A CUBIC TRANSMUTED ISHITA DISTRIBUTION: PROPERTIES, SIMULATION AND APPLICATION TO COVID-19 DATA.

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ABSTRACT:
This study introduces the Cubic Transmuted Ishita Distribution (CuTrID), which is an improvement of the Ishita distribution that was developed by substitution of the Ishita distribution into the cubic transmuted family. The CuTrID provides greater flexibility in modeling positively skewed data. We derived some essential properties of the compound CuTrID, including survival and hazard functions, moments, and generating functions. Parameter estimation was conducted by the method of maximum likelihood estimation, and the performance of the estimators was examined through simulation study under varying sample sizes. The simulation outputs show that the estimators are converging to the true parameters, with decreasing bias and MSE as the sample size increases. We applied the CuTrID distribution to COVID-19 infection data from Nigeria. Some information criteria which include AIC, CAIC, BIC, HQIC, and goodness-of-fit (Kolmogorov-Smirnov) statistics revealed that the CuTrID provides a superior fit over the Ishita distribution (ID), Transmuted Ishita distribution (TrID), and Sine–Ishita distribution (SID). These findings show that CuTrID is a compound distribution that can be used for modeling modern data with increasing failure rates, making it a valuable tool in epidemiological, biomedical, and reliability studies.

Keywords: Ishita distribution, Cubic transmuted family, Cubic transmuted Ishita distribution, Properties, Maximum Likelihood Estimation, Covid-19 data, Applications.

1 INTRODUCTION

Many traditional probability distributions have been used for decades in modeling data, and research has shown that extending these probability distributions lead to compound distributions which are more flexible for modern data application in various fields such as the medical/biological sector, engineering, economics, genetics, agronomy (Rao and Aslam, 2021; Strzelecki, 2021; Tung *et al*., 2021; Reynolds *et al*., 2021; Prataviera, 2022).

Shanker and Shukla (2017) proposed a compound distribution and called it "Ishita distribution". They obtained this compound distribution by two component combination of exponential distribution with parameter (parameter h) and a gamma distribution with parameters $(3,\eta)$ using mixing proportion, $\frac{\eta^3}{\eta^3+2}$.

The Ishita distribution (ID) density and distribution functions are obtained as;

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$$g(x)=\frac{\eta^3}{\eta^3+2}\left(\eta+x^2\right)e^{-\eta x} \quad (1)$$

and

$$G(x)=1-\left[1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right]e^{-\eta x} \quad (2)$$

For $x>0$ with strictly positive scale parameter η the ID distribution demonstrated enhanced performance over the Akash, Lindley, and standard exponential distributions in practical applications, as established by Shanker and Shukla (2017).

We have witnessed an increase of generalized distribution families in statistical literature, which shows us a new method of developing compound distributions. Some of the contributions in this area include the exponentiated T-X (Alzaghal et al., 2013), and Quadratic rank transmutation (QRTM) (Shaw and Buckley, 2007), which clear the way for some of the previous research. Following this, scholars in the field of statistics have developed many of these generalized families, including Weibull-X (Alzaatreh et al., 2013), Sine-G family (Kumar et al., 2015), Lomax generalized class (Cordeiro et al., 2014), Weibull generator (Bourguignon et al., 2014), Lindley Generator (Cakmakyapan and Ozel, 2016), Gompertz Generator (Alizadeh et al., 2017), Odd Lindley Generator (Gomes-Silva et al., 2017), Weibull generalized (Tahir et al., 2016), and the odd lomax generalized family (Cordeiro et al., 2019).

We also have more recent research that has introduced generalizations like the Cubic Transmuted G family (Rahman et al., 2019), the odd Chen-G (Anzagra et al., 2022), and a new sine class (Benchiha et al., 2023). Lately, researchers continued in this trend, by featuring some G families as the bivariate Lomax generalized family (Fayomi et al., 2023), the X-Exponential generalized Family (Mohammad, 2024), and the Fréchet generalized family (Ieren et al., 2024). All these studies put together show the new dimension in distribution theory, demonstrating that the proposed compound distributions offer a more powerful distribution for modeling modern data structures compared to standard distributions. With generalized families' researchers have developed numerous compound distributions. These compound distributions include the Transmuted-kumaraswamy model (Khan et al., 2016), Exponential-lindley model (Ieren and Balogun, 2021), Power-lindley model (Ghitany et al., 2013) distributions, the Lomax-exponential model (Ieren and Kuhe, 2018), Sine-Lomax-exponential model (Joel et al., 2024), and Lomax-inverse exponential model (Abdulkadir et al., 2020). And many more of these compound distributions have been introduced as well, such as; the odd-lindley inverse exponential model (Ieren and Abdullahi, 2020), transmuted Weibull-exponential model (Yahaya and Ieren, 2017), and Weibull-Fréchet model (Afify et al., 2016). Additional contributions encompass the bivariate generalized Rayleigh model (Abdel-Hady, 2013), a Transmuted-Odd Lindley-Rayleigh compound model (Umar et al., 2021), Lomax-Fréchet model (Gupta et al., 2015) and the Transmuted Odd-generalized Exponential-exponential model (Abdullahi et al., 2018).
Motivated by these recent studies, in this paper we introduced a new continuous model, the Cubic Transmuted Ishita Distribution (CuTrID), utilizing the Cubic transmuted generalized class.
We then structured the paper as follows: Section 2 defines the new distribution and provides graphical illustrations. Section 3 derives its key statistical properties. The method we used for parameter estimation of the CuTrID, is maximum likelihood and is detailed in section 4, followed by a simulation study in Section 5. Section 6 we demonstrated the usefulness of the CuTrID on a COVID-19 dataset alongside competing distributions. And in section 7 we offered a concluding summary of the study.

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2. THE CUBIC TRANSMUTED-ISHITA DISTRIBUTION (CuTrID)

We obtained the cubic transmuted Ishita distribution's density and distribution functions using the steps proposed by Rahman *et al.* (2019). According to Rahman *et al.* (2019), the pdf and cdf of cubic transmuted class is respectively given by;

$$F(x) = (1-\lambda)G(x) + 3\lambda\left[G(x)\right]^2 - 2\lambda\left[G(x)\right]^3 \quad (3)$$

and

$$f(x) = g(x)\left[1-\lambda+6\lambda G(x) - 6\lambda\left[G(x)\right]^2\right] \quad (4)$$

The Cubic Transmuted Ishita Distribution (CuTrID) is defined for $x > 0$ and with a parameter λ, where $-1 \le \lambda \le 1$ which is the cubic transmuted parameter within the family. In Equations (3) and (4), *G(x)* represents the distribution function of the standard distribution to be extended, while f(x) and g(x) denote the PDFs associated with the resulting distribution F(x) and the standard G(x), respectively.

