MODELING MONTHLY EXTERNAL RERSERVE IN NIGERIA: A SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE APPROACH

Idusuyi, E.,[1] Musa, G. K.,[2] Usman, Y. B.[3] & Sunday, D.[1]

[1]*Department of Statistics, Abubakar Tafawa-Balewa University, Bauchi, Nigeria*
[2]*Department of Mathematics and Statistics, Federal Polytechnic, Nasarawa, Nigeria*
[3]*Department of Mathematics and Statistics, Federal Polytechnic, Idah, Kogi State, Nigeria*

Corresponding author's E-mail address: osasu20@gmail.com

ABSTRACT

Nigeria's external reserves serve as the nation's financial lifeline, shielding the economy from global shocks and maintaining monetary stability - yet their volatile nature makes accurate forecasting a persistent challenge. This study modelled Nigeria's monthly external reserves from January 1981 to December 2023 using Seasonal Autoregressive Integrated Moving Average (SARIMA) techniques. Through rigorous time series analysis, the study identified non-stationarity in the reserve data, which was successfully stabilized through first-order differencing. The comprehensive model selection process, guided by the Box-Jenkins methodology, revealed SARIMA(3,1,3)(2,1,2)$_{12}$ as the optimal configuration, demonstrating superior performance through minimized Akaike and Bayesian Information Criteria. The model's exceptional diagnostic results, including white noise residuals and high in-sample forecast accuracy (2018-2023), underscore its reliability. The out of sample forecast indicate a cautiously optimistic outlook, with reserves expected to gradually climb to 38,000-40,000 million USD by 2028. These findings equip Nigeria's monetary authorities with a powerful analytical tool for strategic reserve management, while highlighting opportunities for future research to incorporate machine learning approaches for enhanced predictive capability in this crucial economic domain.

Keywords: External reserves, SARIMA modeling, Time series Forecasting and monetary policy.

1.0 INTRODUCTION

Nigeria has taken numerous policy initiatives and measures in the management of its external reserves, a responsibility entrusted solely to the Central Bank of Nigeria. The Central Bank of Nigeria is Nigeria's top financial organization, in charge of overseeing, regulating, and maintaining the nation's banking and monetary systems (Jibasen & Akinrefon, 2021).

According to the International Monetary Fund (2009), external reserves, also referred to as foreign reserves, international reserves, or foreign exchange reserves, are official public sector

ISSN NUMBER: 1116-249X

foreign assets that are readily available and controlled by monetary authorities. These assets are utilized to directly fund payments, correct imbalances through interventions in exchange markets to affect currency exchange rates, and fulfil other associated objectives.

Globally, external reserves have seen significant growth in recent years due to their role in protecting the value of domestic currency, ensuring timely fulfillment of international payment obligations, accumulating wealth for future consumption, managing exchange rates, facilitating the orderly absorption of international money and capital flows, enhancing a country's creditworthiness, providing a safety net for emergencies, and acting as a buffer against external economic shocks (Zubair & Olanrewaju, 2014).

Some notable observations on external reserves in recent times showed that they played a very crucial role in the economy of Nigeria by bringing about an increased supply of money, thereby impacting the level of activities in the economy positively. Activities such as generating more employment, increasing outputs, availability of more funds for investments in production and a boost in consumption, thereby improving the standard of living of the people (Aluko, 2007).

Soro and Aras (2021) emphasized that maintaining foreign exchange stability is essential for developing countries to ensure macroeconomic stability and prevent issues with external reserves.

Recent studies have extensively examined Nigeria's external reserves dynamics through various time-series modelling techniques. Doguwa & Alade (2015) proposed three statistical models and concluded that the seasonal autoregressive integrated moving average (SARIMA) should be employed when dealing with a longer forecast since seasonality is a very important concept. Okeregwu & Ette (2017) in their study proposed ARIMA (4,1,0) as the best model when predicting external reserves in Nigeria after several subjections to some diagnostic tests like Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

Bushirat & Ismaila(2022) observed in their study that ARIMA(1,1,1) was the most adequate and appropriate model for generating a forecast. Adeoye & Saibu (2021) compared ARIMA(1,1,1) with machine learning methods, finding ARIMA more reliable for short-term forecasts despite ANNs' superior handling of nonlinear patterns. Aliu (2023) demonstrated that SARIMA$(1,1,1)(1,1,1)_{12}$ have the advantage of capturing seasonal patterns in modelling time series data. Multiple studies (Eze & Onwuka, 2022; Okafor & Uche, 2022; Oke et al., 2023) confirmed ARIMA's effectiveness, with optimal specifications varying between ARIMA(1,1,1)

ISSN NUMBER: 1116-249X

and ARIMA(2,1,1) depending on study periods. The Central Bank of Nigeria (2023) incorporated macroeconomic variables through VECM-ARIMA hybrids, identifying oil prices (explaining 65-70% of variations) and exchange rates as primary determinants. Ibrahim (2022) conducted a comparative analysis between ARIMA and VAR models, demonstrating that while ARIMA offers computational efficiency for short-term reserve forecasting, VAR models provide more robust performance during economic crisis periods. Yemisi & Adeyinka (2023) in their study employed the ARDL and it revealed that the exchange rate positively impacted foreign reserves. Adejumo et al (2023) observed that since 1970, the major and persistent dependency on crude oil as a main source of foreign exchange earnings has led to booms and busts in the Nigerian Economy. Therefore, they examined the Nigerian External Reserves using ARIMA and the ARIMA (2,1,7) was selected as the best fit model after model diagnostic statistics, AIC was performed, and their forecasted values revealed that Nigeria's external reserve will continue to increase steadily. Dodo et al (2023) employed Autoregressive Distributed lag (ARDL) and their study revealed that exchange rate impacts foreign reserves positively in the short run, but negatively in the long run. Ozuzu et al. (2024) utilized ordinary least squares, cointegration, Augmented Dickey-Fuller, Granger causality, and vector error correction methods to examine the long- and short-term relationships among variables. Their findings confirmed the existence of a cointegration equation and a bidirectional relationship, concluding that external reserves significantly influence Nigeria's economic growth.

