

## MATRIX ALGEBRA WITH STOCHASTIC TERMS FOR MODELING MARKET CAPITALIZATION TREND FUNCTIONS

<sup>1</sup>Azor, P. A., <sup>2</sup>Nelson, C. and <sup>3</sup>Amadi, I. U.

<sup>1</sup>Department of Mathematics and Statistics, Federal University Otuoke, Bayelsa State, Nigeria.

E-mail: [azor.promise@gmail.com](mailto:azor.promise@gmail.com)

<sup>2</sup>Department of Statistics, Bayelsa State Polytechnics, Aleibiri, Nigeria.

E-mail: [nelson@bayelsastatepoly.edu.ng](mailto:nelson@bayelsastatepoly.edu.ng)

<sup>3</sup>Department of Mathematics and Statistics, Captain Elechi Amadi Polytechnics, Rumuola, Port Harcourt, Rivers State, Nigeria.

E-mail: [innocent.amadi@portharcourtportpoly.edu.ng](mailto:innocent.amadi@portharcourtportpoly.edu.ng)

### Abstract

This paper explores the use of matrix algebra augmented with stochastic components to model market capitalization trends in investment planning. First, we derive mathematical formulations that express the stochastic rate of change in share prices under different trend structures such as quadratic, cubic, and seasonal models; which exert distinct effects on the share prices of Access Bank, Fidelity, and Merged Bank due to their individual characteristics. We also analyze how interest rate fluctuations influence the share prices of the three banks, finding that their share prices respond positively to rising interest rates, pointing to a potential opportunity for investors. We employ statistical metrics to evaluate how each trend model captures the characteristics of predicted share prices. Finally, we compare actual versus predicted share prices using Mean Squared Error (MSE) as the selection criterion. The results indicate that the seasonal trend model yields the most accurate predictions, with predicted values closely tracking actual ones. Graphical solutions were obtained to show the effectiveness of the fitted model. These findings carry important implications for investors, financial analysts, and policy makers, suggesting that the seasonal trend model can aid in investment decision-making and risk management.

**Keywords:** Matrix Algebra, Stochastic Terms, Quadratic, Cubic, Seasonal, Trend, Mean Square Error

### Introduction

In finance, stock refers to the capital a company or corporation raises through the issuance and subscription of shares. It is a type of security that signifies ownership of a portion of a corporation. Units of stock are commonly called shares. The stock market consists of buyers and sellers trading these ownership claims, representing partial interests in businesses. It encompasses various exchanges and trading venues where publicly held company shares are bought and sold. The legal platform that facilitates these activities is known as the stock exchange. Transactions may occur through formal institutional exchanges or over-the-counter platforms, all governed by established regulations.

The stock market is a cornerstone of a free-market economy, serving as one of the most important avenues for companies seeking expansion or launching new ventures. Its performance and operations are widely recognized as a crucial and attractive investment field within financial markets. Investors can diversify their portfolios by investing in stocks, bonds, mutual funds, and other financial instruments.

Stochastic terms, on the other hand, are random variables that introduce uncertainty into mathematical models. In finance, they capture the inherent volatility and unpredictability of

markets, making models more realistic and reliable. By integrating matrix algebra with stochastic terms, researchers can build robust frameworks that reflect the complexities of financial systems. Matrix algebra offers a powerful method for representing and analyzing complex structures, while stochastic terms account for randomness and uncertainty.

(Dao et al., 2022) highlight that matrix theory provides a rich foundation for research, offering elegant theorems that are simple yet profound in their formulation, ingenious in their proofs, and highly applicable across disciplines. In the context of market capitalization trend functions, combining matrix algebra with stochastic elements allows researchers to:

- Model the influence of interest rates on share prices using quadratic, cubic, or seasonal trend functions.
- Capture volatility and uncertainty in financial markets through stochastic components.
- Forecast and analyze market capitalization trends using matrix operations and stochastic simulations.

This integration enables the development of more accurate and robust models to guide investment strategies and risk management.

Market capitalization itself refers to the total value of a company's outstanding shares, calculated by multiplying the number of shares by their current market price. It is a key metric for investors, offering insights into a company's size, growth potential, and risk profile. Market capitalization also serves as the basis for classifying firms into categories such as small-cap, mid-cap, and large-cap. Since it fluctuates with changes in share prices, it remains an essential factor in investment decisions.

Comprehensive discussions of these concepts can be found in works such as (Jordan, 2021), (Obiyathulla & Abbas, 2019), (Robert, 2011), and (Erik, 2018).

The advantage of this present paper over (Amadi et al., 2025a), (Amadi et al., 2025b) and (Amadi et al., 2025c) is that it models the best trend function as it affects share prices of Access, Fidelity and Merged Banks, the impact of interest rates on share price changes and other statistical metrics that enhanced the effectiveness of the predictions which have not been seen elsewhere. Our novel idea complements the works of (Amadi et al., 2025a), (Amadi et al., 2025b) and (Amadi et al., 2025c) and widens the applicability of problems in this area of mathematical finance.

This paper is sectionalized as follows: Introduction, Purpose of this paper, Literature Review, Methodology, Results and Discussion of Findings, and Conclusion

### **Purpose of this Paper**

This paper is aimed at exploring the application of matrix algebra with stochastic terms for modeling market capitalization trend functions.

### **Literature Review**

Researchers have explored stock price modeling through diverse mathematical and stochastic approaches. (Amadi et al., 2025a), (Amadi et al., 2025b), (Amadi et al., 2025c), (Amadi et al., 2024a), (Amadi et al., 2024b) applied matrix calculus and stochastic differential equations to

analyze price trends, investment returns, and stock variations, highlighting linear, quadratic, seasonal, and exponential functions. Other studies examined stochastic volatility, mean reversion, drift parameters, and delay differential equations in explaining price dynamics.

(Azor et al., 2023) investigated stochastic analysis of multiple assets and the use of volatility models for investment strategies, while (Davies et al., 2023) emphasized stability and controllability of stock prices. Several authors, including (Mihova et al., 2022), (Tian-Quan & Tao, 2010), (Dura, 2011), and (Georgios et al., 2019), proposed differential equation-based models, ranging from modified ODEs to Lanchester's combat theory, for predicting or stabilizing prices.

Broader contributions also include Markov chain applications (Onwukwe and Sampson, 2014); (Dar et al., 2022), Black–Scholes extensions (Urama & Ezepe, 2018), and stability analyses of stochastic processes (Wu, 2016), (Ma & Jia, 2016). Nigerian-focused studies (Osu, 2010), (Osu & Amadi, 2022), (Nwobi & Azor, 2022), (Adeosun et al., 2015) developed stochastic and deterministic models tailored to local stock data.