By substituting the standard CDF and PDF from equations (1) and (2) into the Cubic Transmuted family given in equations (3) and (4), and then simplified, we obtained the final distribution function and density function of the CuTrID, presented in equations (5) and (6) as follows:

$$F(x) = 1-(1-\lambda)\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)e^{-\eta x} - 3\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^2 e^{-2\eta x} + 2\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^3 e^{-3\eta x} \quad (5)$$

and

$$f(x) = \frac{\eta^3(\eta+x^2)}{\eta^3+2}e^{-\eta x}\left[1-\lambda+6\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)e^{-\eta x} - 6\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^2 e^{-2\eta x}\right] \quad (6)$$

The cubic transmuted Ishita distribution is defined for x strictly positive, and η strictly positive which stands for the scale parameter and λ ranges from -1 to 1 denotes the transmuted parameter.

Figure 1 exhibits the probability density and hazard function graphs for the Cubic Transmuted Ishita Distribution across assigned parameter values.

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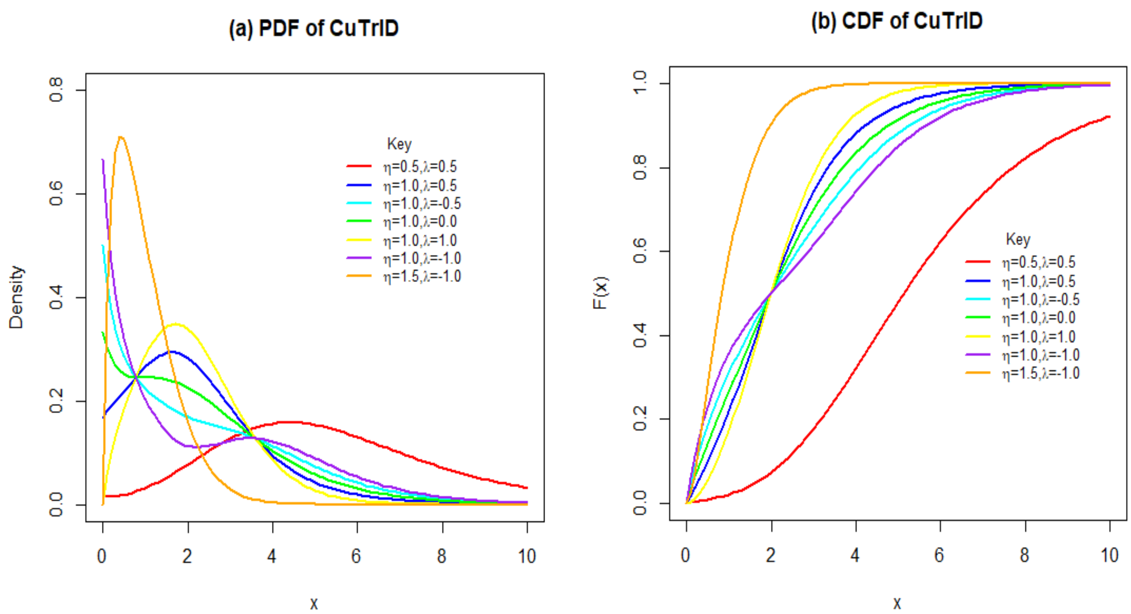

Figure 1: Graphs for PDF and CDF of the CuTrID.

Based on this figure 1, the density function of the CuTrID shows positive skewness and also exihibits shape flexibility across different parameter values. The corresponding distribution function displays the expected asymptotic behavior, converging to 1 as X approaches infinity and equals zero when X decreases to zero as normally expected.

3. SOME ESSENTIAL PROPERTIES OF CuTrID

Here, we extensively derived and discussed some properties of the CuTrID distribution as follows:

3.1 Reliability analysis of the CuTrID.

In this part we presented the derivation and graphs of the survival and hazard rate functions. The survival function represents the probability that a subject survives beyond a specified time. It is obtained as 1 minus the CDF mathematically, and substituting the cdf of the CuTrID, we obtained the CuTrID's survival function:

$$S(x)=1-\left\{1-(1-\lambda)\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)\mathrm{e}^{-\eta x}-3\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^2\mathrm{e}^{-2\eta x}+2\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^3\mathrm{e}^{-3\eta x}\right\}$$

$$S(x)=(1-\lambda)\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)\mathrm{e}^{-\eta x}+3\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^2\mathrm{e}^{-2\eta x}-2\lambda\left(1+\frac{\eta x(\eta x+2)}{\eta^3+2}\right)^3\mathrm{e}^{-3\eta x} \quad (7)$$

The hazard function represents the instantaneous probability of failure per unit time. Likewise, the failure rate function is obtain by inserting the pdf and survival function of the CuTrID, we obtained the CuTrID's hazard function:

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$$h(x)=\frac{\dfrac{\eta^{3}\left(\eta+x^{2}\right)}{\eta^{3}+2}\mathrm{e}^{-\eta x}\left[1-\lambda+6\lambda\left(1+\dfrac{\eta x(\eta x+2)}{\eta^{3}+2}\right)\mathrm{e}^{-\eta x}-6\lambda\left(1+\dfrac{\eta x(\eta x+2)}{\eta^{3}+2}\right)^{2}\mathrm{e}^{-2\eta x}\right]}{(1-\lambda)\left(1+\dfrac{\eta x(\eta x+2)}{\eta^{3}+2}\right)\mathrm{e}^{-\eta x}+3\lambda\left(1+\dfrac{\eta x(\eta x+2)}{\eta^{3}+2}\right)^{2}\mathrm{e}^{-2\eta x}-2\lambda\left(1+\dfrac{\eta x(\eta x+2)}{\eta^{3}+2}\right)^{3}\mathrm{e}^{-3\eta x}} \quad (8)$$

The graphs of the survival function (SF) and hazard function (HF) for few assigned parameter values as can be seen in Figure **2** below:

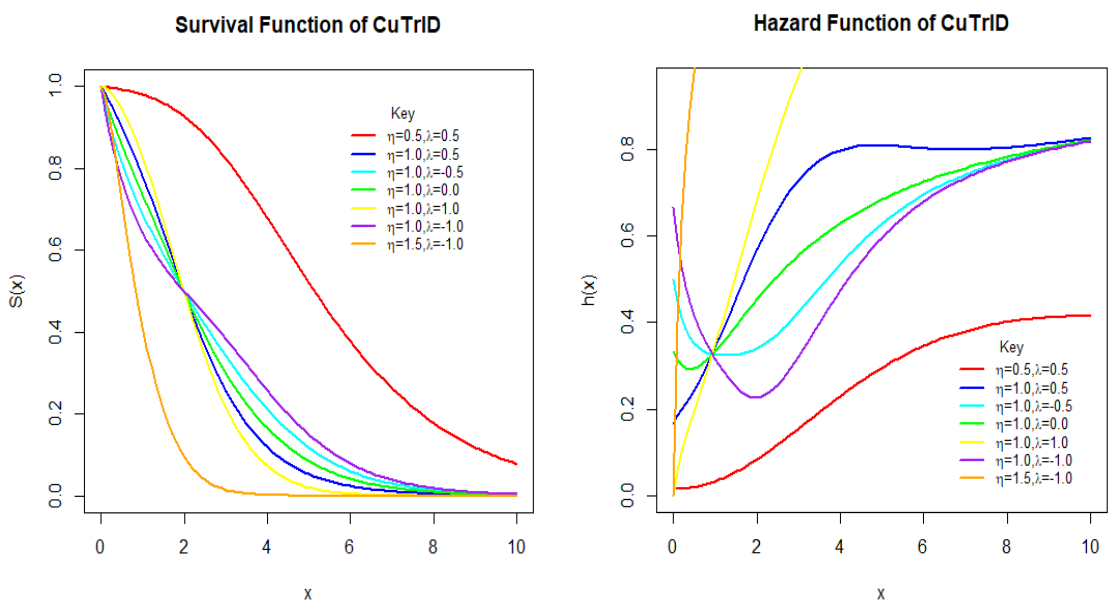

Figure 2: Reliability Characteristics of CuTrID: Survival and Hazard Functions.

Figure 2 demonstrates that the survival probability begins at certainty for initial time and monotonically decreases toward zero as time approaches infinity. This pattern corresponds to an increasing hazard rate, indicating that components following the CuTrID experience higher risk of failure over time, consistent with the behavior of aging.

3.2 Derivation of Moments and Generating Functions for CuTrID

The n^{th} moment raw moment (moment about zero) is given by the expectation;

$$\mu_n^{'}=E\left(X^{n}\right)=\int_{0}^{\infty}x^{n}f\left(x\right)dx \quad (9)$$

and f(x), is the *density function* of the CuTrID is as given in Equation (6) and before we substitute (6) in (11), the expansion, simplification and linear representation of the pdf of the CuTrID is done as follows:

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$$f(x)=\frac{\eta^3(1-\lambda)(\eta+x^2)}{\eta^3+2}e^{-\eta x}+\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-2\eta x}+\frac{\eta x(\eta x+2)}{\eta^3+2}e^{-2\eta x}-\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-3\eta x}-\frac{12\lambda\eta^4x(\eta x+x^2)(\eta x+2)}{(\eta^3+2)^2}e^{-3\eta x}-\frac{6\lambda\eta^5x^2(\eta+x^2)(\eta x+2)^2}{(\eta^3+2)^3}e^{-3\eta x}$$

$$f(x)=\frac{\eta^3(1-\lambda)(\eta+x^2)}{\eta^3+2}e^{-\eta x}+\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-2\eta x}+\frac{\eta x(\eta x+2)}{\eta^3+2}e^{-2\eta x}-\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-3\eta x}-\frac{12\lambda\eta^4x(\eta^2x+\eta x^2+2x^2+2\eta)}{(\eta^3+2)^2}e^{-3\eta x}-\frac{6\lambda\eta^5x^2(\eta x^4+4\eta x^3+4x^2+\eta^3x^2+4\eta^2x+4\eta)}{(\eta^3+2)^3}e^{-3\eta x}$$

$$f(x)=\frac{\eta^3(1-\lambda)(\eta+x^2)}{\eta^3+2}e^{-\eta x}+\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-2\eta x}+\frac{\eta x(\eta x+2)}{\eta^3+2}e^{-2\eta x}-\frac{6\lambda\eta^3(\eta+x^2)}{\eta^3+2}e^{-3\eta x}-\frac{12\lambda\eta^4(\eta^2x^2+\eta x^3+2x^3+2\eta x)}{(\eta^3+2)^2}e^{-3\eta x}-\frac{6\lambda\eta^5(\eta x^6+4\eta x^5+4x^4+\eta^3x^4+4\eta^2x^3+4\eta x^2)}{(\eta^3+2)^3}e^{-3\eta x} \quad (10)$$

Substituting equation (10), the n[th] ordinary/crude moment of the CuTrID is derived as:

$$E(X^n)=\frac{\eta^3(1-\lambda)}{\eta^3+2}\int_0^\infty x^n(\eta+x^2)e^{-\eta x}dx+\frac{6\lambda\eta^3}{\eta^3+2}\int_0^\infty x^n(\eta+x^2)e^{-2\eta x}dx+\frac{\eta}{\eta^3+2}\int_0^\infty x^n(\eta x^2+2x)e^{-2\eta x}dx-\frac{6\lambda\eta^3}{\eta^3+2}\int_0^\infty x^n(\eta+x^2)e^{-3\eta x}dx-\frac{12\lambda\eta^4}{(\eta^3+2)^2}\int_0^\infty x^n(\eta^2x^2+\eta x^3+2x^3+2\eta x)e^{-3\eta x}dx-\frac{6\lambda\eta^5}{(\eta^3+2)^3}\int_0^\infty x^n(\eta x^6+4\eta x^5+4x^4+\eta^3x^4+4\eta^2x^3+4\eta x^2)e^{-3\eta x}dx$$