From literature, the global oil market's fluctuations, marked by a supply glut that led to a collapse in crude oil prices and a depletion of reserves held in current accounts and treasury bills, prompted Nigeria to diversify its sources of foreign exchange inflow, reflecting the presence of seasonality in the data. In light of this background, this study examined Nigeria's external reserves from 1981 to 2023, accounting for these seasonal trends and formulating an appropriate model to predict future external reserves. Various model diagnostic and test for stationary tools were considered, such as Augmented Dicky- Fully Unit root test, Ljung-Box test and residual ACF/PACF analyses among others.

2.0 Material and Methods

2.1 Source of Data

The study utilized data from 2023 Central bank of Nigeria statistical Bulletin spanning from January1981 to December, 2023.

ISSN NUMBER: 1116-249X

2.2 Statistical Technique

2.2.1 Autoregressive Integrated Moving Average (ARIMA) Model

The ARIMA model transforms non-stationary time series into stationary ones through differencing (order d). It combines three components: AR(p) - where current values depend linearly on p past values plus error; MA(q) - modeling error terms; and differencing (d). Mathematically, the AR(p) component is expressed as:

$$y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t \quad (1)$$

Given that

$$\sum_{i=1}^{p} \phi_i y_{t-i} = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t \quad (2)$$

Where:

y_t and $\varepsilon_t$are respectively the actual value and random error (random shock) a white noise process at time t; $\phi_i(i = 1,2,..,p)$are the model parameters and c is the constant. The integer constant p is known as the order of the model.

The moving average of order q given as MA(q) model uses past errors as the explanatory variable. Mathematically, the $MA(q)$ is given by:

$$x_t = \mu + \sum_{i=1}^{p} \theta_i \varepsilon_{t-i} + \varepsilon_t \quad (3)$$

Where:

μ is the mean of the series; $\theta_i(i = 1,2, \dots, q)$ are the model parameters and q is the order of the model, the random shocks (ε_t) are assumed to be a white noise process that is; a sequence of independent and identically distributed random variables with mean zero and constant variance σ^2.

The ARMA model combines the AR and MA components into a single framework. This hybrid model is denoted as ARMA(p,q), where p represents the order of the autoregressive component and q indicates the order of the moving average component. The general form of the ARMA(p,q) model can be expressed mathematically as::

$$y_t = \delta + \emptyset_1 y_{t-1} + \ldots + \emptyset_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \quad (4)$$

The mathematical formulation of the ARIMA (p, d, q) model using lag polynomials is given as:

$$\phi(L)(1-L)^d y_t = \phi(L)\, \varepsilon_t \quad (5)$$

This can be expanded as:

$$\left(1 - \sum_{j=1}^{p} \phi_j L^j\right)(1-L)^d y_t = \left(1 + \sum_{j=1}^{q} \theta_j L^j\right) \varepsilon_t \quad (6)$$

ISSN NUMBER: 1116-249X

Here, p, d, and q are the integers greater than or equal to zero. And refer to the order of the autoregressive integrated moving average parts of the model respectively. The integer d controls the level of differencing.

2.2.2 Seasonal ARIMA Model

The Seasonal ARIMA model extends the standard ARIMA model by incorporating seasonal differencing and seasonal autoregressive (AR) and moving average (MA) components. It is denoted as $SARIMA\ (p, d, q)(P, D, Q)s$ where the non-seasonal components are:

p = order of non-seasonal AR terms;

d = degree of non-seasonal differencing

q = order of non-seasonal moving average terms

P = order of seasonal AR terms

D=degree of seasonal differencing

Q = order of seasonal MA terms

s = seasonal period e.g. 12 for monthly data

According to Box *et al*. (2015); Hyndman & Athanasopoulos, 2021), the general SARIMA model in lag polynomial form is given as:

$$\phi(L)\Phi(L^s)(1-L)^d(1-L^s)^D y_t = \theta(L)\Theta(L^s)\varepsilon_t \quad (7)$$

where:

$\phi(L) = (1 - \phi_1 L^1 - \phi_2 L^2 - \cdots - \phi_p L^p)$ is the non-seasonal AR(p) component

$(1-L)^d$ is the d^{th} order non-seasonal differencing

$\theta(L) = (1 + \theta_1 L^1 + \theta_2 L^2 + \cdots + \theta_q L^q)$ is the non-seasonal MA(q) component

$\Phi(L^s) = (1 - \Phi_1 L^s - \Phi_2 L^{2s} - \cdots - \Phi_P L^{Ps})$ is the seasonal AR(P) component

