the random effect which independently and identically distributed with mean 0 and variance σ_u^2 that is, $n_i \sim N(0, \sigma_n^2)$ while the spatial frailty v_i is define purely based on Intrinsic Conditional Autoregressive (ICAR) to account for the spatial dependency between clusters (states)

2.3 Extended Flexible AFT Models

Both the AFT model in (1) and its spatial random effect extension (3) assume (i) a linear relationship between each continuous covariate and log event time, and (ii) time-invariant log acceleration factors (β), meaning the log time ratios remain constant over time. In this study, we provide a flexible extension of the AFT model to simultaneously relax both these conventional assumptions and to account for possible non-constant acceleration factor (Time-varying

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coefficient) effect on log of time and/or non-linear (NL) effects of continuous covariate(s) while accounting for the spatial random effect.

First, by relaxing the linearity assumption of conventional AFT model, the AFT model in (3) was extended to:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \psi_j m_j + \sum_{k=1}^{q} f_k(X_k) + w_i + \sigma\varepsilon \quad (4)$$

Where:

$\sum_{j=1}^{p} \psi_j m_j$ = Linear effect of categorical covariates m_j

$\sum_{k=1}^{q} f_k X_k$ = Non-linear effects of continuous covariates X_k modelled via B-splines

$f_k(X_k) = \sum_{n=1}^{d} \gamma_{kn} C_{kn}(X_k)$

$C_{kn}(.) = n^{th}$ basis function for X_k (e.g. quadratic spline)

$\gamma_{kn} =$ Coefficients of the spline terms

$d =$ Degrees of freedom (Internal Knots +Spline order)

Secondly, by relaxing the constant acceleration factor assumption, the AFT model in (3) was extended to:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \psi_j m_j + \sum_{i=1}^{p} \alpha_i(t_i) Z_i + w_i + \sigma\varepsilon \quad (5)$$

Where:

$\sum_{j=1}^{p} \psi_j m_j$ = Linear effect of categorical and metrical covariates m_j (constant acceleration factor)

$\sum_{i=1}^{p} \alpha_i(t_i) Z_i =$ Linear effect of time-varying covariates Z_i

$\alpha_i(t) =$ is the time varying coefficient (non-constant acceleration factor) model through random work in INLA. $\alpha(t) = \alpha(t-1) + \eta_t$ evolves stochastically, allowing flexible, data-driven temporal variation. $\eta_t \sim N(0, \tau^2)$ innovations (Gaussian noise) with variance τ^2.

Thirdly, by relaxing both assumptions, the AFT model in (3) was extended to:

$$Log(T) = \beta_0 + \sum_{j=1}^{p} \psi_j m_j + \sum_{i=1}^{p} \alpha_i(t_i) Z_i + \sum_{k=1}^{q} f_k(X_k) + w_i + \sigma\varepsilon \quad (6)$$

Where:

$\sum_{j=1}^{p} \psi_j m_j$ = Linear effect of categorical covariates m_j (constant acceleration factor)

$\sum_{i=1}^{p} \alpha_i(t_i) Z_i =$ Linear effect of time-varying covariates Z_i

$\sum_{k=1}^{q} f_k X_k$ = Non-linear effects of continuous covariates X_k

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$\alpha_i(t) =$ is the time varying coefficient (non-constant acceleration factor) model through random work in INLA. Note that the covariates vectors $m, Z\ and\ X$ are subset of matrix M and, regression coefficient $\beta = \psi, \alpha(t),\ \gamma$.

2.4 Intrinsic Conditional Autoregressive (ICAR) model.

A conditional auto-regressive structure is used to model spatial dependence in the Besag ICAR prior (Besag *et al.* 1991). Specifically, it is modelled that the prior distribution of each observation is conditional on the values of its neighbors. A Gaussian distribution is commonly used to describe this conditional dependence. The variance of the distribution represents the degree of spatial autocorrelation, and the mean is a weighted average of the nearby values. Spatially correlated random effects can be used to account for spatial dependency.

Let $E_{ij} = 1$ if area $E_i\ and\ E_j$ share a nontrivial border and $E_{ij} = 0$ otherwise. Set $E_{ij} = 0$, then the $G \times G$ matrix $E = [E_{ij}]$ is called the adjacency matrix for the region D. The ICAR prior is defined through the set of all conditional distributions as given below:

$$\pi(ICAR\_Prior) = v_j|\{v_i: i \neq j\} \sim N\left(\bar{v}_j, {\vartheta^2}/{a_{j+}}\right), j = 1, \dots, G \quad (7)$$

Equation 3.23 denoted $v \sim ICAR\left({1}/{\vartheta^2}\right)$, where $a_{j+}$is the number of neighbors of area C_j, $\bar{v}_j = \frac{1}{a_{j+}}\sum_{i:a_{ij=1}} \gamma_i$ is the sample mean of the a_{j+}values of the neighboring areal unit frailties, and ${\vartheta^2}/{a_{j+}}$is the conditional variance. In this study, the spatial parameter v_i is a 37×1 vector of spatial effects to account for heterogeneity between states in Nigeria. Let $w = (v_1, \dots, v_{37})$.

