EVALUATING THE EFFICIENCY OF HIERARCHICAL BAYESIAN ESTIMATORS IN DYNAMIC PANEL DATA MODEL

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ABSTRACT

Using simulated datasets under various panel structures (NT), this work assesses the effectiveness of hierarchical Bayesian estimators in dynamic panel data models. The parameters of models with lagged response variables and stochastic errors were estimated under informative priors using Monte Carlo techniques and Markov Chain Monte Carlo (MCMC) simulations.

Across all panel designs, the results show that model precision increases with sample size; the most consistent estimator gains occur when the number of cross-sectional units surpasses time periods (N>T). The computational expense of high-dimensional Bayesian estimation is highlighted by the fact that this gain is accompanied by an increase in numerical standard error (NSE).

This work confirms the robustness of hierarchical Bayesian techniques for dynamic panel analysis, especially in large and imbalanced datasets. The results offer practical insights for researchers and policymakers modeling economic and social processes. Future research can extend this framework to non-linear models, alternative prior structures, and real-world applications.

Keywords: hierarchical, stochastic, Bayesian, estimator, simulation, panel data

1. Introduction

Individual heterogeneity and temporal dynamics across units, such as homes, businesses, or nations, must be captured via panel data, which blends cross-sectional and time-series observations. Compared to strictly cross-sectional or time-series data, its hybrid structure has a number of benefits, including as increased parameter efficiency, less multicollinearity, and the ability to account for unobservable heterogeneity [2,3].

In particular, dynamic panel data models are helpful for examining time-dependent behaviors while taking individual-specific effects into consideration. However, estimation becomes difficult when lagged dependent variables and possibly associated error structures are present. Although they are often used, traditional techniques like the Generalized Method of Moments (GMM) and Ordinary Least Squares (OLS) have significant drawbacks. Because of endogeneity brought on by the lagged dependent variable, OLS produces estimates that are skewed and inconsistent [8]. GMM is known to perform poorly with

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small samples, which can result in weak instruments and a loss of efficiency, particularly when the time dimension (T) is short in relation to the cross-sectional dimension (N), even though it was meant to address endogeneity [1,10].

These drawbacks show how stronger estimating frameworks are required. Because of its adaptability to complex model topologies, managing parameter uncertainty, and incorporating prior information, Bayesian approaches offer a strong substitute. Unlike frequentist methods, Bayesian methods can describe hierarchical dependencies, adjust for missing data, and perform well in small samples [11]. Dynamic panel model estimation is now possible even with enormous datasets thanks to recent developments in Bayesian computation, including variational inference techniques and effective Markov Chain Monte Carlo (MCMC) approaches [7].

The capabilities of Bayesian analysis are further expanded by hierarchical Bayesian techniques, which enable multilevel modeling with parameters that can differ between individuals or groups. This is especially helpful in panel contexts where pooled estimators are unable to capture individual-specific dynamics. The increasing importance of these techniques in econometrics is shown by studies such as Zhang (2023) and Choudhury and Koop (2023), which highlight how well they describe latent structures and cross-sectional heterogeneity [4,11].

The performance of hierarchical Bayesian estimators under different panel structures still requires further empirical analysis, notwithstanding these advancements. In order to fill this gap, this study uses simulation-based experiments to investigate the efficacy of such estimators in three panel configurations: NT. The work offers methodological understanding and useful recommendations for researchers using Bayesian approaches in dynamic panel settings by assessing estimator precision, standard error, and computational cost.

2. Materials and Methods

As previously stated, the lagged variable liberated from the response variable in the model is one of the predictors in this study's dynamic panel data. The study's standard panel data model is represented by the equation

$$y_{it} = X_{it}\theta_{it} + \varepsilon_{it} \tag{1}$$

Where,

$$X_{it} = \left(x_{it}, y_{i,t-1}\right), \theta = (\delta, \theta_i)$$

The dynamic model considered is equation (2)

$$y_{it} = \delta y_{i,t-i} + \beta_i x_{it} + \varepsilon_{it} \tag{2}$$

$$i = 1,\ldots,N \text{ and } t = 1,\ldots,T \qquad \varepsilon_{it} = u_i + v_{it},$$

where y_{it} represents the dependent variable for the i^{th} individual at the t^{th} time period, δ is the coefficient of the lagged dependent variable, x_{it} is the exogenous unit specific regressors, ε_{it} is the error term decomposed as in equation (3);

$$\varepsilon_{it} = \mu_i + v_{it}. \tag{3}$$

Equation (2) can be expressed as equation (4)