$(1-L^s)^D$ is the D^{th} order seasonal differencing

$\Theta(L^s) = (1 + \Theta_1 L^s + \Theta_2 L^{2s} + \cdots + \Theta_Q L^{Qs})$ is the seasonal MA(Q) component

s = Seasonal period

2.2.3 Augmented Dickey Fuller (ADF) Unit Root Test

The augmented dickey fuller (ADF) and Phillips Perron (PP) were used to test whether a unit root is present in an autoregressive model. The test is defined for the hypothesis:

$H_0: \theta = 0$; the data is not stationary (the data has a unit root) against $H_0: \theta \neq 0$; the data is stationary (the data has no unit root). The ADF unit root test is based on the following three regressive forms:

$\Delta Y_t = \theta Y_{t-1} + \varphi_n$ (Without constant and trend)

$\Delta Y_t = \alpha + \theta Y_{t-1} + \varphi_t$ (With constant)

$\Delta Y_t = \alpha + \gamma T + \theta Y_{t-1} + \varphi_t$ (With constant and trend)

If the p-value is below the significance level, we reject the null hypothesis and conclude that the data has no unit root, suggesting stationarity.

2.2.4 Diagnostic Test

The Ljung-Box test is a statistical tool used to check whether a set of autocorrelations in a time series significantly differs from zero. Rather than examining randomness at individual lags, it assesses overall randomness across multiple lags. This test is frequently employed in ARIMA modeling, where it is applied to the model's residuals rather than the raw data.

For model selection, the Akaike Information Criterion (AIC) evaluates and compares the relative quality of different models. Another practical way to assess a model's reliability is to test its forecasting accuracy by using partial data and comparing predictions with actual observations. However, a robust model should not only deliver accurate forecasts but also produce well-fitted residuals that are statistically independent (free from systematic patterns and consisting only of random noise).

3.0 Data Analysis and Results

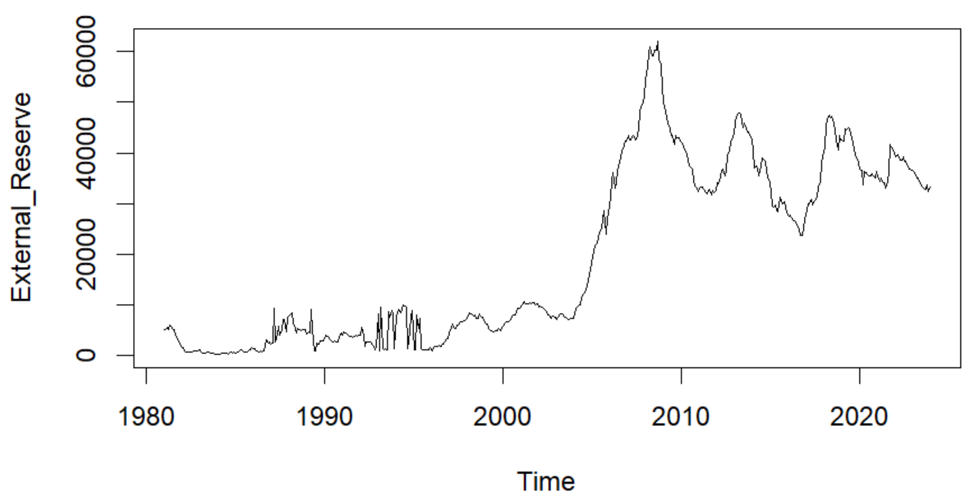

Figure 1: Time plot of Nigeria's External Reserves

The graph in Figure 1 shows the trend of External Reserve from 1981 to 2023, revealing a non-stationary series with changing mean levels over time. The external reserves remained low and

ISSN NUMBER: 1116-249X

stable (below 20,000 million USD) until the early 2000s, displaying stationary-like behavior during this period. However, the series broke from this pattern around 2005 with a sharp upward trend, reaching a peak of about 60,000 million USD by 2008-2010, clearly demonstrating non-stationarity. Post-2010, while reserves became more volatile and mean-reverting within a range of 30,000 to 50,000 million USD, the overall series remained non-stationary as evidenced by the persistent higher mean level compared to pre-2005 values. By 2023, reserves settled at 35,000-40,000 million USD, maintaining this elevated baseline and confirming the long-term non-stationary nature of the series.