(11)

we used integration by substitution and gamma function in equation (11), and simplifying gives the nth ordinary moment of X for the CuTrID is obtained as:

$$E(X^n)=\frac{(\eta^3\Gamma(n+1)+\Gamma(n+3))}{\eta^{2n}(1-\lambda)^{-1}(\eta^3+2)}+\frac{(\eta(2\eta)^{n+3}\Gamma(n+1)+(2\eta)^{n+1}\Gamma(n+3))}{(2\eta)^{2n+4}(6\lambda\eta^3)^{-1}(\eta^3+2)}+\frac{(\eta(2\eta)^{n+2}\Gamma(n+3)+2(2\eta)^{n+3}\Gamma(n+2))}{(2\eta)^{2n+5}\eta^{-1}(\eta^3+2)}-\frac{(\eta(3\eta)^{n+3}\Gamma(n+1)+(3\eta)^{n+1}\Gamma(n+3))}{(3\eta)^{2n+4}(6\lambda\eta^3)^{-1}(\eta^3+2)}-\frac{12\lambda\eta^4}{(\eta^3+2)^2}\left[\frac{\eta^2\Gamma(n+3)}{(3\eta)^{n+3}}+\frac{(\eta+2)\Gamma(n+4)}{(3\eta)^{n+4}}+\frac{2\eta\Gamma(n+2)}{(3\eta)^{n+2}}\right]-\frac{6\lambda\eta^5}{(\eta^3+2)^3}\left[\frac{\eta\Gamma(n+7)}{(3\eta)^{n+7}}+\frac{4\eta\Gamma(n+6)}{(3\eta)^{n+6}}+\frac{(\eta^2+4)\Gamma(n+5)}{(3\eta)^{n+5}}+\frac{4\eta^2\Gamma(n+4)}{(3\eta)^{n+4}}+\frac{4\eta\Gamma(n+3)}{(3\eta)^{n+3}}\right]$$

(12)

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The moment generating function, formal definition, serves as the foundation for our derivation. For the Cubic Transmuted Ishita Distribution, we employ the n-th ordinary moment presented in equation (12) and utilize power series expansion to derive the MGF as seen:

$$M_X(t)=E\left[e^{tx}\right]=E\left[\sum_{n=0}^{\infty}\frac{(tx)^n}{n!}\right]=\sum_{n=0}^{\infty}\frac{t^n}{n!}\int_0^{\infty}x^n f(x)\,dx=\sum_{n=0}^{\infty}\frac{t^n}{n!}E\left(X^n\right) \quad (13)$$

The MGF of CuTrID is obtained through inserting of equation (12) into equation (13) and subsequent algebraic simplification.

The characteristics function (CF) of the CuTrID is also derived in accordance to the nth ordinary moment using power series expansion we had ealier:

$$\phi_X(t)=E\left[e^{itx}\right]=E\left[\sum_{n=0}^{\infty}\frac{(itx)^n}{n!}\right]=\sum_{n=0}^{\infty}\frac{(it)^n}{n!}\int_0^{\infty}x^n f(x)\,dx=\sum_{n=0}^{\infty}\frac{(it)^n}{n!}E\left(X^n\right) \quad (14)$$

Again, substituting for $E\left(X^n\right)$ in equation (12) and simplifying will produce the characteristic function of the CuTrID.

4. THE CuTrID PARAMETERS ESTIMATION BY MAXIMUM LIKELIHOOD METHOD

Let consider a random sample $X_1, X_2, \ldots\ldots\ldots, X_n$ of 'n' independent and identically distributed observations from the Cubic-transmuted Ishita Distribution (CuTrID). The likelihood function for the unknown parameters η and λ based on the density function in equation (6), is given by:

$$L(\underline{X}\mid\eta,\lambda)=\prod_{i=1}^{n}\left(\frac{\eta^3\left(\eta+x_i^2\right)}{\eta^3+2}e^{-\eta x_i}\right)\prod_{i=1}^{n}\left[1-\lambda+6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)e^{-\eta x_i}-6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)^2 e^{-2\eta x_i}\right]$$

(14)

Taking the natural logarithm we obtained:

$$\begin{aligned} l &= 3n\log\eta-n\log\left(\eta^3+2\right)+\sum_{i=1}^{n}\log\left[\eta+x_i^2\right]-\eta\sum_{i=1}^{n}x_i \\ &+\sum_{i=1}^{n}\log\left[1-\lambda+6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)e^{-\eta x_i}-6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)^2 e^{-2\eta x_i}\right] \end{aligned} \quad (15)$$

We differentiate l with respect to η and λ to obtained the following result:

$$\begin{aligned} \frac{\partial l}{\partial\eta} &= \frac{3n}{\eta}-\frac{2n\eta^2}{\left(\eta^3+2\right)}+\sum_{i=1}^{n}\left(\eta+x_i^2\right)^{-1}-\sum_{i=1}^{n}x_i \\ &+\sum_{i=1}^{n}\left\{\frac{6\lambda\eta x_i e^{-\eta x_i}\left(\eta^4+\eta^3x_i^2+3\eta^2x_i+8\eta+2x_i\right)\left(\eta^3+2\right)^{-2}\left[8\left(\frac{\eta^3+2+\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)e^{-\eta x_i}-1\right]}{\left[1-\lambda+6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)e^{-\eta x_i}-6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)^2 e^{-2\eta x_i}\right]}\right\} \end{aligned} \quad (16)$$

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$$\frac{\partial l}{\partial \lambda}=\sum_{i=1}^{n}\left\{\frac{6\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)\mathrm{e}^{-\eta x_i}-6\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)^2\mathrm{e}^{-2\eta x_i}-1}{\left[1-\lambda+6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)\mathrm{e}^{-\eta x_i}-6\lambda\left(1+\frac{\eta x_i\left(\eta x_i+2\right)}{\eta^3+2}\right)^2\mathrm{e}^{-2\eta x_i}\right]}\right\} \quad (17)$$

To obtain likelihood estimates for η and λ, we set equations (16) and (17) equal to zero, forming a system that the equations are non-linear. The analytical complexity of this system requires numerical methods; consequently, we apply the Newton-Raphson iterative procedure using R software.

5 NUMERICAL SIMULATION STUDY FOR CuTrID

This section details a numerical simulation study designed to evaluate the performance of the likelihood-based estimators (MLEs) for the CuTrID parameters. The estimation was implemented using the ‘optim ()’ function in R with the L-BFGS-B method to maximize the model's log-likelihood function.