2.5 Weibull Baseline Distribution

The probability density function (PDF) of the Weibull distribution with the scale parameter φ and the shape parameter γ is denoted as:

$$f(t|\varphi, \gamma) = F'(t: \varphi, \gamma) = \frac{\gamma}{\varphi}\left(\frac{t}{\varphi}\right)^{\gamma-1} \exp\left[-\left(\frac{t}{\varphi}\right)^{\gamma}\right] \quad (8)$$

The survival function of the Weibull distribution is given as:

$$S(t) = 1 - F(t) = \exp\left[-\left(\frac{t}{\varphi}\right)^{\gamma}\right] \quad (9)$$

The hazard function of the Weibull distribution is given as:

$$h(t|\varphi, \gamma) = \frac{\varphi}{\gamma}\left(\frac{t}{\gamma}\right)^{\varphi-1} \quad (10)$$

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2.6 Bayesian Inference

A typical Bayesian workflow includes three major steps that is the prior distribution, Likelihood Function $L(\theta)$ and Posterior distribution $\pi(\theta|D)$.

Step I: Prior Distribution for Parameters and Regression Coefficient

This study employs a non-informative independent gamma distribution as the prior for baseline parameters. Gamma distributions offer flexibility, accommodating both non-informative (uniform) priors and marginal priors for regression coefficients. Similar priors have been widely adopted in prior literature, including studies by Alvares et al. (2021), Muse et al. (2022a, 2022b), and Khan & Basharat (2022). Here,

$$\pi(\rho)\sim G(c_1,d_1)=\frac{d_1^{c_2}}{\Gamma(c_1)}\rho^{c_1-1}e^{-d_1\rho};\ c_1,d_1,\rho>0 \quad (11)$$

$$\pi(\lambda)\sim G(c_2,d_2)=\frac{d_2^{c_2}}{\Gamma(c_2)}\lambda^{c_2-1}e^{-d_2\lambda};\ c_2,d_2,\lambda>0 \quad (12)$$

The hyperparameters of the prior distributions can be easily estimated using historical data from the baseline distribution (Muse et al., 2022b). Assuming a normal prior distribution for the regression coefficients, we have:

$$\pi(\psi)\sim G(c_3,d_3) \quad (13)$$

The Time -varying coefficient $[\alpha(t)]$ was modeled via random work of order one (RW1).

$$\alpha_i(t)\sim N(\alpha_i(t-1),\tau_\alpha^2),\ \tau_\alpha^2\sim Gamma(a_\alpha,b_\alpha). \quad (14)$$

Non-linear effect of continuous covariates modeled via quadratic b-splines (p=2) with one internal knot (m = 1) for estimating $f(X)$. The coefficient:

$$\gamma_k\sim N(0,\tau_k^2P_k^{-1}),\ \tau_k^2\sim Gamma(a_f,b_f) \quad (15)$$

The prior distribution for the spatial parameter w_i in this study is a 37×1 vector of spatial random effects to account for heterogeneity between states in Nigeria. Let $w=(w_1,\dots,w_{37})$. The spatial parameter w was modelled via Baseg-ICAR prior, that is:

$$\pi(w)\sim ICAR \quad (16)$$

By letting $\beta=(\psi,\alpha(t),\gamma_k)$, the joint prior distribution of all unknown parameters is given by:

$$\pi(\rho,\lambda,\beta,w)=\pi(\rho)\pi(\lambda)\pi(\beta)\pi(w) \quad (17)$$

Step II: Likelihood Function $L(\theta)$

Let assumed that subjects are observed at g distinct spatial locations $g_1,\dots,g_k$. Let also assumed that t_{ij} be the (possibly censored) survival time, for subject j at location g_i and M_{ij} be the

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corresponding p-dimensional vector of covariates, $i = 1, \dots, k, j = 1, \dots, n_i$; let $n = \sum_{i=1}^{k} n_i$. Furthermore, let assume that the survival t_{ij} lies in the interval $(c_{ij}, d_{ij}), 0 \leq c_{ij} \leq d_{ij} \leq \infty$. Also, let $\varepsilon = \frac{\log(T_i) - \mu_i}{\sigma}$, where $\mu_i = \beta_0 + \sum_{j=1}^{p} \beta_j m_j + \sum_{i=1}^{p} \alpha_i(t_i) Z_i + \sum_{k=1}^{q} f_k(X_k) + w_i$

The likelihood function of the extended mode is given by,

$$L(\theta|T, \delta) = \prod_{i=1}^{n} [\frac{1}{\sigma T_i} \emptyset(\frac{\log(T_i) - \mu_i}{\sigma})]^{\delta_i} \times [S(\frac{\log(T_i) - \mu_i}{\sigma})]^{1-\delta_i} \quad (18)$$

Where:

$\emptyset(.)$ = probability density function of the error distribution (e.g. Weibuill)

$S(.)$ =corresponding survival function

$\theta = (\rho, \lambda, \beta, w)$ all parameters

Step III: Posterior distribution $\boldsymbol{\pi(\theta|D)}$

Using Bayes' Theorem, the joint posterior density function is expressed as the multiplication of the likelihood function and the prior distribution as given below:

$$p(\theta|T) = L(\theta|T, \delta) \times \pi(\rho, \lambda, \beta, w) \quad (19)$$

The joint posterior density was analytically intractable due to integration challenges, so inference was performed using Integrated Nested Laplace Approximation (INLA) (Martino & Riebler, 2014).