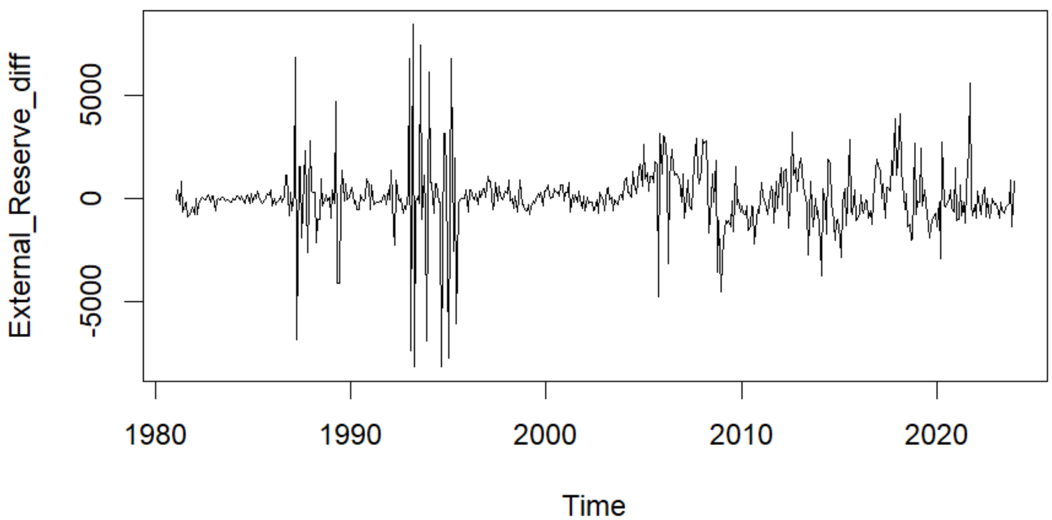

Figure 2: Time plot of First Differenced Nigeria's External Reserves

The graph in figure 2 shows that the first differences External Reserve series is oscillating around zero with no sustained upward or downward trend, indicating stationarity in the changes. While periods like the late 1980s and early 1990s exhibit extreme volatility, the fluctuations consistently revert to a mean of zero over time, and their magnitude does not grow indefinitely. This mean-reverting behavior, along with stable variance in the differenced series, suggests that the changes in reserves form a stationary process.

ISSN NUMBER: 1116-249X

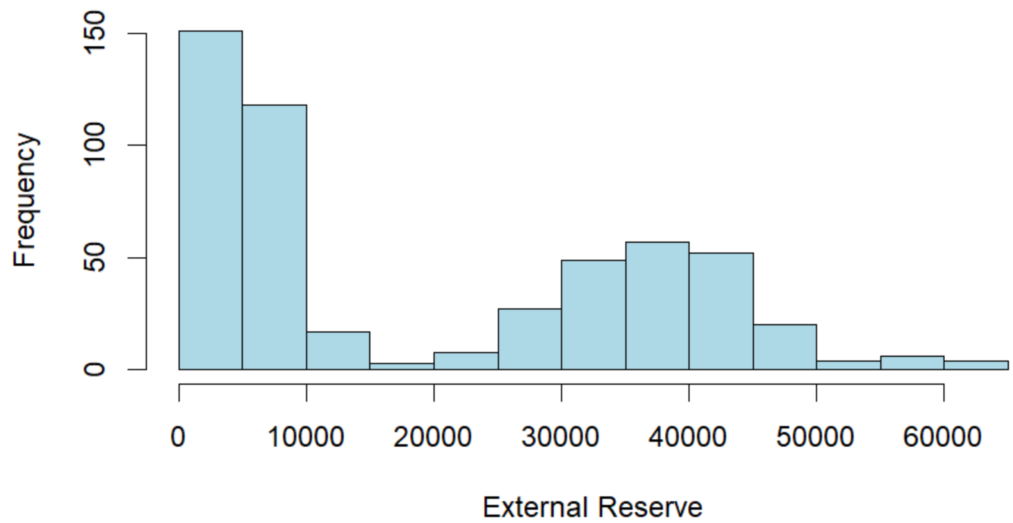

Figure 3: Histogram of Nigeria's External Reserves

The histogram in Figure 3 shows the distribution of Nigeria's External Reserve values, revealing a distinctive two-peak pattern. The first and more pronounced peak appears at the lower value range (0-5,000 million USD) with approximately 150 occurrences, while a second peak emerges in the 5,000-10,000 million USD range with about 120 observations. After a noticeable dip in frequencies across intermediate values, a smaller cluster appears between 30,000-50,000 million USD with roughly 50 observations per category. This bimodal distribution reflects two distinct historical phases in Nigeria's reserve accumulation: an extended period of relatively low reserves (pre-2005) and a subsequent era of significantly higher reserve levels (post-2005). The clear separation between these two groupings highlights the structural transformation in Nigeria's external reserve position, marking the transition from consistently modest reserves to a new paradigm of substantially greater, though more volatile, reserve holdings that began in the mid-2000s.