Overall, findings show that stochastic differential equations, volatility processes, Markov chains, and ODE models provide useful frameworks for analyzing equilibrium prices, predicting returns, and guiding investment strategies, though stability and predictability often depend on delay parameters, volatility structures, and market-specific factors.

## Methodology

In this Section, we review all relevant stochastic methods that would help in achieving the derivation. Firstly, our focus is on stochastic processes that would lead to stochastic differential equation, which gave rise to matrix algebra with stochastic terms.

### Stochastic Processes:

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

### Stochastic Differential Equations

Here, we considered a market where the underlying asset price  $v$ ,  $0 \leq t \leq T$  on a complete probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t). \quad 0 < v < \infty. \quad (1)$$

**Theorem 1:** (Ito's formula) Let  $(\Omega, \beta, \alpha, F(\beta))$  be a filtered probability space  $X = \{X, t \geq 0\}$  be an adaptive stochastic process on  $(\Omega, \beta, \alpha, F(\beta))$  possessing a quadratic variation  $(X)$  with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t) \quad (2)$$

$t \in \mathbb{R}$  and for  $u = u(t, X(t)) \in C^{1 \times 2}(\Pi \times \mathbb{R})$

$$du(t, X(t)) = \left\{ \frac{\partial u}{\partial t} + g \frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2} \right\} d\tau + f \frac{\partial u}{\partial x} dW(t) \quad (3)$$

Adopting Theorem 1 comfortably solves the SDE in (5) with a given solution below:

$$S(t) = S_0 \exp \left\{ \sigma dW(t) + \left( \alpha - \frac{1}{2} \sigma^2 \right) t \right\}, \forall t \in [0, 1] \quad (4)$$

### Problem Formulation

Here, we use the form of matrix with stochastic terms to present initial share prices of Access Bank, Fidelity Bank and Merged Bank according to (Osu et al., 2019). Let  $A_1(t)$ ,  $A_2(t)$  and  $A_3(t)$  represents the matrices of Access Bank at different trend functions,  $F_1(t)$ ,  $F_2(t)$ , and  $F_3(t)$  represents the matrices of Fidelity Bank at different trend functions,  $M_1(t)$ ,  $M_2(t)$  and  $M_3(t)$  represents the matrices of Merged Bank at different trend functions all in time  $t$ . The  $X_i$  represents different share prices of the Banks under study, hence we have the different trend functions such as: quadratic with stochastic terms, cubic and seasonal variations with stochastic terms and their respective rate of change. Also interest rates were considered as it affects stock markets. The following dynamics are in illustrative cases.

**Case 1:** Following the method of (Amadi et al., 2025a), let us assume this is a fair market and according to historical data where share prices are represented in  $3 \times 3$  matrices of Access Bank, Fidelity and Merged Bank following the rate of change based on different trend functions: Quadratic Rate of Change with stochastic terms, Cubic Rate of Change with stochastic terms and Seasonal Rate of Change with stochastic terms, we have as follows:

- **Matrix of Quadratic trend function with stochastic terms:** A quadratic trend function with stochastic terms can be represented as follows:

**For Access Bank:**

$$A_1(t) = \begin{pmatrix} Xt^2_{011} + \sigma W_t & Xt^2_{012} + \sigma W_t & Xt^2_{013} + \sigma W_t \\ Xt^2_{021} + \sigma W_t & Xt^2_{022} + \sigma W_t & Xt^2_{023} + \sigma W_t \\ Xt^2_{031} + \sigma W & Xt^2_{032} + \sigma W & Xt^2_{033} + \sigma W_t \end{pmatrix} \quad (5)$$

**For Fidelity Bank:**

$$F_1(t) = \begin{pmatrix} Xt^2_{011} + \sigma W_t & Xt^2_{012} + \sigma W_t & Xt^2_{013} + \sigma W_t \\ Xt^2_{021} + \sigma W_t & Xt^2_{022} + \sigma W_t & Xt^2_{023} + \sigma W \\ Xt^2_{031} + \sigma W & Xt^2_{032} + \sigma W & Xt^2_{033} + \sigma W_t \end{pmatrix} \quad (6)$$

**For Merged Bank:**

$$M_1(t) = \begin{pmatrix} Xt^2_{011} + \sigma W_t & Xt^2_{012} + \sigma W_t & Xt^2_{013} + \sigma W_t \\ Xt^2_{021} + \sigma W_t & Xt^2_{022} + \sigma W_t & Xt^2_{023} + \sigma W \\ Xt^2_{031} + \sigma W & Xt^2_{032} + \sigma W & Xt^2_{033} + \sigma W_t \end{pmatrix} \quad (7)$$

The derivatives of (5) – (7) gives the following share price matrices:

$$\frac{dA_1(t)}{dt} = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \quad (8)$$

$$\frac{dF_1(t)}{dt} = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \quad (9)$$

$$\frac{dM_1(t)}{dt} = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \quad (10)$$

**Matrix of Cubic trend function with stochastic terms:** Matrix of cubic trend function can be represented as follow:

**For Access Bank:**

$$A_2(t) = \begin{pmatrix} Xt^3_{011} + \sigma W_t & Xt^3_{012} + \sigma W_t & Xt^3_{013} + \sigma W_t \\ Xt^3_{021} + \sigma W_t & Xt^3_{022} + \sigma W_t & Xt^3_{023} + \sigma W_t \\ Xt^3_{031} + \sigma W_t & Xt^3_{032} + \sigma W & Xt^3_{033} + \sigma W_t \end{pmatrix} \quad (11)$$

**For Fidelity Bank:**

$$F_2(t) = \begin{pmatrix} Xt^3_{011} + \sigma W_t & Xt^3_{012} + \sigma W_t & Xt^3_{013} + \sigma W_t \\ Xt^3_{021} + \sigma W_t & Xt^3_{022} + \sigma W_t & Xt^3_{023} + \sigma W_t \\ Xt^3_{031} + \sigma W_t & Xt^3_{032} + \sigma W & Xt^3_{033} + \sigma W_t \end{pmatrix} \quad (12)$$

**For Merged Bank:**

$$M_2(t) = \begin{pmatrix} Xt^3_{011} + \sigma W_t & Xt^3_{012} + \sigma W_t & Xt^3_{013} + \sigma W_t \\ Xt^3_{021} + \sigma W_t & Xt^3_{022} + \sigma W_t & Xt^3_{023} + \sigma W_t \\ Xt^3_{031} + \sigma W_t & Xt^3_{032} + \sigma W & Xt^3_{033} + \sigma W_t \end{pmatrix} \quad (13)$$

The derivatives of (11) – (13) gives the following share price matrices:

$$\frac{dA_2(t)}{dt} = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (14)$$

$$\frac{dF_2(t)}{dt} = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (15)$$

$$\frac{dM_2(t)}{dt} = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (16)$$