The simulation employed 500 replications for sample sizes ranging from n = 25 to 500. We generated the data by using the inverse CDF of the CuTrID. The study investigated four distinct parameter mixture: (i) $\eta = 0.5, \lambda = 0.5$; (ii) $\eta = 1.0, \lambda = 1.0$; (iii) $\eta = 0.5, \lambda = 1.0$; and (iv) $\eta = 1.0, \lambda = 0.5$.

The performance of the estimators $\hat{\eta}_{MLE}$ and $\hat{\lambda}_{MLE}$ was assessed using two key metrics: Mean Square Error (MSE) and Bias. For each sample size, we computed the average MLEs, MSE, Bias, and Absolute Bias. The comprehensive outputs of this simulation are presented in Tables 1-4 and illustrated graphically in Figures 3-6.

Table 1: Simulation outputs for the CuTrID for $\eta = 0.5$ *and* $\lambda = 0.5$

n	Measures/ Criteria	Parameters		n	Measures/ Criteria	Parameters	
		η	λ			η	λ
n=25	MLEs	0.4914	0.7089	n=200	MLEs	0.4862	0.6371
	MSEs	0.0023	0.1286		MSEs	0.0005	0.0358
n=75	MLEs	0.4864	0.6563	n=300	MLEs	0.4859	0.6331
	MSEs	0.0009	0.0691		MSEs	0.0004	0.0291
n=100	MLEs	0.4856	0.6519	n=400	MLEs	0.4857	0.6366
	MSEs	0.0007	0.0598		MSEs	0.0003	0.0272
n=150	MLEs	0.4864	0.6347	n=500	MLEs	0.4859	0.6385
	MSEs	0.0005	0.0434		MSEs	0.0003	0.0260

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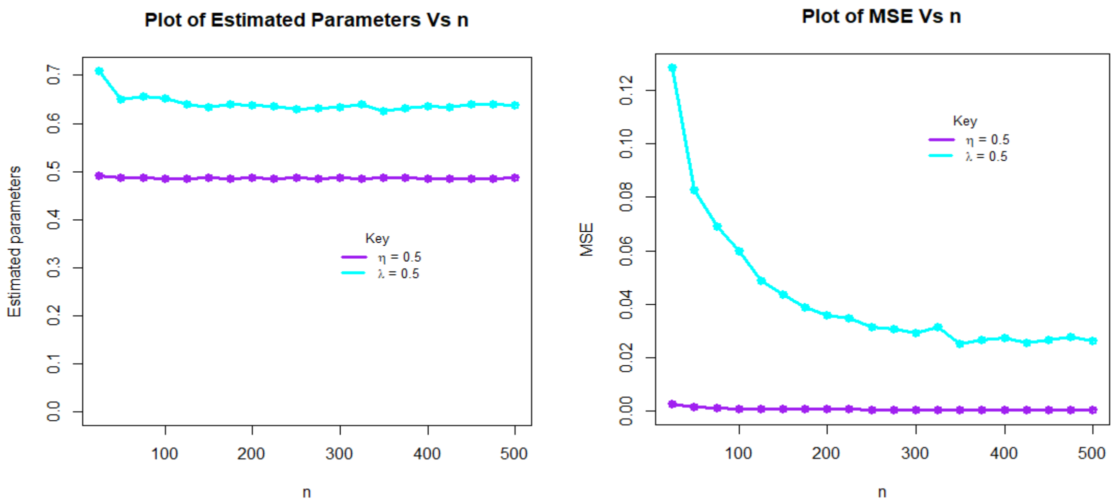

Figure 3: Graphs of MLEs and MSEs of the CuTrID for $\eta = 0.5$ *and* $\lambda = 0.5$

Table 2: Simulation outputs for the CuTrID for $\eta = 1.0$ *and* $\lambda = 1.0$

n	Measures/ Criteria	Parameters		n	Measures/ Criteria	Parameters	
		η	λ			η	λ
n=25	MLEs	1.0702	0.8974	n=200	MLEs	1.0485	0.9510
	MSEs	0.0142	0.0452		MSEs	0.0035	0.0075
n=75	MLEs	1.0531	0.9295	n=300	MLEs	1.0481	0.9556
	MSEs	0.0059	0.0189		MSEs	0.0030	0.0056
n=100	MLEs	1.0506	0.9367	n=400	MLEs	1.0473	0.9613
	MSEs	0.0047	0.0150		MSEs	0.0028	0.0040
n=150	MLEs	1.0510	0.9395	n=500	MLEs	1.0474	0.9640
	MSEs	0.0042	0.0116		MSEs	0.0027	0.0034

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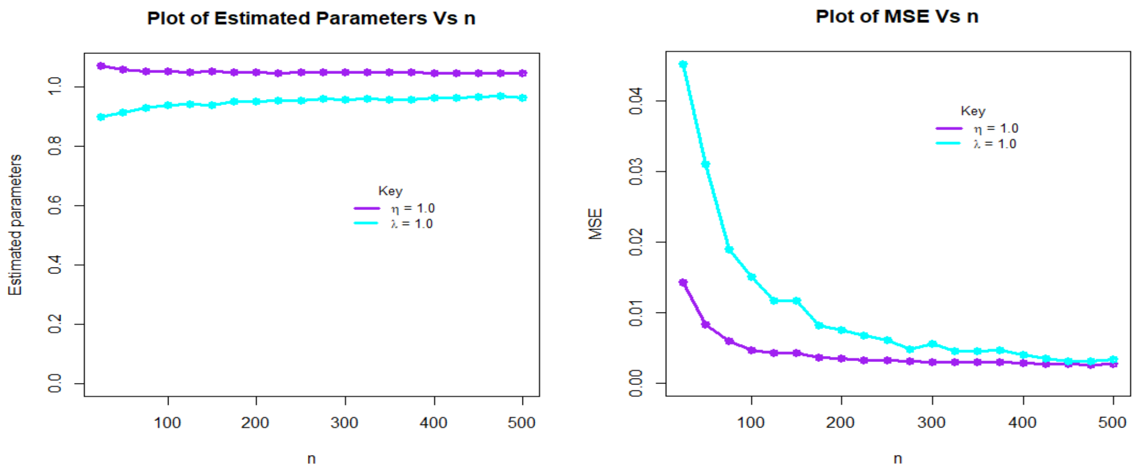

Figure 4: Graphs of MLEs and MSEs of the CuTrID for $\eta = 1.0$ *and* $\lambda = 1.0$