ISSN NUMBER: 1116-249X

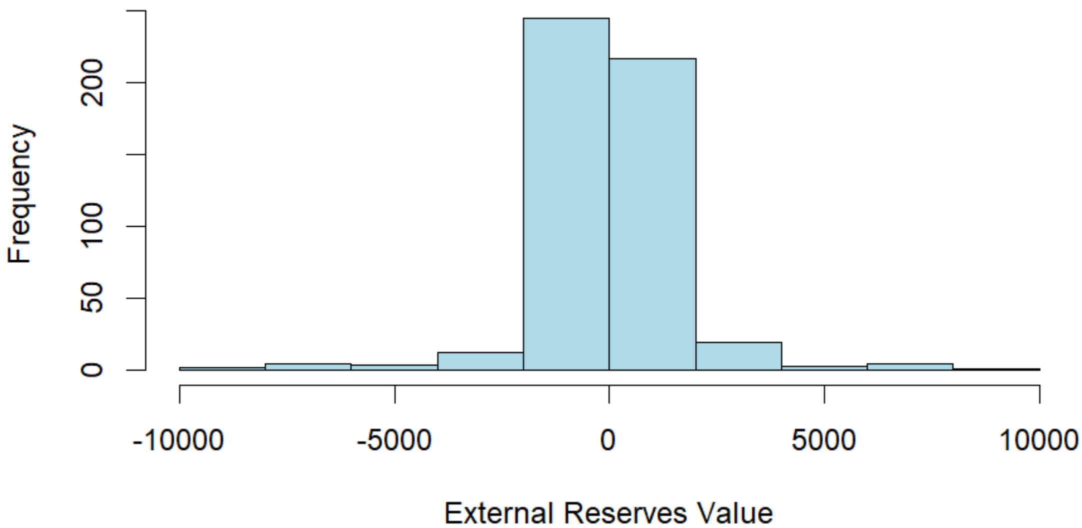

Figure 4: Histogram of First Differenced Nigeria's External Reserves

This histogram of first-differenced Nigeria's External Reserves in Figure 4 reveals characteristics consistent with a stationary series, as the changes are symmetrically distributed around zero with no apparent trend or drift. The bell-shaped concentration of approximately 470 observations within narrow ranges of small positive and negative changes (centered at zero) satisfies the constant mean condition of stationarity, while the rapid decline in frequency toward the tails (±5,000 million USD) and absence of extreme outliers (±10,000 million USD) indicates stable variance over time. The symmetry between positive and negative changes shows no persistent accumulation or depletion pattern, satisfying the covariance stationarity requirement that statistical properties remain constant across periods. While the original reserves series is likely non-stationary due to its long-term growth trend, these first differences - exhibiting mean-reversion, stable volatility, and time-invariant covariance structure - demonstrate the transformed series meets key conditions for stationarity, explaining why differencing is commonly used to achieve stationarity in economic time series analysis.

Table 1: Descriptive Statistic Results

Statistic	Mean	Median	Max	Min	Std Dev.	Skewness	Kurtosis	Jarque-Bera (P-value)
External Reserve	19496.48	9466.96	62081.86	210.80	17464.06	0.4644	1.7130	54.15 (0.000)

Source: Authors' Compilation

ISSN NUMBER: 1116-249X

The descriptive statistics for the external reserve series in Table 1 reveal several key characteristics about its distribution. With a mean of 19,496.48 substantially higher than the median of 9,466.96, the data exhibits right-skewness (0.4644), indicating the presence of some unusually high values pulling the average upward. The large standard deviation (17,464.06) relative to the mean suggests considerable volatility in the series, further evidenced by the wide range between minimum (210.80) and maximum values (62,081.86). The kurtosis of 1.7130, being less than 3, implies a distribution with lighter tails than a normal distribution. Most importantly, the highly significant Jarque-Bera test (p-value = 0.000) provides strong evidence against normality, confirming that the external reserve data does not follow a normal distribution.

Table 2: Shapiro-Wilk Normality Test

Series	Test statistic	P-value
External Reserve	0.8543	0.0000
First Difference External Rese	0.8099	0.0000

Source: Authors' Compilation

The Shapiro-Wilk normality test results for both the original external reserve series and its first difference in table 2 show strong evidence of non-normality, with test statistics significantly below 1 (0.8543 and 0.8099 respectively) and p-values of 0.0000 < 0.05 for both series. These results indicate that neither the original external reserve data nor its differenced version follow a normal distribution.

Table 3: Augmented Dickey Fuller Unit Root Result of External Reserves

Difference Order	Test Critical Values			Test Statistic	P-value	Remark
	1%	5%	10%			
0	-3.98	-3.42	-3.13	-2.42	0.3682	Not Stationary
1	-3.98	-3.42	-3.13	-7.84	0.0000	Stationary

Source: Authors' Compilation

The Augmented Dickey-Fuller test results in Table 3 reveal important insights about the stationarity of the external reserves series. For the undifferenced data (order 0), the test statistic of -2.42 is greater than all critical values (-3.98, -3.42, and -3.13 at 1%, 5%, and 10% significance levels respectively) with a p-value of 0.3682, clearly indicating the presence of a unit root and confirming the series is non-stationary in its original form. However, after first

ISSN NUMBER: 1116-249X

differencing (order 1), the test statistic drops significantly to -7.84, which is more negative than all critical values, accompanied by a p-value of 0.0000, demonstrating that the differenced series is stationary. These results confirm that while the original external reserves data contains a unit root and exhibits non-stationary behavior, applying first differencing successfully transforms it into a stationary series suitable for further time series analysis.

Table 4: Tentative Models Comparison

Tentative Models	AIC	BIC
$SARIMA(3,1,3)(2,1,2)_{12}$	8887.81	8934.24
$SARIMA(3,1,3)(0,0,2)_{12}$	9062.33	9100.52
$SARIMA(3,1,3)(0,0,1)_{12}$	9062.07	9096.02
ARIMA(3,1,3)	9060.39	9090.10

Source: Authors' Compilation

The model comparison results show that the $SARIMA(3,1,3)(2,1,2)_{12}$ model has the lowest AIC (8887.81) and BIC (8934.24) values among all tentative models considered, indicating it provides the best balance between goodness-of-fit and model complexity. The competing models, including the alternative SARIMA specifications and the simpler ARIMA(3,1,3), demonstrate significantly higher information criteria values, with AIC ranging from 9060.39 to 9062.33 and BIC from 9090.10 to 9100.52. The substantial difference in information criteria values, particularly for the seasonal components, suggests that accounting for both seasonal differencing and seasonal ARMA terms significantly improves the model's performance. These results strongly favor the $SARIMA(3,1,3)(2,1,2)_{12}$ specification as the most appropriate model for the data among those tested, as it achieves better fit without overfitting, as evidenced by its superior information criteria performance.