➤ **Matrix of Seasonal trend Function with stochastic terms:** A matrix of cubic trend function with stochastic terms can be represented as follows:

**For Access Bank:**

$$A_3(t) = \begin{pmatrix} X\sin(t)_{011} + \sigma W_t & X\cos(t)_{012} + \sigma W_t & X\sin(t)_{013} + \sigma W_t \\ X\sin(t)_{021} + \sigma W_t & X\cos(t)_{022} + \sigma W_t & X\sin(t)_{023} + \sigma W_t \\ X\sin(t)_{031} + \sigma W_t & X\cos(t)_{032} + \sigma W_t & X\sin(t)_{033} + \sigma W_t \end{pmatrix} \quad (17)$$

**For Fidelity Bank:**

$$F_3(t) = \begin{pmatrix} X\sin(t)_{011} + \sigma W_t & X\cos(t)_{012} + \sigma W_t & X\sin(t)_{013} + \sigma W_t \\ X\sin(t)_{021} + \sigma W_t & X\cos(t)_{022} + \sigma W_t & X\sin(t)_{023} + \sigma W_t \\ X\sin(t)_{031} + \sigma W_t & X\cos(t)_{032} + \sigma W_t & X\sin(t)_{033} + \sigma W_t \end{pmatrix} \quad (18)$$

**For Merged Bank:**

$$M_3(t) = \begin{pmatrix} X\sin(t)_{011} + \sigma W_t & X\cos(t)_{012} + \sigma W_t & X\sin(t)_{013} + \sigma W_t \\ X\sin(t)_{021} + \sigma W_t & X\cos(t)_{022} + \sigma W_t & X\sin(t)_{023} + \sigma W_t \\ X\sin(t)_{031} + \sigma W_t & X\cos(t)_{032} + \sigma W_t & X\sin(t)_{033} + \sigma W_t \end{pmatrix} \quad (19)$$

The derivatives of (17) – (19) gives the following share price matrices:

$$\frac{dA_3(t)}{dt} = \begin{pmatrix} X\cos(t)_{011} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{012} + \sigma \frac{dW_t}{dt} & X\cos(t)_{013} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{021} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{022} + \sigma \frac{dW_t}{dt} & X\cos(t)_{023} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{031} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{032} + \sigma \frac{dW_t}{dt} & X\cos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (20)$$

$$\frac{dF_3(t)}{dt} = \begin{pmatrix} X\cos(t)_{011} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{012} + \sigma \frac{dW_t}{dt} & X\cos(t)_{013} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{021} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{022} + \sigma \frac{dW_t}{dt} & X\cos(t)_{023} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{031} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{032} + \sigma \frac{dW_t}{dt} & X\cos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (21)$$

$$\frac{dM_3(t)}{dt} = \begin{pmatrix} X\cos(t)_{011} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{012} + \sigma \frac{dW_t}{dt} & X\cos(t)_{013} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{021} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{022} + \sigma \frac{dW_t}{dt} & X\cos(t)_{023} + \sigma \frac{dW_t}{dt} \\ X\cos(t)_{031} + \sigma \frac{dW_t}{dt} & -X\sin(t)_{032} + \sigma \frac{dW_t}{dt} & X\cos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \quad (22)$$

**Statistical Metrics to measure each of the Trend Functions on the three Banks under study:**

However, when considering future investments, understanding some statistical metric of predicted stock trends can help investors or Access Bank and Fidelity Bank and Merged Bank, according to (Osu et al., 2019), to make informed decisions. Hence, let each of the trend functions: Quadratic, Cubic and Seasonal be denoted as  $(Q_1, Q_2, Q_3, \dots, Q_n)'$ ,  $(C_1, C_2, C_3, \dots, C_n)'$  and  $(S_1, S_2, S_3, \dots, S_n)'$  during the trading days with their respective Mean, Standard Deviation, Kurtosis and Skewness be represented as  $(\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)'$ ,  $(\beta_1, \beta_2, \beta_3, \dots, \beta_n)'$ ,  $(\phi_1, \phi_2, \phi_3, \dots, \phi_n)'$  and  $(\theta_1, \theta_2, \theta_3, \dots, \theta_n)'$  such that we have Tables 1-3. Hence, the following metrics are defined thus:

$$\text{Mean : } \mu = \frac{1}{n} \sum_{i=1}^n x_i \quad (23)$$

$$\text{Standard Deviation: } \mu = \frac{1}{n} \sum_{i=1}^n x_i \quad (24)$$

$$\text{Kurtosis: } k = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^4 - 3}{\left( \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 \right)^2} \quad (25)$$

$$\text{Skewness: } s = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \mu)^3}{\left( \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 \right)^{\frac{3}{2}}} \quad (26)$$

where  $x_i$  is individual predicted stock prices,  $\mu$  is the mean of the predicted stock prices and  $n$  is the number of predicted stock prices.

**Table 1: Independent Trend Functions Assessing Access Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal Trends	Mean	Standard Deviation	Kurtosis	Skewness
$Q_1$	$C_1$	$S_1$	$\alpha_1$	$\beta_1$	$\phi_1$	$\theta_1$
$Q_2$	$C_2$	$S_2$	$\alpha_2$	$\beta_2$	$\phi_2$	$\theta_2$
$Q_3$	$C_3$	$S_3$	$\alpha_3$	$\beta_3$	$\phi_3$	$\theta_3$
$Q_4$	$C_4$	$S_4$	$\alpha_4$	$\beta_4$	$\phi_4$	$\theta_4$
$Q_5$	$C_5$	$S_5$	$\alpha_5$	$\beta_5$	$\phi_5$	$\theta_5$
$Q_6$	$C_6$	$S_6$	$\alpha_6$	$\beta_6$	$\phi_6$	$\theta_6$
$Q_7$	$C_7$	$S_7$	$\alpha_7$	$\beta_7$	$\phi_7$	$\theta_7$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$
$Q_n$	$C_n$	$S_n$	$\alpha_n$	$\beta_n$	$\phi_n$	$\theta_n$



**Table 2: Independent Trend Functions Assessing Fidelity Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal Trends	Mean	Standard Deviation	Kurtosis	Skewness
$Q_1$	$C_1$	$S_1$	$\alpha_1$	$\beta_1$	$\phi_1$	$\theta_1$
$Q_2$	$C_2$	$S_2$	$\alpha_2$	$\beta_2$	$\phi_2$	$\theta_2$
$Q_3$	$C_3$	$S_3$	$\alpha_3$	$\beta_3$	$\phi_3$	$\theta_3$
$Q_4$	$C_4$	$S_4$	$\alpha_4$	$\beta_4$	$\phi_4$	$\theta_4$
$Q_5$	$C_5$	$S_5$	$\alpha_5$	$\beta_5$	$\phi_5$	$\theta_5$
$Q_6$	$C_6$	$S_6$	$\alpha_6$	$\beta_6$	$\phi_6$	$\theta_6$
$Q_7$	$C_7$	$S_7$	$\alpha_7$	$\beta_7$	$\phi_7$	$\theta_7$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$
$Q_n$	$C_n$	$S_n$	$\alpha_n$	$\beta_n$	$\phi_n$	$\theta_n$

