Comparative Analysis on the Performance of MARFIMA and ARTFIMA in Forecasting the Nigerian All Share Index

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Abstract

The Nigeria All Share Index (NASI) is a critical benchmark for the country's stock market performance, exhibiting complex dynamics characterized by nonstationarity, and long range dependence (LRD). Traditional time series models often failed to capture these features adequately. This study conducts a comparative analysis of two advanced fractional integration models Modified Autoregressive Fractionally Integrated Moving Average (MARFIMA) and Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) in modeling and forecasting NASI. The MARFIMA model introduces a sequential differencing filter to address the limitations of classical ARFIMA models, such as slow convergence and truncation errors, while ARTFIMA incorporates tempered fractional differencing to handle heavy-tailed distributions. Using daily NASI data from January 2000 to January 2019, we estimate model parameters via Whittle estimator and evaluate performance using Akaike Information Criterion (AIC), Schwarz Bayesian Information Criterion (SBIC), Root Mean Square Error (RMSE), and Normalized Mean Square Error (NMSE). Results indicate that MARFIMA (2, 1.2, 2) outperforms ARTFIMA (2, -0.4, 2) in model fit (AIC: 53,225.23 vs. 72,046.49) and forecast accuracy (RMSE: 217.155 vs. 435.9115). The superior performance of MARFIMA is attributed to its ability to efficiently remove trends while preserving long memory, making it a robust tool for financial market analysis. These findings have significant implications for investors and policymakers seeking accurate market forecasts in emerging economies.

Keywords: MARFIMA, ARTFIMA, Nigeria All share Index, long memory, fractional differencing and forecasting.

1. Introduction

The Nigerian All-Share Index (NASI) is a key benchmark for tracking the performance of the Nigerian Stock Exchange (NSE), representing the aggregate value of listed equities (Maxwell *et al*., 2018). Its fluctuations are driven by macroeconomic conditions, geopolitical events, and shifts in investor sentiment, resulting in complex time series behaviour with nonstationarity and long-range dependence (LRD) (Afix, 2018). Accurate NASI modeling is vital for portfolio optimization, risk management, and policymaking. However, traditional approaches like ARIMA often underperform due to their inability to fully capture LRD and nonlinear market dynamics (Zhang, 2009). Fractional integration models, such as the Autoregressive Fractionally Integrated Moving Average (ARFIMA), were developed to address these limitations by allowing for fractional differencing parameters ($0<d<1$) to model long memory (Granger & Joyeux, 1980; Hosking, 1981). However, ARFIMA models suffer from computational inefficiencies, particularly with large datasets, and truncation errors due to their reliance on binomial series expansion. Recent advancements include the Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model, which introduces tempered fractional differencing to handle heavy-tailed distributions (Meerschaert et al., 2014), and the Modified ARFIMA (MARFIMA) model, adopted in this study, which employs a sequential differencing filter to improve convergence and memory retention for $1<d<1.5$. The MARFIMA model, adopted in this study, builds on these foundations by introducing a sequential differencing filter that recursively removes trends while preserving memory, addressing the over-differencing problem inherent in classical ARFIMA. This innovation is particularly relevant for high frequency financial data, where computational efficiency and accuracy are paramount. The modeling of financial time series with long-range dependence has been extensively studied, with seminal contributions from Granger and Joyeux (1980) and Hosking (1981), who introduced the ARFIMA model to capture LRD through fractional differencing. Subsequent research has explored various extensions to address ARFIMA model limitations. For instance, the ARTFIMA model (Meerschaert et al., 2014) incorporates tempered fractional differencing to better model series with heavy-tailed innovations, making it suitable for turbulent financial markets. In the context of stock market indices, studies such as Sensoy and Tabak (2013) and Caporale (2017) have demonstrated the prevalence of LRD in both developed and emerging markets. However, these studies primarily focused on ARFIMA and its variants, leaving a gap in the comparative analysis of newer models like MARFIMA and ARTFIMA. Recent work by Rahman and Jibrin (2019) highlighted the truncation issues of ARFIMA in crude oil price modeling, suggesting the need for more robust alternatives. Swarnalatha *et al*. (2024) conducted a comprehensive evaluation of long range dependence modeling in agricultural commodity pricing, employing the Geweke and Porter-Hudak estimation technique within an ARFIMA framework. Their analysis focused on two decades of monthly groundnut price data (2002-2023) from Andhra Pradesh's agricultural markets. Through rigorous model comparison, the researchers identified ARFIMA(1,0.43,1) as the optimal specification, demonstrating superior short term predictive accuracy relative to seasonal ARIMA alternatives. Bello *et al*. (2025) developed an enhanced autoregressive fractionally integrated moving average framework, designated as MARFIMA (p,d,q), specifically designed to analyze nonstationary time series with fractional differencing parameters in the 1 to 1.5 range. The study validated this innovative model's efficacy through comprehensive comparisons with standard ARFIMA specifications, employing both simulated data and actual financial market indicators. Evaluation incorporated established model selection metrics like the Akaike and Schwarz Bayesian Information Criteria, along with forecast accuracy assessments using measures including root mean squared error and normalized mean squared error. Empirical results consistently showed MARFIMA's superior predictive performance when applied to four key Nigerian economic metrics: crude oil price dynamics, stock exchange performance, the all-shares index, and the food and beverage sector index. This paper is organized as follows: Section 2 details the methodology and model specifications. Section 3 presents the empirical results. Section 4 discusses the findings, while Section 5 concludes with policy recommendations and directions for future research.