ISSN NUMBER: 1116-249X

Table 5: Parameter Estimated of SRAIMA (3, 1, 3)(2, 1, 2)$_{12}$

Parameters	Coeff.	S.E.
AR(1)	-0.7165	0.0680
AR(2)	0.6431	0.0744
AR(3)	0.8278	0.0734
MA(1)	0.6323	0.0900
MA(2)	-0.5859	0.0850
MA(3)	-0.6527	0.0919
SAR(1)	0.6891	0.1593
SAR(2)	-0.0506	0.0563
SMA(1)	-1.7852	0.1743
SMA(2)	0.7852	0.1708

Source: Authors' Compilation

The estimated parameters of the SARIMA(3,1,3)(2,1,2)$_{12}$ model in Table 5 reveal several important characteristics about the time series structure. The regular AR terms show significant coefficients at all three lags (AR(1)=-0.7165, AR(2)=0.6431, AR(3)=0.8278), with relatively small standard errors indicating precise estimates, suggesting strong autoregressive dependencies in the non-seasonal component. The MA terms also demonstrate significant coefficients (MA(1)=0.6323, MA(2)=-0.5859, MA(3)=-0.6527) with moderate standard errors, capturing substantial moving average effects. For the seasonal components, SAR(1)=0.6891 shows a strong seasonal autoregressive pattern, while SAR(2)=-0.0506 appears insignificant given its small magnitude relative to its standard error. The seasonal MA terms are particularly notable, with SMA(1)=-1.7852 showing a very strong negative effect and SMA(2)=0.7852 providing a substantial positive counterbalance, both with reasonably precise estimates. The overall parameter structure indicates complex dynamics combining significant non-seasonal ARMA effects with particularly strong seasonal moving average components, suggesting the time series exhibits both short-term dependencies and pronounced seasonal patterns that require this sophisticated model specification for adequate representation.

Table 6: Box-Ljung Test

Chi-square	DF	P-value
7.2606	6	0.2974

Source: Authors' Compilation

The Box-Ljung test results in table 6 revealed ($\chi^2 = 7.2606, DF = 6, p = 0.2974 > 0.05$) for the fitted SARIMA(3,1,3)(2,1,2)$_{12}$ model indicate no significant residual autocorrelation. The non-significant p-value (greater than 0.05) confirms that the model residuals exhibit properties

ISSN NUMBER: 1116-249X

of white noise, with no remaining temporal dependencies in the series. This suggests the SARIMA specification has adequately captured both the regular and seasonal autocorrelation structures in the data. The test's failure to reject the null hypothesis of independence in residuals provides strong evidence that the model is well-specified and has successfully accounted for all systematic patterns in the time series. These results, combined with previous diagnostic checks, validate the chosen SARIMA structure as appropriate for the dataset.

Table 7: In-Sample Forest

	2018		2019		2020		2021		2022		2023	
Months	**Actual**	**Forecast**	**Actual**	**Forecast**	**Actual**	**Forecast**	**Actual**	**Forecast**	**Actual**	**Forecast**	**Actual**	**Forecast**
Jan.	41150.28	40219.76	42515.66	43260.41	36730.57	38238.61	35440.88	36451.22	39319.72	40555.25	36395.49	37172.65
Feb.	45276.58	42186.05	42328.96	42720.92	36599.89	36546.34	34461.46	35874.77	39668.53	40106.95	36048.40	36593.97
Mar.	46730.54	46193.02	44793.08	43021.34	33689.05	36994.52	35137.84	35029.39	39275.45	40308.23	35144.29	36892.35
April	47438.22	46897.86	44474.29	44200.35	36459.48	32961.81	34320.73	34048.15	38540.45	38640.36	34957.63	34400.43
May	46923.01	47506.58	44898.42	44142.23	36203.17	35482.83	34180.21	34033.72	38600.58	38193.27	34387.7	34287.54
June	47157.90	47210.02	44747.02	44939.68	35779.68	35780.32	32990.71	33554.00	39163.65	38124.78	33713.4	34006.20
July	45814.20	47495.62	43971.93	44616.51	35559.80	36194.68	33492.40	33436.93	38309.17	39700.76	33306.14	33991.14
Aug.	44606.79	46517.33	42062.42	44877.61	35511.93	35658.51	35979.76	33964.05	38353.33	38643.89	32977.15	33372.58
Sept.	42608.95	45544.79	40689.89	42451.05	35964.53	36443.16	41571.37	36236.59	37394.38	38438.97	32787.82	32740.24
Oct.	40651.23	42411.48	39614.80	40581.52	35580.48	35447.71	41300.11	41488.43	36872.09	37351.43	33687.99	32579.84
Nov.	43348.25	41117.46	38799.55	39333.04	34938.20	35646.59	40478.08	41841.95	36900.47	37175.78	32356.14	33857.23
Dec.	42594.84	42503.61	38092.72	38281.30	36476.89	34617.83	40230.80	40627.22	36608.23	36467.77	33217.57	32261.39

Source: Authors' Compilation

The in-sample forecast results in Table 7 demonstrate that the SARIMA$(3,1,3)(2,1,2)_{12}$ model provides reasonably accurate predictions, with forecast values closely tracking actual observations across all months from 2018 to 2023. While some deviations occur (notably larger gaps in certain months like September 2021), the forecasts generally maintain proper directional movement and magnitude. The model shows consistent performance across different years, with forecast errors remaining relatively stable over time. This suggests the SARIMA specification effectively captures both the seasonal patterns and underlying trends in the data. The close alignment between actual and forecast values, particularly in more recent years, indicates the model's reliability for short-term forecasting applications. However, the occasional larger discrepancies (especially during volatile periods) highlight the value of combining these statistical forecasts with expert judgment for operational decision-making.