3.0 Results

Table 1: Constant Hazard Postulation

Covariates	Chi-Square	DF	P-value
Maternal Age at Birth	116.129	1	2×10^{-16}
Duration of Breastfeeding	10.280	1	0.0013
Preceding Birth Intervals	4.829	2	0.0754
Maternal Education Qualification	4.777	3	0.0653
Wealth Index	2.702	4	0.0945
Body Mass Index	0.497	2	0.7801
Antenatal Care Utilization	3.039	1	0.0813
Sex of Child	1.781	1	0.1821
Type of Place of Residence	4.106	1	0.0427
Mosquito net use	9.412	1	0.0022
Source of Drinking Water	0.164	1	0.6852
Global Test	207.675	18	2×10^{-16}

Source: Author's Compilation

The results of the test of proportional hazards assumption are shown in Table 1. Significant violations ($p < 0.05$) were found for maternal age at birth, breastfeeding duration, mosquito net

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use, and place of residence, indicating these covariates have time-varying effects on under-five mortality. The remaining variables - including maternal education, wealth index, birth intervals, antenatal care, maternal BMI, child sex, and water source - showed no significant violations ($p > 0.05$). The global test was highly significant ($\chi^2 = 207.675$, $p < 0.05$), confirming overall non-proportionality in the model. Consequently, time-varying coefficients were incorporated for covariates that violated the proportional hazards assumption.

Table 2: Flexible AFT Model Comparison

Model	**DIC**	**WAIC**
Weibull AFT Model	152397.19	152393.06
Weibull AFT Model with Spatial Random Effect	152355.10	152345.08
Spline based Weibull AFT Model	152392.07	152387.76
Spline based Weibull AFT Model with Spatial Random Effect	152350.49	152340.33
Weibull AFT Model with NCAF	104896.59	104892.87
Weibull AFT Model with NCAF and Spatial Random Effect	104895.00	104891.48
Flexible Weibull AFT Model	104876.99	104872.86
Flexible Weibull AFT Model with Spatial Random Effect*	104875.47	104870.92

Source: Authors Compilation

The results in Table 2 compare the performance of various flexible Accelerated Failure Time (AFT) models using Deviance Information Criterion (DIC) and Watanabe-Akaike Information Criterion (WAIC). The standard Weibull AFT model has the highest DIC (152397.19) and WAIC (152393.06), indicating the poorest fit among the models. Incorporating spatial random effects improves model performance, as seen in the reduced DIC (152355.10) and WAIC (152345.08) for the spatial Weibull AFT model. The spline-based Weibull AFT models show marginal improvements over the standard Weibull but remain inferior to models accounting for non-constant acceleration factors (NCAF). The most substantial improvement occurs with the introduction of NCAF, where the Weibull AFT model with NCAF achieves a significantly lower DIC (104896.59) and WAIC (104892.87). The best-performing model is the flexible Weibull AFT (model that simultaneously incorporate non-linear effect of continuous covariates and non-constant acceleration factor) with spatial random effects, which yields the lowest DIC (104875.47) and WAIC (104870.92), demonstrating that simultaneously accounting for both nonlinear effects of continuous covariates, non-constant acceleration factor and spatial dependencies provides the optimal fit.

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Table 3: Posterior Mean Estimates from Flexible Weibull AFT Model with Spatial Random Effect

Covariates	Posterior Mean	Time Ratio	Posterior SD	95% CI
Maternal Age at Birth				
Mage bs1	0.445	1.560	0.110	0.229, 0.662
Mage bs2	0.512	1.669	0.084	0.347, 0.677
Mage bs3	0.516	1.675	0.095	0.329, 0.703
Breast Feeding Status				
Not Breast Fed	***Ref***			
Breast Fed	1.784	5.954	0.026	1.733, 1.834
Preceding Birth Intervals				
< 24	***Ref***			
24 – 33	0.032	1.033	0.017	0.001, 0.065
> 33	0.160	1.174	0.023	0.115, 0.205
Maternal Educ. Qualification				
No Formal Education	***Ref***			
Primary	-0.011	0.989	0.022	-0.032, 0.055
Secondary	0.058	1.060	0.027	0.003, 0.110
Higher Education	0.109	1.115	0.052	0.008, 0.210
Wealth Index				
Poorest	***Ref***			
Poorer	0.035	1.036	0.038	0.039, 0.109
Middle	0.096	1.101	0.038	0.023, 0.170
Richer	0.090	1.094	0.039	0.014, 0.167
Richest	0.118	1.125	0.040	0.039, 0.197
Maternal Body Mass Index				
Underweight	***Ref***			
Normal	0.018	1.018	0.039	-0.058, 0.094
Overweight/Obese	-0.007	0.993	0.045	-0.094, 0.081
Antenatal Care Utilization				
Incomplete	***Ref***			
Complete	0.761	2.140	0.072	0.620, 0.901
Sex of Child				
Male	***Ref***			
Female	-0.014	0.986	0.015	-0.043, 0.015
Type of Place of Residence				
Rural	***Ref***			
Urban	0.010	1.010	0.020	0.029, 0.049
Mosquito Net Use				
No	***Ref***			
Yes	0.039	1.040	0.016	0.006, 0.070
Source of Drinking Water				
Improved	***Ref***			
Unimproved	0.022	1.022	0.019	-0.015, 0.058
Spatial Effect Precision	6.080		1.213	2.642, 7.050