2. Methodology

2.1 ARTFIMA (p, d, q) model

The Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model extends traditional ARFIMA specifications by incorporating tempered fractional differencing to better capture long memory (LRD) and heavy-tailed behaviour in time series

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data. Unlike standard ARFIMA models constrained to 0 < d < 1, ARTFIMA introduces an additional tempering parameter (λ > 0), which controls the rate of hyperbolic decay in autocorrelations. The model is presented as:

$$\phi(L)(1-e^{-\lambda}L)^d Y_m = \theta(L)\varepsilon_m \quad 0 < d < 1, \quad \lambda > 0. \tag{1}$$

Where L is the lag operator, ϕ(L) and θ(L) are AR/MA polynomials, λ is a tempering parameter to handle heavy tailed distributions and d is the fractional differencing parameter capturing long memory (Meerschaert *et al*., 2014).

2.2 MARFIMA (p, d, q) model

The Modified Autoregressive Fractionally Integrated Moving Average (MARFIMA) model represents a significant advancement in time series analysis, particularly for datasets exhibiting persistent memory and nonstationary behaviour. Unlike conventional ARFIMA models which are typically constrained to fractional differencing parameters ($0 < d < 1$), this innovative approach emphasizes the importance of a recursive sequential fractional differencing operator. This enhanced methodology effectively processes extensive time series datasets where the differencing parameter d falls within the extended range of $1 < d < 1.5$.

$$\phi(L)\{(1-L)(1-dL)\}Y_m = \theta(L)\varepsilon_m \quad 1 < d < 1.5 \tag{2}$$

Where $\phi(L) = 1-\phi_1 L-\phi_2 L^2-\cdots-\phi_p L^p$ and $\theta(L) = 1-\theta_1 L-\theta_2 L^2-\cdots-\theta_q L^q$ are characteristic polynomials of AR and MA process, d is the fractional differencing filter, L is the backward shift operator, (1-L)(1-dL) is the recursive sequence fractional differencing filter, and ε_m is a white noise (Bello *et al.* 2025).

2.3 Model selection method

In statistical modeling, researchers frequently employ two key metrics for evaluating model performance: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC/SBIC). These criteria serve as essential tools in regression analysis and other modeling approaches by simultaneously assessing how well a model fits the data while penalizing excessive complexity. Models with lower AIC and BIC scores are generally preferred, as they achieve an optimal trade-off between accuracy and simplicity (Musa *et al*., 2014; Tasi'u *et al*., 2022).

The Akaike Information Criteria is

$$AIC = M\ln\left[\frac{\hat{\sigma}_e^2}{M}\right] + 2P \tag{3}$$

Where M is the number of observations, $\hat{\sigma}_e^2$ is the variance of the error term, and P is the number of parameters of the model. The Bayesian information criteria is an extension of the AIC that imposes a large penalty for additional coefficients. It is given as:

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$$SBIC = M \ln\left[\frac{\hat{\sigma}_e^2}{M}\right] + P + P\ln(M) \tag{4}$$

Where $\hat{\sigma}_e^2$ is the variance of the error term, *ln(M)* where ***M*** is the log of the number of observations in the dataset and *P* is the number of parameters of the model.