Table 3: Simulation outputs for the CuTrID for $\eta = 0.5$ *and* $\lambda = 1.0$

n	Measures/ Criteria	Parameters		n	Measures/ Criteria	Parameters	
		η	λ			η	λ
n=25	MLEs	0.4923	0.9668	n=200	MLEs	0.4860	0.9954
	MSEs	0.0014	0.0101		MSEs	0.0004	0.0003
n=75	MLEs	0.4867	0.9860	n=300	MLEs	0.4859	0.9965
	MSEs	0.0006	0.0019		MSEs	0.0003	0.0002
n=100	MLEs	0.4859	0.9899	n=400	MLEs	0.4857	0.9981
	MSEs	0.0005	0.0014		MSEs	0.0003	0.0001
n=150	MLEs	0.4862	0.9933	n=500	MLEs	0.4859	0.9986
	MSEs	0.0004	0.0005		MSEs	0.0003	0.0000

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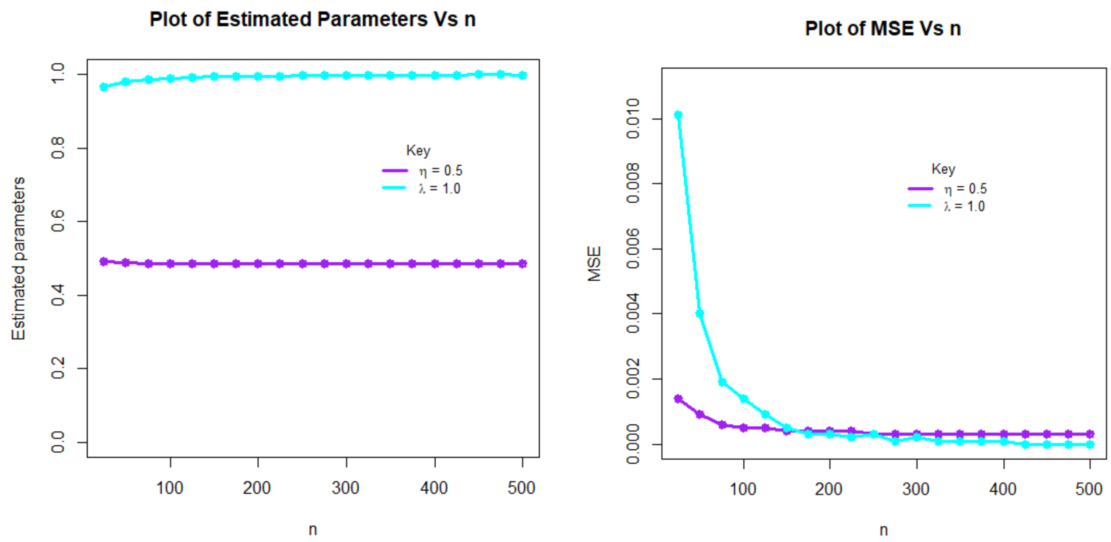

Figure 5: Graphs of MLEs and MSEs of the CuTrID for $\eta = 0.5 \quad and \quad \lambda = 1.0$

Table 4: Simulation outputs for the CuTrID for $\eta = 1.0 \quad and \quad \lambda = 0.5$

n	Measures/ Criteria	Parameters		n	Measures/ Criteria	Parameters	
		η	λ			η	λ
n=25	MLEs	1.0729	0.6063	n=200	MLEs	1.0882	0.4011
	MSEs	0.0199	0.1051		MSEs	0.0104	0.0364
n=75	MLEs	1.0770	0.4559	n=300	MLEs	1.0867	0.3881
	MSEs	0.0120	0.0568		MSEs	0.0092	0.0314
n=100	MLEs	1.0865	0.4286	n=400	MLEs	1.0846	0.3908
	MSEs	0.0119	0.0482		MSEs	0.0086	0.0263
n=150	MLEs	1.0847	0.4151	n=500	MLEs	1.0853	0.4005
	MSEs	0.0105	0.0420		MSEs	0.0085	0.0220

ISSN NUMBER: 1116-249X

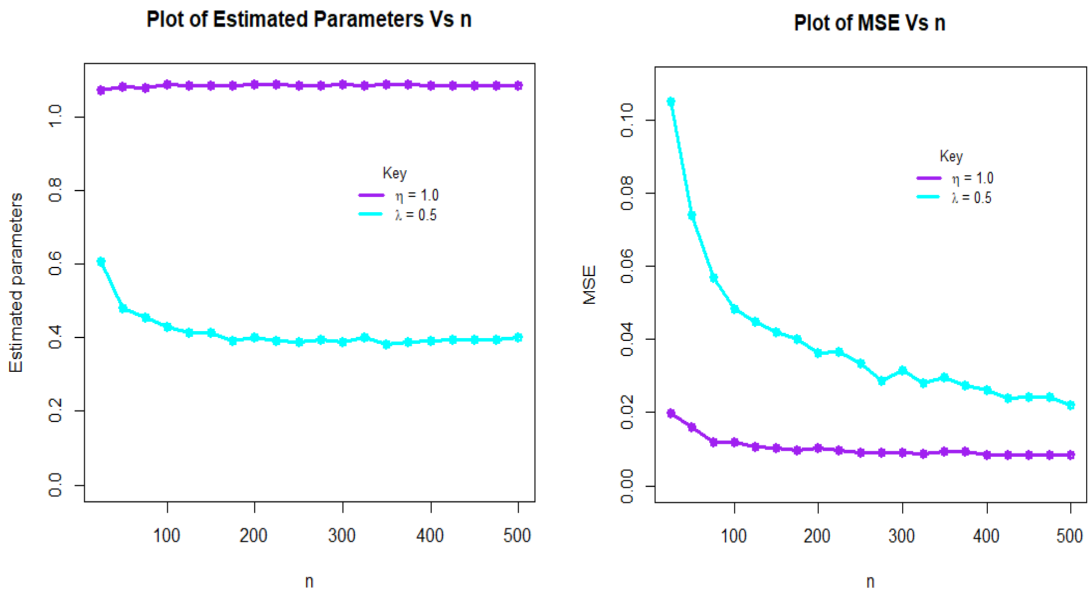

Figure 6: Graphs of MLEs and MSEs of the CuTrID for $\eta = 1.0$ *and* $\lambda = 0.5$

Tables 1-4 and Figures 3-6 contain the simulation results for the CuTrID model, and we also present the performance of the maximum likelihood estimates through their average values, absolute bias, and mean square errors. The result clearly shows that as the sample size grows; the parameter estimates approach nearer to their true values. Likewise, both the bias and mean square error consistently become smaller across increasing sample sizes.