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$$y_{it} = X_{it}\theta_{it} + \varepsilon_{it}, \tag{4}$$

where $X_{it} = \left(x_{it}, y_{i,t-1}\right)'$ and $\theta = (\gamma, \beta_i)'$

Expressing equation (4) in matrix form:

$$y = \begin{bmatrix} y_{i,1} \\ y_{i,2} \\ \vdots \\ y_{i,T} \end{bmatrix}, X = \begin{bmatrix} y_{i,0} & x_{0i1} & \cdots & x_{ki1} \\ y_{i,1} & x_{0i2} & \cdots & x_{ki2} \\ \vdots & \vdots & \vdots & \vdots \\ y_{i,T-1} & x_{0iT} & \cdots & x_{kiT} \end{bmatrix}, \theta = \begin{bmatrix} \delta \\ \beta_i \\ \vdots \\ \beta_k \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,T} \end{bmatrix}.$$

So, y is $(NT \times 1)$ vector of dependent variable, X is $(NT \times NK)$ matrix of unit specific regressors, β is $(NK \times 1)$ vector of parameters and ε is $(NT \times 1)$ vector of error terms in equation (5);

$$y^* = X^*\theta + \varepsilon^* \tag{5}$$

where y^* is the transformed dependent variable, X^* is the transformed regressor, θ is the coefficient of predictor and ε^* is transformed error term

2.1 Bayesian Linear Regression and Model Estimation

2.1.1 The Likelihood Function

Error term assumption is mostly used to establish the structure of the likelihood function. Thus, from assumptions above and the multivariate normal density definition, the likelihood is expressed thus in equation (6)

$$p(y|\theta, h, \Phi) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left|\Phi^{-\frac{1}{2}}\right| \left\{exp\left[-\frac{h}{2}(y - X\theta)'\Phi^{-1}(y - X\theta)\right]\right\} \tag{6}$$

or we use the transformed data in equation (7);

$$p(y^*|\theta, h, \Phi) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{exp\left[-\frac{h}{2}(y^* - X^*\theta)'(y^* - X^*\theta)\right]\right\}. \tag{7}$$

Substituting for $h = \frac{1}{\sigma^2} \Rightarrow \sigma^2 = h^{-1}$ in equation (7) we have equation (8)

$$P(y*/\theta, \sigma^2, \Phi) = \frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N} exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(y^* - X^*\theta)^2\right]. \tag{8}$$

Simplifying further equation (8) becomes equation (9);

$$P(y*/\theta, \sigma^2, \Phi) = \frac{1}{(2\pi)^{\frac{N}{2}}h^{-\frac{N}{2}}} exp\left[-\frac{1}{2h^{-1}}\sum(y^* - X^*\theta)'(y^* - X^*\theta)\right]. \tag{9}$$

Expressing the exponent $(y^* - X^*\theta)'(y^* - X^*\theta)$ in equation (9) in terms of OLS estimator $\hat{\theta}$ to equation (10), where,

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$$\hat{\theta}(\Phi) = (X^{*\prime}X)^{-1}X^{*\prime}y^{*} = (X'\Phi^{-1}X)^{-1}X'\Phi^{-1}y \quad (10)$$

And s^2 the estimator of the variance of the model σ^2 is

$$s^2(\Phi) = \frac{\left(y^{*}-X^{*}\hat{\theta}(\Phi)\right)'\left(y^{*}-X^{*}\hat{\theta}(\Phi)\right)}{v} = \frac{\left(y-X\hat{\theta}(\Phi)\right)'\Phi^{-1}\left(y-X\hat{\theta}(\Phi)\right)}{v}$$

Also, in equation (11)

$$\sum(y^{*} - X^{*}\hat{\theta})'(y^{*} - X^{*}\hat{\theta}) = vs^2 + \left(\theta - \hat{\theta}\right)'X^{*\prime}X^{*}\left(\theta - \hat{\theta}\right) \quad (11)$$