2.4 Measures of Forecast Accuracy

The measures of forecast accuracy used in this article are Root Mean Square Error (RMSE) and Normalize Mean Square Error (NMSE).

2.4.1 Root mean square error (RMSE)

The root mean square error (RMSE) serves as a key metric for evaluating prediction accuracy in statistical models. This measure is calculated by determining the square root of the average squared differences between actual observed values and model predictions. A lower RMSE value indicates greater predictive precision, reflecting better model performance (Chai & Draxler, 2014; Hyndman & Koehler, 2006).

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(y_i - \hat{y}_i)^2} \tag{5}$$

Where m is the number of observations, y_i is the actual observed value, $\hat{y}_i$ is the predicted value.

2.4.2 Normalize mean square error (NMSE)

The normalized mean square error (NMSE) serves as a scaled version of the mean square error (MSE), adjusting for dataset size. This standardized metric enables more meaningful performance comparisons across different datasets or when working with variables that have substantially different measure scale (Smith and Jones, 2020: Taylor *et al*., 2021).

$$NMSE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{y_i - \hat{y}_i}{std(y)}\right|^2 \tag{6}$$

Where *m* is the number of observations, y_i is the actual observed value, $\hat{y}_i$ is the predicted value and $Std = \sqrt{\sum_{i=1}^{M}\frac{(y_i - \mu)^2}{M}}$ is the standard deviation of the actual values.

3. Results

Daily Nigeria All Share Index data were used to illustrate the proposed MARFIMA model. The data span from January 14, 2000, to January 2, 2019 with 4946 data points.

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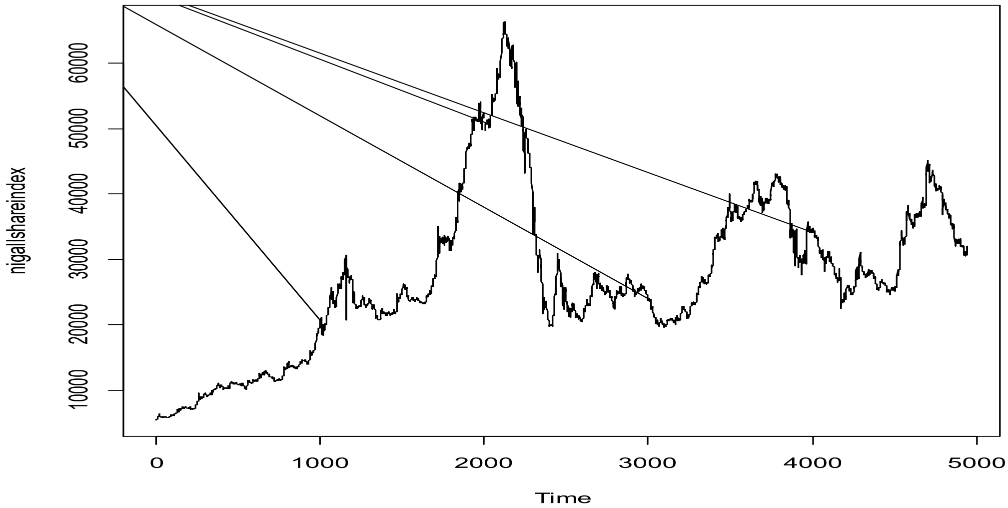
Table 1: Descriptive analysis on Nigerian All Shares Index

Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis	Prob.
27185.82	25606	66371.19	5437	2.3421	0.56609	0.37747	0.0000

Table 1 above shows that Nigerian All Shares Index is positively skewed (Skewness=0.5661) and (Kurtosis=0.3775) indicates flatter distribution than normal.