6. APPLICATIONS OF CuTrID TO COVID-19 DATA

This section demonstrates the validation of the Cubic Transmuted Ishita Distribution (CuTrID) by applying it to a COVID-19 dataset. The CuTrID's parameters are estimated using maximum likelihood estimation (MLE), and its goodness-of-fit is evaluated against three competing distributions which are the Transmuted Ishita (TrID), Sine-Ishita (SID), and the standard Ishita (ID) distributions.

We compared these distributions, by using some standard information criteria [AIC, BIC, CAIC, HQIC], as well as the Kolmogorov-Smirnov (K-S) statistic. These evaluation metrics, as discussed in Chen and Balakrishnan (1995), serve as benchmarks for model performance, where lower values indicate a superior fit.

The dataset consists of the daily reported COVID-19 infections in Nigeria, covering a seven-month period from March 20 to October 19, 2020. Sourced from the 'Nigeria Centre for Disease Control' and previously analyzed by Osatohanmwen et al. (2022), this dataset provides a relevant case study for assessing the proposed distribution's performance in modeling epidemiological data. A statistical summary of the data is provided below:

Table 5: Summary Statistics for the Covid-19 dataset

parameters	n	Min	Q_1	Media	Q_3	Mean	Max	Variance	Skewness	Kurtosis
Data value	214	4.0	125.2	239.0	452.8	287.3	790.0	43047.4	0.46378	-0.89878

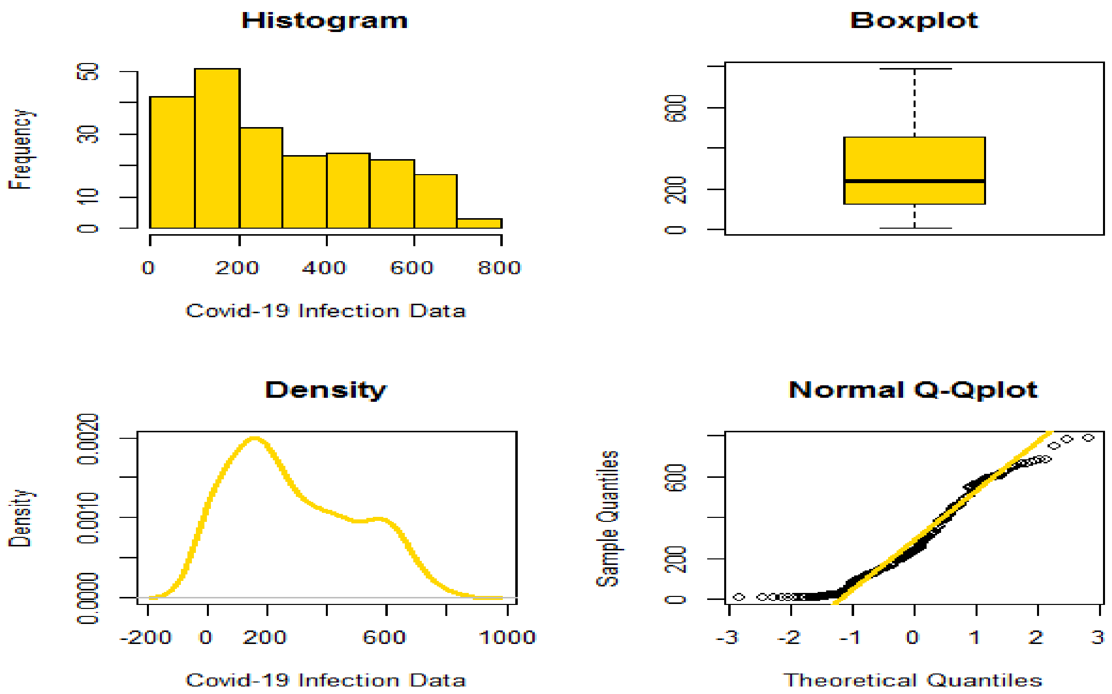

Figure 7: A graphical summary of Nigeria's COVID-19 daily infection data

The descriptive statistics (Table 5) and graphical displays in Figure 7, including the histogram, box plot, density plot, and normal Q-Q plot, indicate that the daily COVID-19 infection data for Nigeria is unimodal, right-skewed, and platykurtic.

Table 6: Likelihood-Based Parameter Estimates for COVID-19 Dataset Analysis

Distribution	Parameter Estimates	
CuTrID	$\hat{\eta}=0.010949363$	$\hat{\lambda}=-0.15617912$
TrID	$\hat{\eta}=0.010577555$	$\hat{\lambda}=0.07514299$
SID	$\hat{\eta}=0.006797698$	-
ID	$\hat{\eta}=0.017926177$	-

Table 7: Model Selection Criteria for COVID-19 Dataset (ℓ, AIC, CAIC, BIC, HQIC)

ISSN NUMBER: 1116-249X

Distribution	$\hat{\ell}$	*AIC*	*CAIC*	*BIC*	*HQIC*	*K-S*	*P-Value*
CuTrID	1492.972	2989.945	2990.002	2996.677	2992.665	0.1300297	0.001439645
TrID	1500.063	3004.125	3004.182	3010.857	3006.845	0.1315569	0.001213381
SID	1522.515	3047.031	3047.050	3050.397	3048.391	0.1993665	8.183391e-08
ID	1613.332	3228.663	3228.682	3232.029	3230.024	0.3323278	5.920191e-21

Graphical representation of estimated probability density functions and cumulative distribution functions for models fitted to the COVID-19 dataset.