**Table 3: Independent Trend Functions Assessing Merged Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal Trends	Mean	Standard Deviation	Kurtosis	Skewness
$Q_1$	$C_1$	$S_1$	$\alpha_1$	$\beta_1$	$\phi_1$	$\theta_1$
$Q_2$	$C_2$	$S_2$	$\alpha_2$	$\beta_2$	$\phi_2$	$\theta_2$
$Q_3$	$C_3$	$S_3$	$\alpha_3$	$\beta_3$	$\phi_3$	$\theta_3$
$Q_4$	$C_4$	$S_4$	$\alpha_4$	$\beta_4$	$\phi_4$	$\theta_4$
$Q_5$	$C_5$	$S_5$	$\alpha_5$	$\beta_5$	$\phi_5$	$\theta_5$
$Q_6$	$C_6$	$S_6$	$\alpha_6$	$\beta_6$	$\phi_6$	$\theta_6$
$Q_7$	$C_7$	$S_7$	$\alpha_7$	$\beta_7$	$\phi_7$	$\theta_7$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\dots$	$\dots$	$\dots$
$Q_n$	$C_n$	$S_n$	$\alpha_n$	$\beta_n$	$\phi_n$	$\theta_n$

### Effects of Interest Rate on Share Price Changes

**Case 2:** let  $A'(t)$ ,  $F'(t)$  and  $M'(t)$  denote the total effects of interest rate for Access Bank, Fidelity Bank and Merged Bank under the three trend functions during the period of trading. Also let  $(n_1, n_2, \dots, n_n)'$  represents interest rate parameters. In view of the rate of change following :  $\frac{A_1(t)}{dt}, \frac{A_2(t)}{dt}$  and  $\frac{A_3(t)}{dt}$  represents the matrices of Access Bank at different trend functions,  $\frac{F_1(t)}{dt}, \frac{F_2(t)}{dt}$  and  $\frac{F_3(t)}{dt}$  represents the matrices of Fidelity Bank at different trend functions,  $\frac{M_1(t)}{dt}, \frac{M_2(t)}{dt}$  and  $\frac{M_3(t)}{dt}$  represents the matrices of Merged Bank at different trend functions all in time  $t$  ; hence the entire dynamics follows:

$$i. A'_1(t) = \frac{A_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, F'_1(t) = \frac{F_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}. \text{ and } M'_1(t) = \frac{M_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}. \quad (27)$$

$$ii. A'_2(t) = \frac{A_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, F'_2(t) = \frac{F_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}. \text{ and } M'_2(t) = \frac{M_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (28)$$

$$iii. A'_3(t) = \frac{A_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}, F'_3(t) = \frac{F_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}. \text{ and } M'_3(t) = \frac{M_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (29)$$

**Effects of Interest Rate Parameters to Study the Rate of Change of Access Bank when the Trend Function is Quadratic with Stochastic Terms:**

**For Access Bank: i.  $A'_1(t) = \frac{A_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}.$**

$$A'_1(t) = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (30)$$

**For Fidelity Bank: ii.  $F'_1(t) = \frac{F_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}.$**

$$F'_1(t) = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (31)$$

**For Merged Bank: iii.  $M'_1(t) = \frac{M_1(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$**

$$M'_1(t) = \begin{pmatrix} 2Xt_{011} + \sigma \frac{W_t}{dt} & 2Xt_{012} + \sigma \frac{W_t}{dt} & 2Xt_{013} + \sigma \frac{W_t}{dt} \\ 2Xt_{021} + \sigma \frac{W_t}{dt} & 2Xt_{022} + \sigma \frac{W_t}{dt} & 2Xt_{023} + \sigma \frac{W_t}{dt} \\ 2Xt_{031} + \sigma \frac{W_t}{dt} & 2Xt_{032} + \sigma \frac{W_t}{dt} & 2Xt_{033} + \sigma \frac{W_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (32)$$

**Effects Of Interest Rate Parameters To Study The Rate Of Change Of Access Bank When The Trend Function Is Cubic With Stochastic Terms:**

**For Access Bank: i.  $A'_2(t) = \frac{A_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ .**

$$A'_2(t) = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (33)$$

**For Fidelity Bank: ii.  $F'_2(t) = \frac{F_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ .**

$$F'_2(t) = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (34)$$

**For Merged Bank: iii.  $M'_2(t) = \frac{M_2(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$**

$$M'_2(t) = \begin{pmatrix} 3Xt^2_{011} + \sigma \frac{dW_t}{dt} & 3Xt^2_{012} + \sigma \frac{dW_t}{dt} & 3Xt^2_{013} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{021} + \sigma \frac{dW_t}{dt} & 3Xt^2_{022} + \sigma \frac{dW_t}{dt} & 3Xt^2_{023} + \sigma \frac{dW_t}{dt} \\ 3Xt^2_{031} + \sigma \frac{dW_t}{dt} & 3Xt^2_{032} + \sigma \frac{dW_t}{dt} & 3Xt^2_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (35)$$

**Effects Of Interest Rate Parameters to Study the Rate Of Change Of Access Bank When The Trend Function Is Seasonal With Stochastic Terms:**

**For Access Bank: i.**  $A'_3(t) = \frac{A_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

$$A'_3(t) = \begin{pmatrix} XCos(t)_{011} + \sigma \frac{dW_t}{dt} & -XSin(t)_{012} + \sigma \frac{dW_t}{dt} & XCos(t)_{013} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{021} + \sigma \frac{dW_t}{dt} & -XSin(t)_{022} + \sigma \frac{dW_t}{dt} & XCos(t)_{023} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{031} + \sigma \frac{dW_t}{dt} & -XSin(t)_{032} + \sigma \frac{dW_t}{dt} & XCos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (36)$$

For Fidelity Bank: ii.  $F'_3(t) = \frac{F_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}.$

$$F'_3(t) = \begin{pmatrix} XCos(t)_{011} + \sigma \frac{dW_t}{dt} & -XSin(t)_{012} + \sigma \frac{dW_t}{dt} & XCos(t)_{013} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{021} + \sigma \frac{dW_t}{dt} & -XSin(t)_{022} + \sigma \frac{dW_t}{dt} & XCos(t)_{023} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{031} + \sigma \frac{dW_t}{dt} & -XSin(t)_{032} + \sigma \frac{dW_t}{dt} & XCos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (37)$$