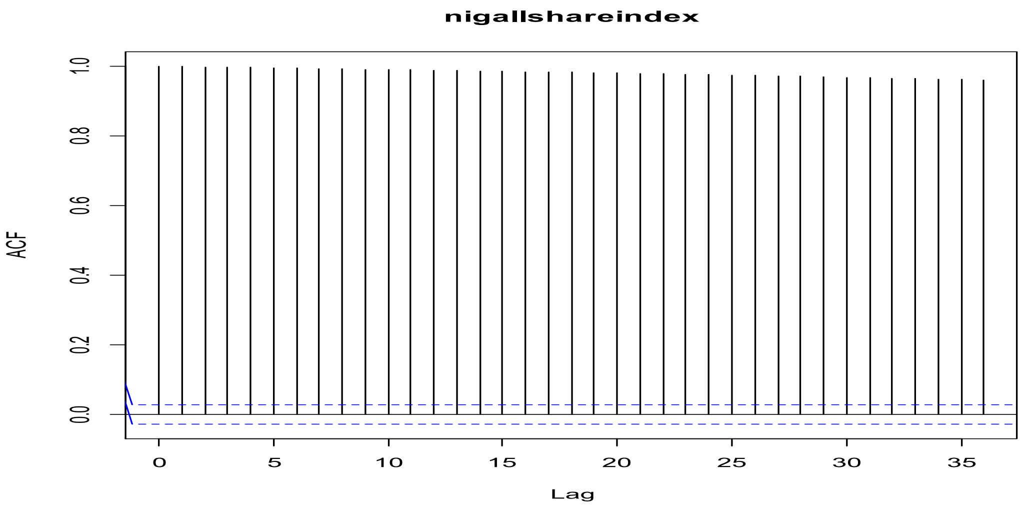


Figure 1: Plot for daily Nigerian All Shares Index and its autocorrelation function (ACF) for original series.

Figure 1 presents the time series plot and corresponding autocorrelation function (ACF). The plot reveals minimal observable trend with random fluctuations, visually suggesting nonstationarity in the data. Furthermore, the ACF demonstrates a slow hyperbolic decay pattern, providing graphical evidence of long range dependence in the time series.

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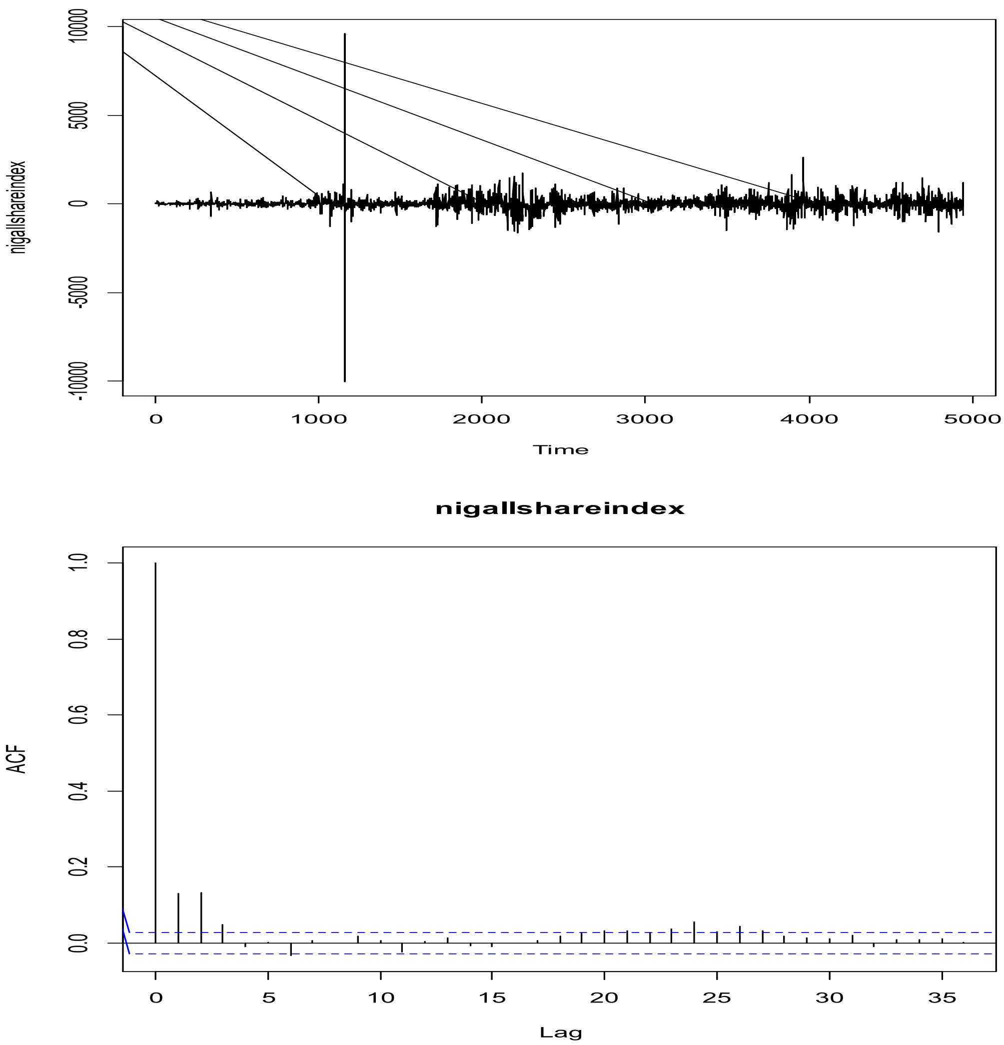