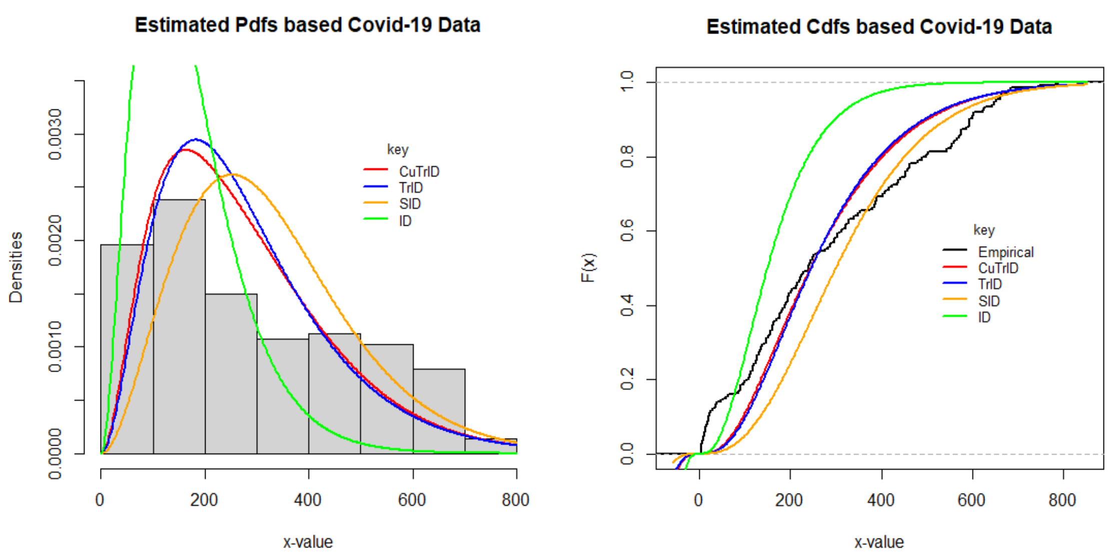

Figure 8: Fitted density and cumulative distribution functions for the competing models applied to the COVID-19 dataset.

ISSN NUMBER: 1116-249X

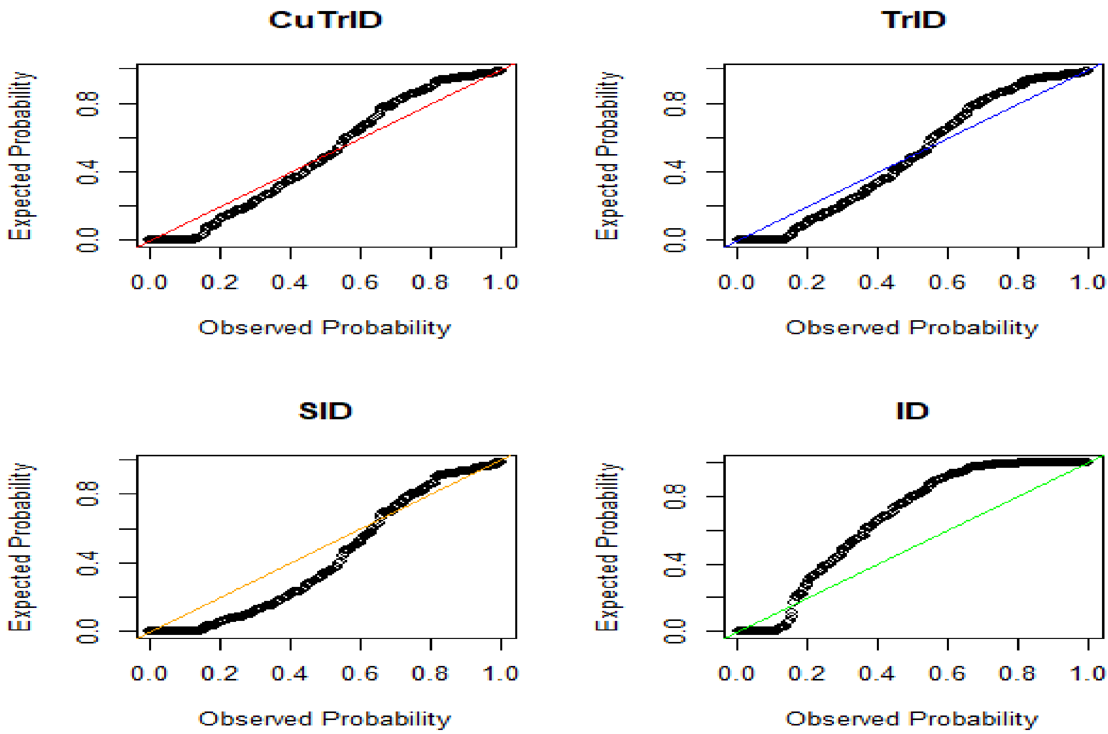

Figure 9: Distribution Fit Assessment via Probability Plots - COVID-19 Data.

Table 6 is a visual displays of the likelihood-based estimates for the parameters of the four distributions fitted to the COVID-19 dataset, while Table 7 provides the corresponding goodness-of-fit statistics, including AIC, CAIC, BIC, HQIC, and the K-S test with its p-value. For visual comparison, Figure 8 illustrates the estimated probability density functions (PDFs) and cumulative distribution functions (CDFs) of the CuTrID alongside its competing models, and Figure 9 presents the probability plots for all distributions.

As shown in Table 7, the proposed CuTrID model achieves lower values across all information criteria [AIC, CAIC, BIC, HQIC] and the K-S statistic compared to the other three distributions. This consistently indicates a superior fit to the data. The quantitative results are further corroborated by the visual evidence in Figures 8 and 9, where the CuTrID's estimated PDF and CDF align more closely with the empirical data distribution. The strong performance of the CuTrID in this application confirms the effectiveness of the Cubic transmuted framework for enhancing continuous distributions, a finding that aligns with earlier work by Rahman et al. (2019).

6. SUMMARY AND CONCLUSION

This study introduced a novel compound distribution known as the Cubic Transmuted Ishita Distribution (CuTrID) which is an integration of the Ishita distribution with Cubic Transmuted generalized family. We derived some key statistical properties of this proposed CuTrID and examined them. The compound distribution parameters were estimated using the maximum likelihood method. A simulation study was performed and its shows that the model parameter is stable and analysis of the probability density function for various parameter values demonstrates that the CuTrID is both skewed and highly flexible, with its shape being directly influenced by its parameters. Furthermore, the survival function exhibits a monotonically non-increasing pattern, indicating a decreasing probability of survival over time. The corresponding hazard function displays an increasing trend for all parameter combinations, suggesting the proposed compound distribution's suitability for analyzing survival data where the risk of failure rises over time. The usefulness of the CuTrID was

ISSN NUMBER: 1116-249X

tested using a Covid-19 dataset, where it showed a superior fit compared to three competing distributions. This enhanced performance confirms that the Cubic Transmuted Ishita Distribution is a vital contribution to the statistical distribution theory, with promising potential for application in both theoretical and applied statistical fields.

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