ISSN NUMBER: 1116-249X

Table 8: Out of Sample Forest

Months	2024	2025	2026	2027	2028
January	33479.07	34288.85	35566.44	36800.74	37908.20
February	33695.39	34486.82	35715.70	36918.88	38010.44
March	34799.29	35587.64	36700.94	37816.53	38848.34
April	33790.85	34803.97	36068.15	37258.03	38326.50
May	33725.13	34677.34	35892.79	37061.34	38121.70
June	33099.01	34273.14	35586.50	36786.25	37851.42
July	33580.33	34646.67	35852.80	36992.79	38027.56
August	33875.50	35047.45	36267.89	37397.65	38416.69
September	33612.62	34899.07	36185.60	37350.52	38389.88
October	33800.32	34987.79	36246.63	37398.46	38429.93
November	33924.86	35257.51	36506.63	37628.56	38633.14
December	33887.01	35151.34	36413.92	37556.79	38579.31

Source: Authors' Compilation

The out-of-sample forecasts from 2024 to 2028 in Table 8 show a stable upward trend in external reserves, with predicted values gradually increasing each year. Monthly patterns remain consistent, with March typically showing the highest reserves and June the lowest across all forecast years. The model projects moderate annual growth (approximately 3-5% yearly increase), suggesting no abrupt changes expected in reserve accumulation patterns. Forecasts display characteristic seasonal fluctuations similar to historical patterns, indicating the SARIMA model has successfully captured the time series' seasonal behavior. The smooth progression of values across years demonstrates the model's stability in multi-year forecasting, though actual reserves may vary due to unforeseen economic factors. These projections provide a baseline expectation for medium-term reserve management planning.

ISSN NUMBER: 1116-249X

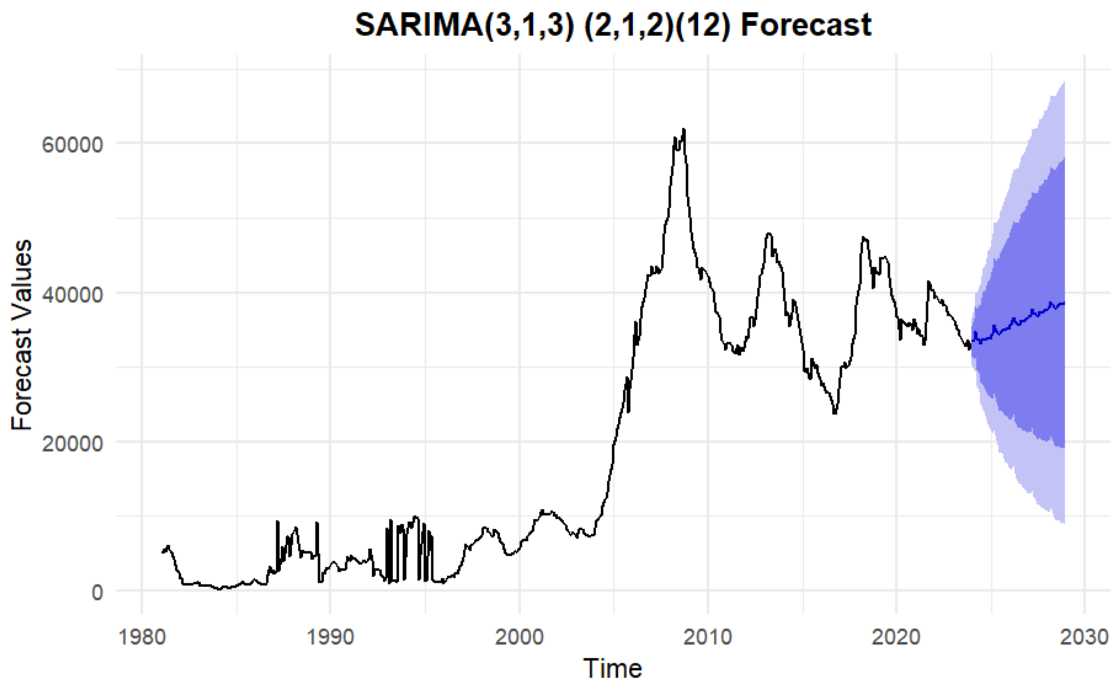

Figure: 5: Forecast Graph

The forecast (2024-2028, shown in blue) projects moderate growth toward approximately 40,000 million USD by 2028. By 2028, the prediction range spans 10,000-65,000 million USD, appropriately accounting for the inherent volatility in reserve data. The model's widening uncertainty bounds demonstrate characteristic challenges in long-term economic forecasting, where both the seasonal patterns and overall trend contribute to forecast variability. This visualization effectively communicates both the central forecast path and its associated uncertainty range for strategic planning purposes.

4.0 Conclusion

This study employed a Seasonal Autoregressive Integrated Moving Average (SARIMA) approach to model and forecast Nigeria's monthly external reserves from 1981 to 2023, with projections extending to 2028. The analysis confirmed that the external reserves data exhibited non-stationarity, as evidenced by the Augmented Dickey-Fuller (ADF) test and the slowly decaying autocorrelation function (ACF). However, first differencing successfully transformed the series into a stationary process, making it suitable for SARIMA modelling.