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Non-Constant Accelerator Factor Precision				
	Mean	Variance	Posterior SD	95% CI
Mage	552	0.0018	2050	117.1, 3480.0
Breastfed	674	0.0015	4570	86.00, 4120.0
Type of Place of Resident	0.034	29.4117	0.008	0.021, 0.530
Mosquito Net Use	6620	0.0002	3050	145.9, 3870.0

Source: Authors Compilation

Table 3 displays the posterior estimates from the flexible Weibull AFT Model with Spatial Random Effect, providing insights into how each covariate influences under-five survival time. Time ratios greater than 1 indicate prolonged survival (lower risk of early death), while values less than 1 suggest shortened survival (higher risk of U5M).

Maternal age shows nonlinear effects through its basis spline terms, with all three splines (bs1, bs2, bs3) demonstrating significant protective effects on child survival. The first spline (TR=1.560) indicates that initial increases in maternal age provide substantial benefits, while the second (TR=1.669) and third (TR=1.675) splines show these protective effects strengthen further at higher maternal ages, suggesting maternal maturity consistently enhances child survival prospects.

Breastfeeding status emerges as the single most influential protective factor, with breastfed children showing nearly six times longer survival times (TR=5.954) compared to non-breastfed children. Preceding birth intervals demonstrate a clear dose-response relationship with child survival. While moderate intervals of 24-33 months show modest benefits (TR=1.033), longer intervals exceeding 33 months provide substantially increase survival time (TR=1.174), highlighting the importance of adequate birth spacing for child health outcomes.

Maternal education presents a graduated protective effect, where primary education shows no significant benefit (TR=0.989), secondary education (TR=1.060) provides measurable advantages, and higher education offers the strongest protection (TR=1.115). This pattern suggests education's benefits for child survival time accumulate with each additional level attained. The wealth index reveals a progressive protective effect. From poorer (TR=1.036) to richest (TR=1.125) households, each wealth quintile shows incrementally better child survival time (lower U5M risk), demonstrating how economic resources consistently translate to improved child health.

Maternal body mass index shows essentially neutral effects on child survival, with normal weight (TR=1.018) and overweight/obese (TR=0.993) mothers showing no statistically significant differences from underweight mothers in terms of child survival outcomes. Complete antenatal

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care utilization demonstrates strong protective effects (TR=2.140), more than doubling child survival time compared to incomplete care. This finding underscores the critical importance of comprehensive prenatal healthcare services for improving child survival.

Sex differences show minimal impact, with female children having marginally lower survival (TR=0.986) than males, though this difference is not statistically significant as the confidence interval includes zero. Type of place of residence was significant predictor of U5M with urban residence provides a small but measurable survival advantage (TR=1.010) over rural areas, likely reflecting better access to healthcare and other resources in urban settings. Mosquito net use increase child survival time (TR=1.040), confirming the importance of malaria prevention measures for child survival in endemic areas. Drinking water source shows negligible impact on survival time (TR=1.022) though statistically not significant.

The spatial random effects (precision=6.080) confirm significant geographic variation in mortality patterns, indicating important regional differences not explained by the measured covariates.

The non-constant acceleration factor analysis reveals how covariate effects on child survival vary across developmental stages, with breastfeeding demonstrating a minute influence (variance=0.0015) as its strong protective effect (TR=5.954) remains slightly consistently high throughout all childhood periods, while residence type shows extreme variability (variance=29.4117) indicating the urban advantage fluctuates dramatically - potentially offering strong protection during specific vulnerable periods but minimal benefits at other ages. Mosquito net use maintains very consistent effects (variance=0.0002) with steady protection across all ages, whereas maternal age exhibits moderate variation (variance=0.0018) as its benefits may strengthen during certain developmental phases when maternal experience becomes particularly crucial, demonstrating that while some factors like breastfeeding provide continuous protection, others like urban residence offer age-specific advantages that could inform targeted intervention strategies.