Figure 2: Plot for daily Nigerian All Shares Index and its autocorrelation function (ACF) for difference series.

Figure 2 displays the fractionally differenced series of the daily Nigerian All Share Index. The plot demonstrates successful trend elimination, indicating stationarity has been achieved throu gh the differencing transformation. Correspondingly, the autocorrelation function (ACF) reve als the absence of persistent autocorrelations, confirming the removal of long range dependen ce characteristics from the transformed series.

Table 2: Stationarity Tests for Nigerian All Shares Index

Original series		First difference series	
ADF	**KPSS**	**ADF**	**KPSS**
-1.6000(0.7632)	8.0100(0.01)	-13.734(0.01)	0.1751(0.1)

Table 2 presents the results of stationarity tests conducted on the Nigerian All Shares Index u sing two different tests: the ADF test and the KPSS test. The tests were performed on both th e original series and the first-differenced series. The original series is nonstationary (the ADF test fails to reject the unit root, and the KPSS test rejects stationarity), whereas the first-differ enced series is confirmed to be stationary by both tests.

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Table 3: Model Fit for Nigerian All Shares Index

Model	AIC	SBIC
MARFIMA(1,1.2,1)	5830.99	5831.51
MARFIMA(1,1.2,2)	5441.42	5443.44
MARFIMA(2,1.2,1)	5503.52	5506.55
MARFIMA(2,1.2,2)	5323.23	5326.76
Model	**AIC**	**SBIC**
ARTFIMA(1,-0.4,1)	72096.07	72135.10
ARTFIMA(1,-0.4,2)	72051.98	72097.52
ARTFIMA(2,-0.4,1)	72051.98	72097.52
ARTFIMA(2,-0.4,2)	72046.49	72098.54

Table 3 both AIC and SBIC are significantly lower for the MARFIMA model compared to the ARTFIMA model. This suggests that the MARFIMA model provides a better fit for the Nigerian All Shares Index data.

Table 4: Forecast Error Measures for Nigerian All Shares Index

Model	RMSE	NMSE
MARFIMA(1, 1.2, 1)	362.719	0.5418
MARFIMA(1, 1.2, 2)	244.752	0.2660
MARFIMA(2, 1.2, 1)	260.747	0.4040
MARFIMA(2, 1.2, 2)	217.155	0.2404
Model	**RMSE**	**NMSE**
ARTFIMA(1, -0.4, 1)	422.6902	0.9218
ARTFIMA(1, -0.4, 2)	427.8625	0.9249
ARTFIMA(2, -0.4, 1)	427.8625	0.9249
ARTFIMA(2, -0.4, 2)	435.9115	0.9281

Table 4 shows that the MARFIMA model (RMSE = 217.155) has less than half the RMSE of the ARTFIMA model (435.9115), indicating that MARFIMA forecasts are significantly more accurate in absolute terms. Similarly, in terms of NMSE, MARFIMA (0.2404) demonstrates substantially better normalized performance compared to ARTFIMA (0.9281).