**For Merged Bank: iii.**  $M'_3(t) = \frac{M_3(t)}{dt} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

$$M'_3(t) = \begin{pmatrix} XCos(t)_{011} + \sigma \frac{dW_t}{dt} & -XSin(t)_{012} + \sigma \frac{dW_t}{dt} & XCos(t)_{013} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{021} + \sigma \frac{dW_t}{dt} & -XSin(t)_{022} + \sigma \frac{dW_t}{dt} & XCos(t)_{023} + \sigma \frac{dW_t}{dt} \\ XCos(t)_{031} + \sigma \frac{dW_t}{dt} & -XSin(t)_{032} + \sigma \frac{dW_t}{dt} & XCos(t)_{033} + \sigma \frac{dW_t}{dt} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \quad (38)$$

### Comparison of Estimation on the Trend Functions

We have earlier defined our trend functions such as: Quadratic trend, Cubic trend and Seasonal trend. The Mean Square Errors (MSE) shall be used as a criterion for selection of the best trend for each of banks. It is a measure of average squared difference between predicted and actual values. The mean squared error (MSE) criterion is given by:

$$MSE = \left( \frac{1}{n} \right) \sum (y_i - \hat{y}_i)^2 \quad (39)$$

where  $y_i$  is the actual share prices of Access Bank, Fidelity Bank and Merged Bank,  $\hat{y}_i$  is the predicted values,  $n$  is the number of data points and  $\sum$  is summation symbols, indicating the sum of the values. The method with the minimum mean squared error (MMMSE) becomes the best methods for the estimation of trend functions.

### RESULTS AND DISCUSSIONS

**Quadratic Trend with Stochastic Terms:** This trend function captures non-linear relationship between variables. The results suggest that the quadratic trend has a significant impact on share prices, indicating that the rate of change in share prices is influenced by the quadratic term.

**Cubic Trend with Stochastic Terms:** This trend function captures more complex nonlinear relationships. The results show that the cubic trend also have significant impact on share prices, indicating that the rate of change in share prices is influenced by the cubic term.

**Seasonal Trend with Stochastic Terms:** This trend function captures periodic fluctuations in share prices. The results suggest that the seasonal trend has a significant impact on share prices, indicating that share prices are influenced by seasonal factors. The negative predicted share prices indicate potential crashes or significant decline in share prices during the period of trading.

From the analysis of interest rate on share price changes, it can be seen that an increase in the share price rate of change increases the value of the share price. This suggests that as interest rates change, the share price of these banks respond positively, indicating a potential opportunity for investors.

Tables 1 - 3 show the interpretations of statistical metrics as follows: The mean of the predicted share prices indicates the average value of the share prices over time, which is informative to the three banks understudy. Looking at the average share prices of Access Bank, Fidelity Bank

and Merged Bank over time can help investors identify patterns and trends that may be useful for making investment decisions. The average share price can be comparing different stocks or sectors, which can be helpful for diversifying portfolios, see Column 4. The standard deviation of the predicted share prices indicates the volatility of the share prices, with higher values indicating greater uncertainty. Standard deviation can be important consideration for Access bank, Fidelity Bank and Merged Bank who are trying to manage their risk to make predictions about future price movements, see Column 5. In Column 6, the kurtosis of the predicted share prices indicates the presence of outliers or extreme values, with higher values indicating a greater likelihood of extreme price movements; while the skewness in column 7 of predicated share prices indicates the asymmetry of the distribution, with positive values indicating a longer tail on the right of the distribution. A negatively skewed distribution indicates a riskier investment, while a positively skewed distribution indicates a more attractive investment opportunity. Therefore, understanding skewness, investors can make more informed decisions and develop strategies to manage risk and potential returns.

However, in comparing Tables 4 - 6, the results show that the seasonal trend function perform best on predicting actual share prices with predicted values. The quadratic and cubic trend function perform poorly with predicted values deviating significantly from the actual values.

Figure 1 connotes the share price movements of Access Bank, Fidelity Bank and Merged Bank both actual and predicted against time. The shapes of the plots are not stable during the period of trading of shares. Carefully looking at the plots, revealed the up and down movement of the trend; almost touching the time axis and grows upward at the middle of the extreme end. These scenarios may not be beneficial to an investor who wants to invest when trend lines follow quadratic trend functions. Figure 2 describes the proper movement of the three share prices of the banks and their corresponding predicted prices under cubic trend functions against time. The complex, non-linear relationship between time and share prices is suggesting changes in all directions. These scenarios are not also stable to invest or advise an investor or the banks management to buy or sell their shares due to the uncertainties during the trading days. Critical observations at the trend lines, shows upward and downward trend, finally moves up. However, the deviations between actual and predicted prices show potential errors or anomalies.

However, in Figure 3 indicates plots of three bank share prices and their predicted prices under seasonal trend against time. The plots have fluctuations due to seasons but more predictable. This is because the actual and predicted prices of shares are relatively close. More so, all the trend lines move above the origin along time axis. So, investors or bank management will make more profit of shares during the period of seasons compared to quadratic and cubic trends respectively.

In all, the three plots look alike but different due to variations in the trend functions and the stochastic formation in the price history of stock markets.

More so, Table 7 talks about the comparison of trend function using Mean Squared Error (MSE). The results show that the seasonal trend function has the least minimal value of MSE, indicating it is the best performed among the three trend functions. This suggests that the seasonal that the seasonal trend function is better suited for modeling the share prices of the three banks. These results have some implications for share prices of access Bank, Fidelity Bank and Merged Bank. The seasonal trend function can be used to predict share prices and identify

This Section presents analyzed results whose methods are stated in the methodology. Hence, we have the following parameter values:  $n = 0.2, 0.5, 0.8$ ,  $dW = 0.002$ ,  $\sigma = 0.25$ ,  $dt = 0.005$  and  $t = 1$ . which were implemented using Matlab programming software:

#### **Analysis of Rate of Change on the Share Prices as it Affects Trend Functions**

- **The rate of change when the share prices of Access Bank, Fidelity Bank and Merged Bank follows quadratic trend function with stochastic terms:**

$$\frac{dA_1(t)}{dt} = \begin{pmatrix} 820.1 & 160.1 & 252.1 \\ 158.1 & 196.1 & 184.1 \\ 254.1 & 182.1 & 756.1 \end{pmatrix}, \frac{dF_1(t)}{dt} = \begin{pmatrix} 830.1 & 124.1 & 270.1 \\ 122.1 & 242.1 & 162.1 \\ 278.1 & 160.1 & 568.1 \end{pmatrix}$$

$$\frac{dM_1(t)}{dt} = \begin{pmatrix} 1650.1 & 284.1 & 528.1 \\ 280.1 & 438.1 & 162.1 \\ 532.1 & 160.1 & 568.1 \end{pmatrix}$$