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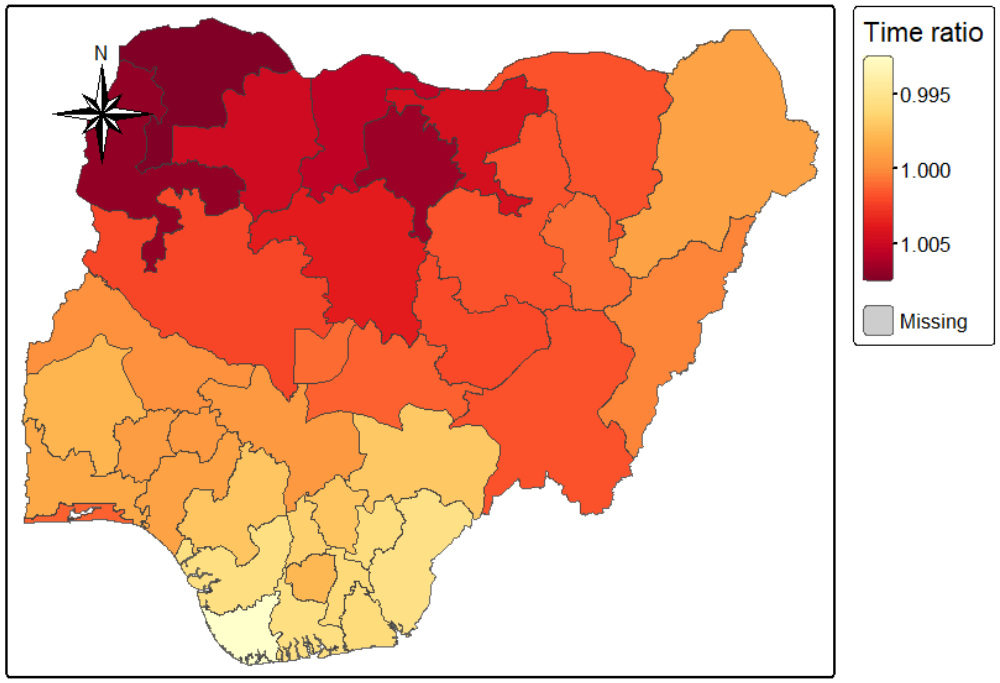

Fig. 1: Posterior mean (Average) Spatial Effect of State Based Survival Risk of U5M from Flexible Weibull AFT Model with Spatial Random Effect

The map of Nigeria in figure 1 illustrates the geographic distribution of U5M using time ratios. The regions colored with darker red/burgundy (northern state) had time ratios greater than 1 indicating increase in survival time (lower U5M risk), while regions with lighter yellow areas (southern states) had time ration ratios less than 1 represent decreased survival time (higher U5M risk). This reveals that some northern states experience better child survival outcomes compared to some states in southern regions.

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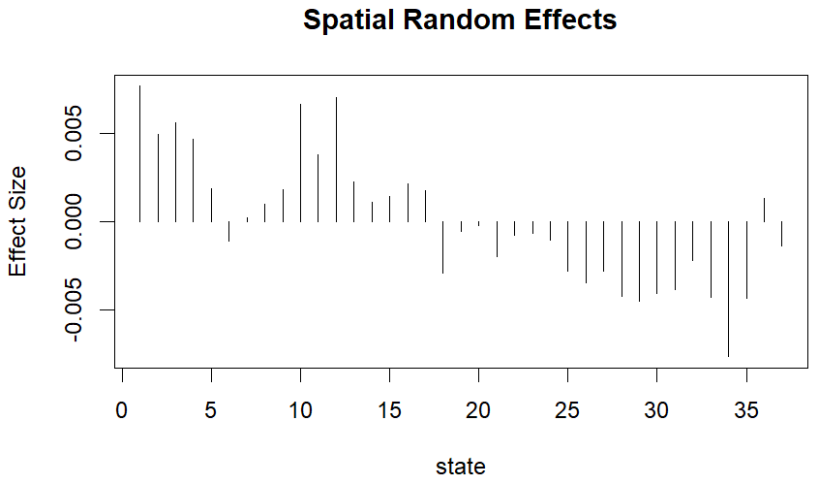

Figure 2: State based Spatial Random Effect size of U5M from Flexible Weibull AFT Model with Spatial Random Effect

The graph in figure 2 depicts the spatial random effects influencing U5M across Nigeria's states, with effect sizes ranging from approximately -0.005 to +0.005 along the y-axis. Northern and north-central states (numbers 1-13, including Benue, Abuja, Kogi, Kwara, Nasarawa, Niger, Plateau, Adamawa, Bauchi, Borno, Gombe, Taraba, and Yobe) predominantly display positive effect sizes, with states like Borno (10) and Gombe (11) showing particularly strong positive effects. These positive values indicate spatial factors in these regions contribute to increased child survival time (reduced U5M risk).

Southern states (approximately numbers 21-36, encompassing states like Abia, Anambra, Ebonyi, Enugu, Imo, Akwa-Ibom, Bayelsa, Cross River, Delta, Edo, Rivers, Ekiti, Lagos, Ogun, Ondo, and Osun) generally exhibit negative effect sizes, with several states in the 28-31 range showing some of the strongest negative effects. The final state, Oyo (37), continues this southern pattern with a negative effect size. These negative values suggest decrease survival time (increase U5M risk).

Figure 3 depicts the Non-Constant Acceleration Factor for Type of Place of Residence on U5M, showing a clear downward trend in mean effect (log of time ratio) over five-time bins spanning from 0 to 59 months (Under five years). The time ratio steadily decreases from approximately 18 at baseline to around 9 at the 40^{th} months of the child age. This declining pattern suggests that the impact of residence type (urban versus rural settings) on U5M diminishes as time progresses,

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indicating that geographical disparities in child survival outcomes become less pronounced as the child ages.