Table 5: Residual Diagnostics for Nigerian All Shares Index

Model	Ljung-Box (p-value)	Jarque-Bera (p-value)
MARFIMA(2,1.2,2)	0.1267	0.7223
ARTFIMA(2,-0.4,2)	0.0241	0.0015

Table 5 presents diagnostic tests on the residuals (forecast errors) of the two competing models for the Nigerian All Shares Index: MARFIMA and ARTFIMA. The tests assess whether the model residuals exhibit desirable statistical properties. The residual diagnostics strongly favour MARFIMA over ARTFIMA, consistent with: (1) its better model fit (Table 3: lower AIC/SBIC), and (2) its superior forecast accuracy (Table 4: lower RMSE/NMSE).

4. Discussion of Results

The comparative analysis between MARFIMA and ARTFIMA models for the Nigerian All Share Index (NASI) yields several important findings that advance our understanding of

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modeling financial time series with long range dependence (LRD) and nonstationarity characteristics. Table 1, the descriptive statistics reveal that NASI exhibits significant non-normality (p=0.0000) with positive skewness (0.5661) and platykurtic distribution (kurtosis=0.3775). These characteristics, combined with the stationarity tests confirming the series' nonstationarity and long memory properties (d=1.2 in MARFIMA), underscore the complexity of modeling this financial index and the inadequacy of traditional ARTFIMA model. Table 2, presents the results of stationarity tests conducted on the Nigerian All Shares Index using two different tests: the ADF test and the KPSS test. The tests were performed on both the original series and the first-differenced series. The original series is nonstationary (the ADF test fails to reject the unit root, and the KPSS test rejects stationarity), whereas the first-differenced series is confirmed to be stationary by both tests. Table 3, the model comparison results demonstrate MARFIMA clear superiority across all evaluation metrics. The information criteria (AIC=5323.23; SBIC=5326.76) show MARFIMA provides a substantially better fit than ARTFIMA (AIC=72,046.49; SBIC=72,098.54). This performance gap is particularly noteworthy as it suggests MARFIMA sequential differencing filter more effectively captures the index's persistent trends while maintaining model parsimony. Table 4, forecast accuracy measures reveal even more striking differences. MARFIMA achieves an RMSE (217.155) less than half that of ARTFIMA (435.9115), with its NMSE (0.2404) representing a 74% improvement over ARTFIMA (0.9281). These results indicate MARFIMA not only fits the historical data better but also generates more reliable out-of-sample predictions a crucial consideration for financial analysts and policymakers. Table 5, residual diagnostics further validate MARFIMA robustness. The model satisfactory Ljung-Box (p=0.1267) and Jarque-Bera (p=0.7223) test results confirm its residuals are uncorrelated and normally distributed, meeting key assumptions for valid statistical inference. In contrast, ARTFIMA significant test statistics (p=0.0241 and p=0.0015 respectively) reveal problematic residual patterns that undermine its reliability. The superior performance of MARFIMA can be attributed to its innovative sequential differencing approach, which effectively handles the d>1 case (d=1.2) that characterizes NASI strong persistence. This contrasts with ARTFIMA tempered fractional approach (d=-0.4), with heavy-tailed innovations but proves inadequate for NASI long memory dynamics.

5. Conclusion

This study presents a comprehensive comparative analysis of two advanced fractional integration models MARFIMA and ARTFIMA in modeling the Nigerian All Share Index (NASI). Our findings demonstrate that the proposed MARFIMA(2,1.2,2) model significantly outperforms ARTFIMA(2,-0.4,2) across all evaluation metrics, establishing its superiority for modeling financial time series with strong persistence characteristics. The empirical results reveal several key insights. First, MARFIMA sequential differencing filter proves particularly effective in handling NASI's nonstationarity and long range dependence (d=1.2), while maintaining computational efficiency. Second, the model shows remarkable forecasting accuracy, with RMSE values less than half those of ARTFIMA, suggesting its strong potential for practical financial applications. Third, diagnostic tests confirm MARFIMA statistical robustness, with well-behaved residuals that satisfy key modeling assumptions. These findings have important implications for both academic research and financial practice. For researchers, the study extends the theoretical framework of fractional integration models by introducing an effective approach for cases where d>1. For market practitioners and policymakers in emerging economies, MARFIMA offers a powerful tool for more accurate market forecasting and risk assessment.

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