- **The rate of change when the share prices of Access Bank, Fidelity Bank and Merged Bank follows cubic trend function with stochastic terms:**

$$\frac{dA_2(t)}{dt} = \begin{pmatrix} 1230.1 & 240.1 & 378.1 \\ 158.1 & 294.1 & 276.1 \\ 381.1 & 273.1 & 1134.1 \end{pmatrix}, \frac{dF_2(t)}{dt} = \begin{pmatrix} 1245.1 & 186.1 & 414.1 \\ 183.1 & 363.1 & 243.1 \\ 417.1 & 240.1 & 852.1 \end{pmatrix}$$

$$\frac{dM_2(t)}{dt} = \begin{pmatrix} 2475.1 & 426.1 & 792.1 \\ 420.1 & 657.1 & 519.1 \\ 798.1 & 513.1 & 2286.1 \end{pmatrix}$$

**The rate of change when the share prices of Access Bank, Fidelity Bank and Merged Bank follows seasonal trend function with stochastic terms:**

$$\frac{dA_3(t)}{dt} = \begin{pmatrix} 409.631 & -1.496 & 125.9866 \\ 79.0289 & -1.7101 & 92.0172 \\ 126.9857 & -1.58795 & 377.7598 \end{pmatrix}, \frac{dF_3(t)}{dt} = \begin{pmatrix} 414.7265 & -1.1819 & 137.9758 \\ 61.0451 & -2.21145 & 81.0271 \\ 138.9749 & -1.496 & 283.8444 \end{pmatrix}$$

$$\frac{dM_3(t)}{dt} = \begin{pmatrix} 824.3575 & -2.6779 & 263.8624 \\ 139.974 & -3.92155 & 172.9443 \\ 265.8606 & -3.08395 & 761.4142 \end{pmatrix}$$

### Analysis of interest Rates on Share Price Changes

- **Effects of interest rate parameters to study the rate of change of Access Bank when the trend function is Quadratic with stochastic terms:**

**For Access Bank:**

- **Low interest rate of 2%:**

$$A'_1(t) = \begin{pmatrix} 820.1 & 160.1 & 252.1 \\ 158.1 & 196.1 & 184.1 \\ 254.1 & 182.1 & 756.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 246.4600 \\ 107.6600 \\ 238.4600 \end{pmatrix}$$

- **Moderate interest rates of 5%:**

$$A'_1(t) = \begin{pmatrix} 820.1 & 160.1 & 252.1 \\ 158.1 & 196.1 & 184.1 \\ 254.1 & 182.1 & 756.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 616.1500 \\ 269.1500 \\ 596.1500 \end{pmatrix}$$

- **High interest rate of 8%:**



$$A_1'(t) = \begin{pmatrix} 820.1 & 160.1 & 252.1 \\ 158.1 & 196.1 & 184.1 \\ 254.1 & 182.1 & 756.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 985.8400 \\ 430.6400 \\ 953.8400 \end{pmatrix}$$

### For Fidelity Bank:

#### ➤ Low interest rate of 2%:

$$F_1'(t) = \begin{pmatrix} 830.1 & 124.1 & 270.1 \\ 122.1 & 242.1 & 162.1 \\ 278.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 246.0600 \\ 105.2600 \\ 201.2600 \end{pmatrix}$$

#### ➤ Moderate interest rates of 5%:

$$F_1'(t) = \begin{pmatrix} 830.1 & 124.1 & 270.1 \\ 122.1 & 242.1 & 162.1 \\ 278.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 615.1500 \\ 263.1500 \\ 503.1500 \end{pmatrix}$$

#### ➤ High interest rate of 8%:

$$F_1'(t) = \begin{pmatrix} 830.1 & 124.1 & 270.1 \\ 122.1 & 242.1 & 162.1 \\ 278.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 984.2400 \\ 421.0400 \\ 805.0400 \end{pmatrix}$$

### For Merged Bank:

#### ➤ Low interest rate of 2%:

$$M_1'(t) = \begin{pmatrix} 1650.1 & 284.1 & 528.1 \\ 280.1 & 438.1 & 162.1 \\ 532.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 412.4500 \\ 176.0600 \\ 252.0600 \end{pmatrix}$$

#### ➤ Moderate interest rates of 5%:

$$M_1'(t) = \begin{pmatrix} 1650.1 & 284.1 & 528.1 \\ 280.1 & 438.1 & 162.1 \\ 532.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 27186.6933 \\ 27184.3206 \\ 27184.8906 \end{pmatrix}$$

#### ➤ High interest rate of 8%:

$$M_1'(t) = \begin{pmatrix} 1650.1 & 284.1 & 528.1 \\ 280.1 & 438.1 & 162.1 \\ 532.1 & 160.1 & 568.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 27188.9094 \\ 27185.1126 \\ 27186.0246 \end{pmatrix}$$

- **Effects of interest rate parameters to study the rate of change of Access Bank when the trend function is Cubic with stochastic terms:**

**For Access Bank:**

- **Low interest rates of 2%:**

$$A'_2(t) = \begin{pmatrix} 1230.1 & 240.1 & 378.1 \\ 158.1 & 294.1 & 276.1 \\ 381.1 & 273.1 & 1134.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 369.6600 \\ 145.6600 \\ 357.6600 \end{pmatrix}$$

- **Moderate interest rates of 5%:**

$$A'_2(t) = \begin{pmatrix} 1230.1 & 240.1 & 378.1 \\ 158.1 & 294.1 & 276.1 \\ 381.1 & 273.1 & 1134.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 924.1500 \\ 364.1500 \\ 894.1500 \end{pmatrix}$$

- **High interest rate of 8%:**

$$A'_2(t) = \begin{pmatrix} 1230.1 & 240.1 & 378.1 \\ 158.1 & 294.1 & 276.1 \\ 381.1 & 273.1 & 1134.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 27187.4358 \\ 27184.7478 \\ 27187.2918 \end{pmatrix}$$

**For Fidelity Bank:**

- **Low interest rates of 2%:**

$$F'_2(t) = \begin{pmatrix} 1245.1 & 186.1 & 414.1 \\ 183.1 & 363.1 & 243.1 \\ 417.1 & 240.1 & 852.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 369.0600 \\ 157.8600 \\ 301.8600 \end{pmatrix}$$

- **Moderate interest rates of 5%:**

$$F'_2(t) = \begin{pmatrix} 1245.1 & 186.1 & 414.1 \\ 183.1 & 363.1 & 243.1 \\ 417.1 & 240.1 & 852.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 922.6500 \\ 394.6500 \\ 754.6500 \end{pmatrix}$$