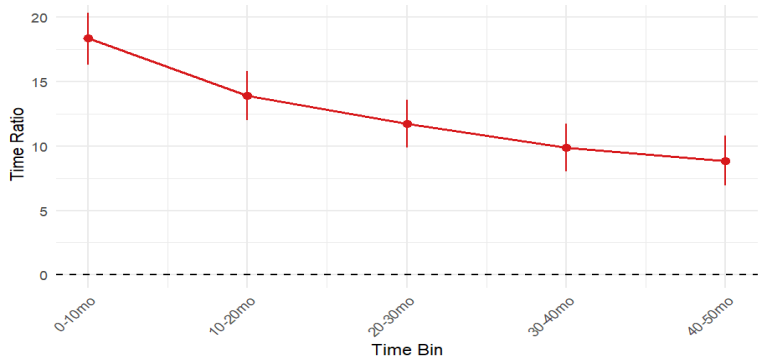

Figure 3: Non-Constant Acceleration Factor for Type of Place of Resident on U5M

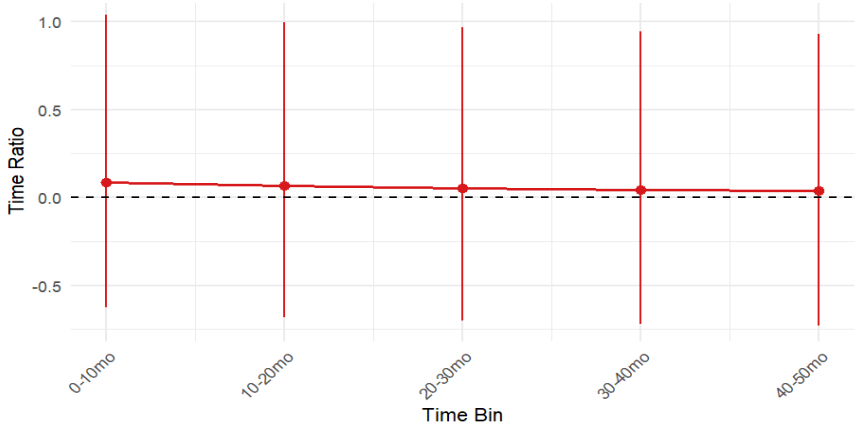

Figure 4: Non-Constant Acceleration Factor for Mosquito Net Use on U5M

Figure 4 examined the Non-Constant Acceleration Factor for Mosquito Net Use on U5M reveals that the protective effect of mosquito nets on child survival remains fairly consistent from early infancy through 59 months of child age.

Figure 5 illustrates the Non-Constant Acceleration Factor for Maternal Age at Birth on U5M across five-time bins ranging from 0 to 59 months. The plot shows central estimates that remain slightly above zero but very close to the reference line throughout all time periods, indicating a minimal and consistent effect of maternal age on child mortality as children age. The flat trajectory of these estimates suggests that whatever minor influence maternal age at birth might have on child survival

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remains fairly constant throughout early childhood development, without meaningful increases or decreases in its impact over time.