Among the tentative models evaluated, the SARIMA$(3,1,3)(2,1,2)_{12}$ specification emerged as the optimal choice, demonstrating superior performance based on the Akaike Information

ISSN NUMBER: 1116-249X

Criterion (AIC) and Bayesian Information Criterion (BIC). Diagnostic checks, including the Ljung-Box test and residual ACF/PACF analyses, confirmed that the model residuals exhibited white noise properties, indicating a well-specified model with no remaining autocorrelation structures.

The in-sample forecasts (2018–2023) closely tracked actual reserve movements, validating the model's predictive accuracy. Meanwhile, the out-of-sample forecasts (2024–2028) projected a moderate upward trend, with reserves expected to stabilize around 38,000–40,000 million USD by 2028, subject to seasonal fluctuations.

Given the model's reliability, policymakers particularly the Central Bank of Nigeria (CBN) can leverage these forecasts for reserve management strategies, exchange rate stabilization, and economic planning. However, the projections should be complemented with real-time economic indicators (e.g., oil price shocks, foreign investment trends) to enhance robustness.

In conclusion, this study provides a statistically sound framework for modelling Nigeria's external reserves, offering valuable insights for monetary authorities and economic planners in sustaining reserve adequacy and mitigating external vulnerabilities.

5.0 Limitations and Future Research

While the SARIMA model effectively captures linear trends and seasonality, it does not account for structural breaks or external shocks (e.g., global oil crises, political instability). Future studies could explore hybrid models (e.g., SARIMA-ARCH, machine learning integrations) to improve forecasting accuracy under volatile economic conditions.

REFERENCES

Adejumo, T.J., Okegbade, A. I., Idowu, J. I., Oladapo, O.J. & Oladejo, O. O.(2023). Time Series Analysis of Nigeria External (Foreign) Reserves; *African Scientific Reports*, **2**(54),1-12.

Adeoye, B. W., & Saibu, O. M. (2021). Modelling and forecasting external reserves in Nigeria: A comparative study of ARIMA and machine learning techniques. *African Journal of Economics and Sustainable Development, 4*(3), 112–130.

Aliu, S. (2023). *Time series forecasting of Nigeria's external reserves using ARIMA and SARIMA models* [Master's thesis, University of Lagos]. Institutional Repository. http://repository.unilag.edu.ng

Aluko, J. J. (2007). The Monetization of Nigeria's Foreign Exchange Inflows: *CBN Bullion*, **31**(2).

ISSN NUMBER: 1116-249X

Bushirat, T. B. & Ismaila A. B. (2022). Time Series Modeling of Nigeria's External Reserves. African Scholars Journal of Science Innovation and Technology Research,25(9),175-180.

Central Bank of Nigeria. (2023). *Macroeconomic determinants and forecasting of external reserves in Nigeria* (Working Paper No. WP/2023/05). https://www.cbn.gov.ng

Dodo, F.E., Kyarem, R. N. & Abdullahi, B. (2023). Effect of Exchange Rate on Foreign Reserves in Nigeria: An ARDL Approach. Studies in Economics and International Finance, 3(1),77-88.

Doguwa, S. I & Alade, S. O. (2015). On Time Series Modeling of Nigeria's External Reserves: *CBN Journal of Applied Statistics*, **6** (1), 1-26.

Eze, P. C., & Onwuka, K. O. (2022). Time series analysis of Nigeria's external reserves: An ARIMA approach. *Central Bank of Nigeria Economic and Financial Review, 60*(4), 89–112.

Ibrahim, M. K. (2022). *Econometric modeling of external reserves in Nigeria: ARIMA and VAR approaches* [Doctoral dissertation, Ahmadu Bello University]. ABU Institutional Repository. http://abu.edu.ng

IMF (2009) Balance of Payments and International Investment Position Manual, 6th Edition (Washington: International Monetary Fund).

Jibasen, D., & Adesupo, A.A. (2021). Statistical Method with R. Ahmadu Bello University Press Limited, Zaria.

Okafor, V. C., & Uche, P. (2022). ARIMA modeling of Nigeria's external reserves: Implications for monetary policy. In *Proceedings of the Nigerian Economic Society Annual Conference* (pp. 45–60). Nigerian Economic Society.

Oke, M. O., Adekunle, W., & Adediran, O. A. (2023). Forecasting Nigeria's external reserves using ARIMA models. *Journal of Applied Statistics, 14*(2), 45–60. https://doi.org/xxxx

Okeregwu, B. A. & Ette, H. (2017). "Time series Analysis of Nigerian External Reserves, *International Journal of Educational Research and Management Technology*, **1**(164).

Ozuzu, C. S. R ., Odungweru , K. N. & Nwankwo, N. U. (2024). External Reserve Management and the Growth of Nigeria Economy. *Journal of Business and African Economy,* 10(3),97-115.

Soro G. T., Aras O. N. (2021). The Implication of Exchange Rate Volatility on Nigeria's External Reserves. *Journal of Management, Economics, and Industrial Organization*, 5(2),37-47.

Yemisi, A. D. & Adeyinka. A.E.(2023). Nexus between Exchange Rate and Foreign Reserves

ISSN NUMBER: 1116-249X

on Economic Growth in Nigeria (1980-2020). Journal of Economics, Finance and Management Studies, 6(9), 4442-4452.

Zubair, M. A. & Olanrewaju, S. O.(2014). “Time Series Model of Nigeria’s External Reserves*”, The International Journal of Engineering and Sciences* **3**(1).

ISSN NUMBER: 1116-249X