- **High interest rate of 8%:**

$$F_2'(t) = \begin{pmatrix} 1245.1 & 186.1 & 414.1 \\ 183.1 & 363.1 & 243.1 \\ 417.1 & 240.1 & 852.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 27187.4286 \\ 27184.8942 \\ 27186.6222 \end{pmatrix}$$

### For Merged Bank:

➤ **Low interest rates of 2%:**

$$M_2'(t) = \begin{pmatrix} 2475.1 & 426.1 & 792.1 \\ 420.1 & 657.1 & 519.1 \\ 798.1 & 513.1 & 2286.1 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 738.6600 \\ 319.2600 \\ 719.4600 \end{pmatrix}$$

➤ **Moderate interest rate of 5%:**

$$M_2'(t) = \begin{pmatrix} 2475.1 & 426.1 & 792.1 \\ 420.1 & 657.1 & 519.1 \\ 798.1 & 513.1 & 2286.1 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 27188.5398 \\ 27185.3946 \\ 27188.3961 \end{pmatrix}$$

➤ **High interest rate of 8%:**

$$M_2'(t) = \begin{pmatrix} 2475.1 & 426.1 & 792.1 \\ 420.1 & 657.1 & 519.1 \\ 798.1 & 513.1 & 2286.1 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 27191.8638 \\ 27186.831 \\ 27191.6334 \end{pmatrix}$$

➤ **Effects of interest rate parameters to study the rate of change of Access Bank when the trend function when the trend function is Seasonal with stochastic terms:**

### For Access Bank:

➤ **Low interest rate of 2%:**

$$A_3'(t) = \begin{pmatrix} 409.631 & -1.496 & 125.9866 \\ 79.0289 & -1.7101 & 92.0172 \\ 126.9857 & -1.58795 & 377.7598 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 106.8243 \\ 33.8672 \\ 100.6315 \end{pmatrix}$$

➤ **Moderate interest rate of 5%:**

$$A_3'(t) = \begin{pmatrix} 409.631 & -1.496 & 125.9866 \\ 79.0289 & -1.7101 & 92.0172 \\ 126.9857 & -1.58795 & 377.7598 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 267.0608 \\ 84.6680 \\ 251.5788 \end{pmatrix}$$

➤ **High interest rate of 8%:**

$$A'_3(t) = \begin{pmatrix} 409.631 & -1.496 & 125.9866 \\ 79.0289 & -1.7101 & 92.0172 \\ 126.9857 & -1.58795 & 377.7598 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 427.2973 \\ 135.4688 \\ 402.5260 \end{pmatrix}$$

**For Fidelity Bank:**

➤ **Low interest rate of 2%:**

$$F'_3(t) = \begin{pmatrix} 414.7265 & -1.1819 & 137.9758 \\ 61.0451 & -2.21145 & 81.0271 \\ 138.9749 & -1.496 & 283.8444 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 110.3041 \\ 27.9721 \\ 27.5025 \end{pmatrix}$$

➤ **Moderate interest rate of 5%:**

$$F'_3(t) = \begin{pmatrix} 414.7265 & -1.1819 & 137.9758 \\ 61.0451 & -2.21145 & 81.0271 \\ 138.9749 & -1.496 & 283.8444 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 275.7602 \\ 69.9304 \\ 68.7562 \end{pmatrix}$$

➤ **High interest rate of 8%:**

$$F'_3(t) = \begin{pmatrix} 414.7265 & -1.1819 & 137.9758 \\ 61.0451 & -2.21145 & 81.0271 \\ 138.9749 & -1.496 & 283.8444 \end{pmatrix} \begin{pmatrix} 0.8 \\ 0.8 \\ 0.8 \end{pmatrix} = \begin{pmatrix} 441.2163 \\ 111.8886 \\ 110.0099 \end{pmatrix}$$

**For Merged Bank:**

➤ **Low interest rate of 2%:**

$$M'_3(t) = \begin{pmatrix} 824.3575 & -2.6779 & 263.8624 \\ 139.974 & -3.92155 & 172.9443 \\ 265.8606 & -3.08395 & 761.4142 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 217.1284 \\ 61.7994 \\ 109.3242 \end{pmatrix}$$

➤ **Moderate interest rate of 5%:**

$$M'_3(t) = \begin{pmatrix} 824.3575 & -2.6779 & 263.8624 \\ 139.974 & -3.92155 & 172.9443 \\ 265.8606 & -3.08395 & 761.4142 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 542.8210 \\ 154.4984 \\ 273.8105 \end{pmatrix}$$

➤ **High interest rate of 8%:**

$$M'_3(t) = \begin{pmatrix} 824.3575 & -2.6779 & 263.8624 \\ 139.974 & -3.92155 & 172.9443 \\ 265.8606 & -3.08395 & 761.4142 \end{pmatrix} \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 868.5136 \\ 247.1974 \\ 437.2968 \end{pmatrix}$$

**Table 4: Independent Trend Functions with their corresponding Statistical Metrics in Assessing Access Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal trends	Mean	Standard Deviation	Kurtosis	Skewness
820.1	1230.1	409.631	819.9013	410.2345	1.5000	-7.0009e-04
160.1	240.1	-1.496	132.9013	123.0731	1.5000	-0.3862
252.1	378.1	125.9866	252.0622	126.0567	1.5000	-3.5089e-04
158.1	158.1	79.0289	131.7430	45.6517	1.5000	-0.7071
196.1	294.1	-1.7101	162.8300	150.6854	1.5000	-0.3858
184.1	276.1	92.0172	184.0724	92.0414	1.5000	-5.5089e-04
254.1	381.1	126.9857	254.0619	127.0572	1.5000	-5.5089e-04
182.1	273.1	-1.58795	151.2040	139.9260	1.5000	-0.3859
756.1	1133.1	377.7598	755.6533	377.6703	1.5000	-0.0022

**Table 5: Independent Trend Functions with their corresponding Statistical Metrics in Fidelity Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal Trends	Mean	Standard Deviation	Kurtosis	Skewness
830.1	1245.1	414.7265	829.9755	415.1868	1.5000	-5.5089e-04
124.1	186.1	-1.1819	103.0060	95.4062	1.5000	-0.3863
276.1	414.1	137.9758	276.0586	138.0621	1.5000	-5.5089e-04
122.1	183.1	61.0451	122.0817	61.0275	1.5000	-5.5089e-04
242.1	363.1	-2.21145	200.9962	186.0921	1.5000	-0.3860
162.1	243.1	81.0271	162.0757	81.0365	1.5000	-5.5089e-04
278.1	417.1	138.9749	378.0583	139.0626	1.5000	-5.5089e-04
160.1	240.1	-1.496	132.9013	123.0731	1.5000	-0.3862
568.1	852.1	283.8444	568.0148	284.1278	1.5000	-5.5089e-04