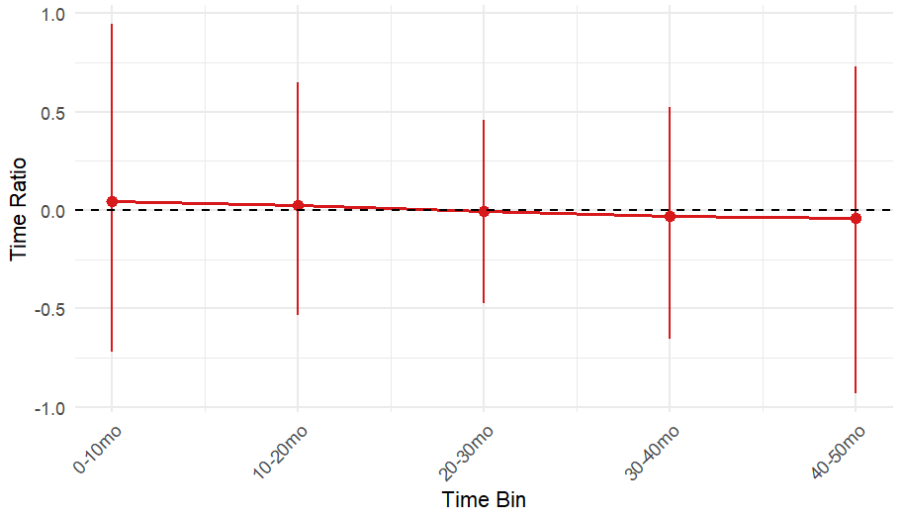

Figure 5: Non-Constant Acceleration Factor for Maternal Age at Birth on U5M

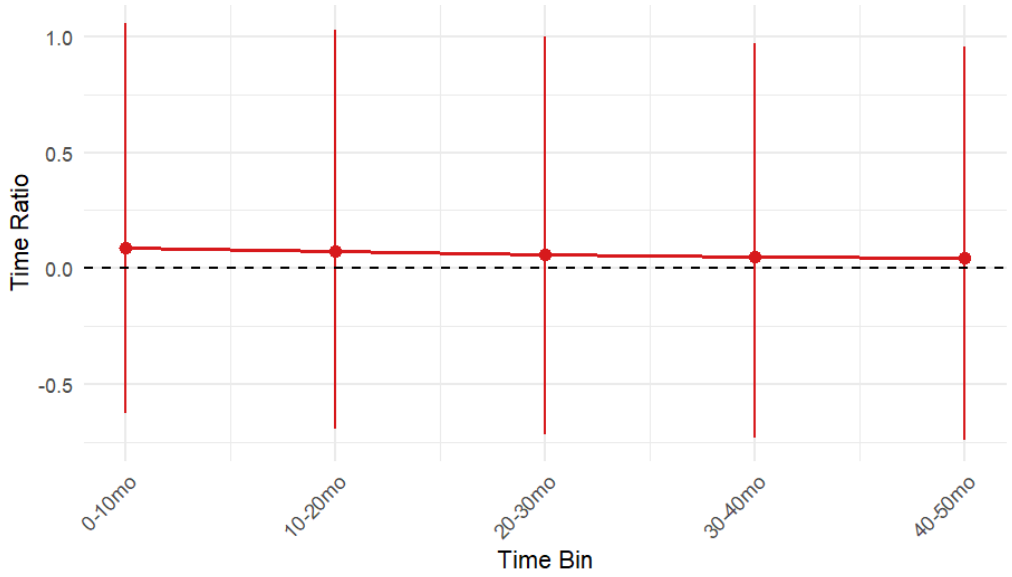

Figure 6: Non-Constant Acceleration Factor for Breastfeeding on U5M

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Figure 6 depicts the Non-Constant Acceleration Factor for Breastfeeding on U5M across five-time bins ranging from 0 to 59 months. The plotted points show a slight downward trend starting at approximately 0.1 in the earliest time bin (0-10 months) and gradually declining to near zero by the 40-59month time bin. The subtle downward slope suggests that whatever protective effect breastfeeding might have on child survival slightly diminishes as children age progresses.

4.0 Discussion of Findings

The parametric Accelerated Failure Time (AFT) model provides an alternative to the Proportional Hazards (PH) model by directly estimating covariate effects on event time through log time ratios rather than hazard ratios (Wei, 1992; Kalbfleisch, 2011). The classical AFT model assumes constant time ratios (acceleration factors) and linear covariate effects on log event time (Cox & Oakes, 1984), but these assumptions often fail in multivariable settings. While flexible PH extensions have extensively addressed non-proportional hazards and nonlinear effects (Faivre, 1999; Inaba et al., 2012; Remontet et al., 2007), similar advancements in AFT models remain underdeveloped, leaving a gap in handling complex survival data where proportionality assumptions are violated. Therefore, this study presents an improved accelerated failure time model that simultaneously incorporates non-constant acceleration factor, non-linear effect (NL) of continuous covariates and spatial random effects. These models were applied to Under-5 Mortality data extracted from the Nigerian Demographic and Health Survey. The NL effects of continuous covariate was modeled using quadratic Basis-splines (B-spline) with one interior knot while the non-constant acceleration factor was modelled using random work. The NL estimate describes how the time ratio varies with an increasing value of a continuous covariate, whereas the non-constant acceleration factor estimate informs how the strength of the covariate effect changes during the follow-up. To capture the clustering nature of the data, the Besag Intrinsic Conditional Autoregressive (ICAR ICAR) spatial random effects was utilized. The performance of the extended model was compared with existing AFT models using model comparison metrics: Deviance Information Criterion (DIC) and Watanabe-Akaike Information Criterion (WAIC).

The results of the model comparison showed that the extended parametric AFT model that simultaneous incorporate NL effect of continuous covariate, non-constant acceleration factor and spatial random effect demonstrate better performance (Table 1). This finding is agreement with findings from previous study by Pang et al. (2021).

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The study's findings align with recent methodological advancements in flexible Accelerated Failure Time (AFT) models, particularly in capturing nonlinear and time-varying effects of covariates on child survival. The nonlinear effects of maternal age, modeled via basis splines, resonate with Zhang et al. (2018), who demonstrated that penalized spline AFT models effectively capture complex relationships, such as U-shaped or threshold effects, in survival data. The protective effects of maternal age increasing with maturity suggest that traditional linear AFT models may underestimate the nuanced influence of maternal factors, reinforcing the need for flexible modeling approaches. Similarly, the strong protective effect of breastfeeding (TR=5.954) supports findings from Kang et al. (2020), whose neural network AFT model highlighted how dominant predictors like breastfeeding can overshadow other covariates, necessitating advanced methods to disentangle their effects. The dose-response relationship of birth intervals also mirrors Wu et al. (2021), who found that spline-based AFT models better detect J-shaped relationships than linear models, emphasizing the importance of flexible modeling in uncovering threshold effects in child survival.

The study's spatial random effects (precision=6.080) corroborate findings from Daniel (2021) and Fenta et al. (2025), who identified significant geographic clustering in U5M using spatial survival models. The unexplained regional heterogeneity suggests that unobserved contextual factors—such as healthcare access or environmental conditions—play a critical role, reinforcing the need for spatially explicit survival models. Additionally, the non-constant acceleration factor analysis aligns with Chen and Zhou (2007) and Orbe et al. (2020), who demonstrated that time-varying effects are common in survival analysis. The extreme variability in urban residence effects (variance=29.4117) suggests that its protective role fluctuates across childhood stages, possibly aligning with periods of heightened vulnerability (e.g., neonatal phase). This finding supports the argument by Lin et al. (2021) that standard AFT models may miss critical age-specific covariate interactions, necessitating flexible approaches like Bayesian Additive Regression Trees (BART) or time-varying coefficient models.

The socioeconomic gradients observed particularly in maternal education and wealth echo findings from Wegbom et al. (2019) and Fagbamigbe & Nnanatu (2022), who identified education and wealth as key determinants of U5M in Nigeria. The graduated protective effect of education (stronger at higher levels) aligns with Kunnuji et al. (2022), suggesting that policies targeting maternal education beyond primary schooling could yield substantial survival benefits. The

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negligible impact of maternal BMI contrasts with some literature but may reflect regional differences in nutritional dynamics, as seen in Yalew et al. (2022), where biological factors like birth weight were more predictive than maternal anthropometrics. The strong protective effect of antenatal care (TR=2.140) supports Ahmed et al. (2020) and Egbon et al. (2022), reinforcing the importance of healthcare access—a finding particularly relevant for sub-Saharan Africa, where gaps in prenatal care persist.

Finally, the study's methodological implications resonate with broader trends in survival analysis. The use of flexible splines and spatial effects aligns with Ghilagaber et al. (2013) and Fenta et al. (2025), who advocate for geo-additive and spatiotemporal models to account for unobserved heterogeneity. The findings also parallel Koissi et al. (2005) and Jaiswal et al. (2024), who emphasized the role of multilevel and frailty models in addressing clustered survival data. However, the study's novel application of non-constant acceleration factors extends existing work by quantifying how covariate effects vary across developmental stages a dimension underexplored in traditional U5M studies.

5.0 Summary and Conclusion

5.1 Summary

The work strengthens child survival analysis by applying Bayesian AFT modeling to show the effects of nonlinear terms, changing speed of change over time, and heterogeneity in space. Based on data from the 2018 Nigeria Demographic and Health Survey, B-splines, variable coefficients, and ICAR spatial effects are all included in the model. The results are better than those from traditional methods (DIC and WAIC), with breastfeeding, antenatal check-ups, and waiting between births all being important for children's survival. The analysis of spatial data suggests that more kids survive in northern Nigeria. While the advantage of living in an urban area gets weaker as time goes on, continuing to breastfeed remains a safe way to protect infants. Using this flexible model, it is possible to analyze under-five mortality in detail and shape better, data-based health actions.

5.2 Conclusion

This study extends the Accelerated Failure Time model to incorporate nonlinear effects, time-varying coefficients, and spatial dependencies for under-five mortality analysis. The extended Weibull AFT model outperformed traditional versions (lower DIC/WAIC values), identifying breastfeeding, maternal education, birth intervals, and antenatal care as key survival determinants

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with significant spatial clustering across Nigerian states. Time-varying effects revealed age-dependent protection from urban residence. These findings highlight the value of flexible modeling for capturing complex mortality patterns often missed by conventional methods. The approach provides policymakers with a robust tool for targeted interventions in high-burden regions. Future work could integrate machine learning to further improve predictions and uncover latent risk patterns.

5.3 Strength of the Study

This study offers several key strengths, including extending Accelerated Failure Time (AFT) model that simultaneously accounts for nonlinear effects of continuous covariate, non-constant acceleration factors, and spatial dependencies a methodological advancement over traditional survival models. By utilizing Bayesian inference with Integrated Nested Laplace Approximation (INLA), the study efficiently handles complex spatial survival data while providing interpretable time ratio estimates. The incorporation of both nonlinear splines and time-varying coefficients allows for a more nuanced understanding of how risk factors like breastfeeding, maternal education, and healthcare access differentially impact child survival across developmental stages and geographic regions. Additionally, the use of nationally representative Demographic and Health Survey (DHS) data enhances the generalizability of findings to similar low-resource settings. The model's superior performance, as demonstrated by lower DIC and WAIC values compared to conventional AFT models, underscores its robustness in analyzing clustered survival data with violated assumptions of conventional AFT model, making it a valuable tool for both researchers and policymakers targeting under-five mortality reduction.

5.4 Limitations of the Study

This study has several limitations. Firstly, its reliance on self-reported data from the 2018 Nigeria Demographic and Health Survey may introduce biases and fail to fully capture the complexities of under-five mortality rates. Additionally, the model may not control for all potential confounding factors, such as genetic factors, that could impact under-five mortality. Finally, while the study offers valuable insights into the Nigerian context, its findings may not be universally applicable due to Nigeria's unique socio-cultural and demographic characteristics.

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