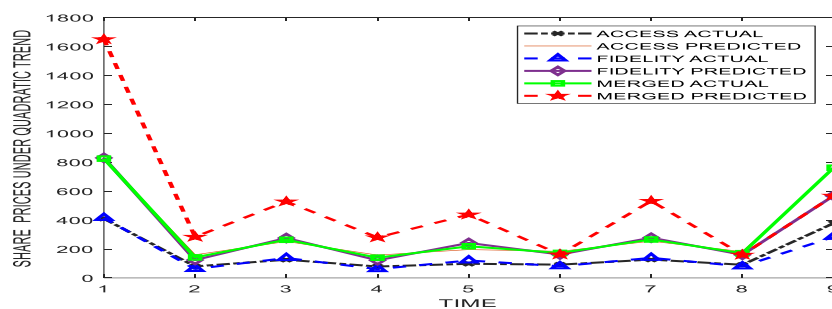
**Table 6: Independent Trend Functions with their corresponding Statistical Metrics in Assessing Merged Bank Share Price Changes**

Quadratic trends	Cubic trends	Seasonal Trends	Mean	Standard Deviation	Kurtosis	Skewness
1650.1	2475.1	824.3375	1.6498e+03	825.3813	1.5000	-5.6572e-04
284.1	426.1	-2.5779	235.8740	218.3701	1.5000	-0.3859

528.1	792.1	263.8624	528.0208	264.1188	1.5000	-5.5089e-04
280.1	420.1	139.974	280.0580	140.0630	1.5000	-5.5089e-04
438.1	657.1	-3.92155	363.7595	336.7228	1.5000	-0.3858
162.1	519.1	172.9443	284.7148	203.0560	1.5000	0.7048
532.1	798.1	265.8606	532.0202	266.1197	1.5000	-5.5089e-04
160.1	513.1	-3.08395	223.3720	263.8446	1.5000	0.4152
568.1	2286.1	283.8444	1.0460e+03	1.0833e+03	1.5000	0.6527

**Table 7: Comparison of Results under Quadratic Trend Functions**

Access Actual Share prices	Predicted Prices	Fidelity Actual Share prices	Predicted Prices	Merged Actual Share prices	Predicted Prices
410	820.1	415	830.1	825	1650.1
80	160.1	62	124.1	142	284.1
126	252.1	138	276.1	264	528.1
79	158.1	61	122.1	140	280.1
98	196.1	121	242.1	219	438.1
92	184.1	81	162.1	173	162.1
127	254.1	139	278.1	266	532.1
91	182.1	80	160.1	171	160.1
378	756.1	284	568.1	762	568.1

**Figure 1: Plots of the three banks Actual share prices and their Predicted prices against Time under Quadratic Trend Function****Table 8: Comparison of Results under Cubic Trend Functions**

Access Actual Share prices	Predicted Prices	Fidelity Actual Share prices	Predicted Prices	Merged Actual Share prices	Predicted Prices
410	1230.1	415	1245.1	825	2475.1
80	240.1	62	186.1	142	426.1
126	378.1	138	414.1	264	792.1

79	158.1	61	183.1	140	420.1
98	294.1	121	363.1	219	657.1
92	276.1	81	243.1	173	519.1
127	381.1	139	417.1	266	798.1
91	273.1	80	240.1	171	513.1
378	1133.1	284	852.1	762	2286.1

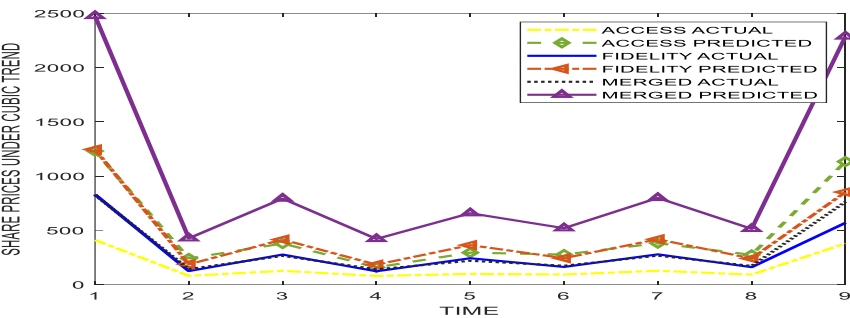
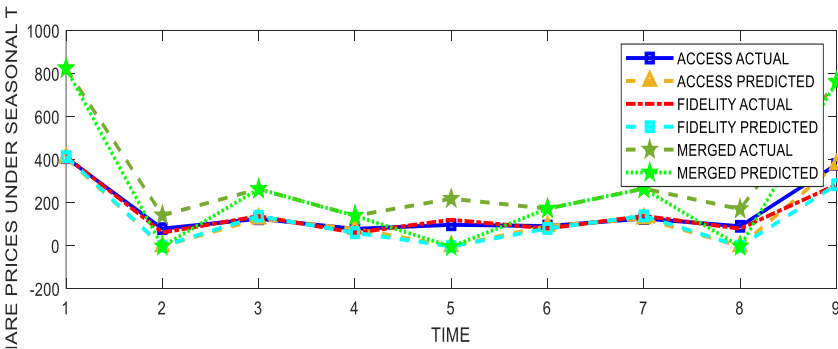


Figure 2: Plots of the three banks Actual share prices and their Predicted prices against Time under Cubic Trend Function

Table 9: Comparison of Results under Seasonal Trend Functions

Access Actual Share prices	Predicted prices	Fidelity Actual Share prices	Predicted Prices	Merged Actual Share prices	Predicted prices
410	410	415	415	825	825
80	-1.50	62	-1.18	142	-2.58
126	126	138	138	264	264
79	79	61	61	140	140
98	-1.7	121	-2.21	219	-3.92
92	92	81	81	173	173
127	127	139	139	266	266
91	-1.6	80	-1.50	171	-3.08
378	378	284	284	762	762



**Figure 3: Plots of the three banks Actual share prices and their Predicted prices against Time under Seasonal Trend Function**

**Table 10: Summary of Best Trend Function for Trading of Shares**

QUADRATIC TREND FUNCTION			
Banks	Access Bank	Fidelity	Merged Bank
MSE	9.2346e+05	2.1218e+05	2.9917e+05
CUBIC TREND FUNCTION			
MSE	2.1160e+06	8.4818e+05	6.2598e+06
SEASONAL TREND FUNCTION			
MSE	8.3662e+03*	7.9983e+03*	1.1578e+05*

### Conclusion

The study concludes that the seasonal trend function is most suitable for modeling share prices of Access Bank, Fidelity and Merged Bank. The results show that the seasonal trend of function provides a good fit to the data, while the Quadratic and Cubic trend functions perform poorly. The study's findings have implications for investors, financial analysts, and policy makers, and suggest that the seasonal trend function can be used to inform investment decision and risk management strategies.

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