Then it gives equation (12)

$$\therefore P(y/\theta, \sigma^2, \Phi) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} exp\left\{-\frac{h}{2}\left[vs^2 + \left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\} \quad (12)$$

Using $v = N - k \Rightarrow N = v + k$ in equation (12) we have equation (13)

$$P(y/\theta, \sigma^2, \Phi) = \frac{h^{\frac{v+k}{2}}}{(2\pi)^{\frac{N}{2}}} exp\left\{-\frac{h}{2}\left[vs^2 + \left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\} \quad (13)$$

Simplifying further, equation (13) gives equation (14)

$$P(y/\theta, \sigma^2, \Phi) = \frac{h^{\frac{v}{2}}h^{\frac{k}{2}}}{(2\pi)^{\frac{N}{2}}} exp\left\{-\frac{h}{2}\left[vs^2 + \left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\} \quad (14)$$

Separating equation (14) into two equation (15) and equation (16)

$$P(y/\theta, \sigma^2, \Phi) = \frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}} exp\left[-\frac{h}{2}\left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\}\left\{h^{\frac{v}{2}} exp - \frac{h}{2}vs^2\right\}\right) \quad (15)$$

$$P(y/\theta, \sigma^2, \Phi) = \frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}} exp\left[-\frac{h}{2}\left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\}\left\{h^{\frac{v}{2}} exp - \frac{hv}{2s^{-2}}\right\}\right) \quad (16)$$

$$P(y/\theta, \sigma^2, \Phi) = \frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}} exp\left[-\frac{h}{2}\left(\theta - \hat{\theta}(\Phi)\right)' X'\Phi^{-1}X\left(\theta - \hat{\theta}(\Phi)\right)\right]\right\}\left\{h^{\frac{v}{2}} exp - \frac{hv}{2s(\Phi)^{-2}}\right\}\right)$$

Equation (16) gives the likelihood of the model.

The quantity $\left\{h^{\frac{k}{2}} exp\left[-\frac{h}{2}\left(\theta - \hat{\theta}\right)'X^{*\prime}X\left(\theta - \hat{\theta}\right)\right]\right\}$ in equation (16) resembles the kernel of the multivariate normal density and $\left\{h^{v/2} \exp - \frac{hv}{2s^{-2}}\right\}$ also resembles the kernel of the gamma density. The results clearly indicate a Normal-Gamma prior for the likelihood function.

2.1.2 The Prior Specification of Dynamic Panel Data Model

The prior distribution in Bayesian analysis captures the researcher's knowledge or presumptions on the model parameters prior to the inclusion of any empirical data. The parameter prior is denoted by $p(\theta, h, \Phi)$. In this

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section, we employ an independent Normal-Gamma prior as suggested by the form of the likelihood function in equation (16). The typical notation $p(\Phi)$ is used to denote the prior for Φ. Using the law of independence of random variables, then: $p(\beta, k, \Phi) = p(\beta)p(h)p(\Phi)$

From equation (16), we now have equation (17)

$$p(\theta) = f_N(\theta|\underline{\theta}, \underline{V})$$

$$p(\theta) = \frac{1}{2\pi}|\underline{V}|^{-\frac{1}{2}}\left\{exp\left[-\frac{1}{2}(\theta-\underline{\theta})'\underline{V}^{-1}(\theta-\underline{\theta})\right]\right\} \quad (17)$$

$$p(k) = f_G(k|\underline{v}, \underline{s}^{-2})$$

Simplifying further, we have equation (18)

$$p(k) = \left\{k^{\frac{\underline{v}}{2}} exp\left[\frac{k\underline{v}}{2(\underline{s})^{-2}}\right]\right\} \quad (18)$$

$$p(\Phi) \propto f_N\left(\rho\,|\,\underline{\rho}, \underline{V}_\rho\right)1\left(\rho\in\varphi\right)$$

$$p(\Phi) = \frac{1}{2\pi}\left|\underline{V}_\rho\right|^{-\frac{1}{2}}\left\{\exp\left[-\frac{1}{2}\left(\rho-\underline{\rho}\right)'\underline{V}_\rho^{-1}\left(\rho-\underline{\rho}\right)\right]\right\} \quad (19)$$

Where $1\left(\rho\in\varphi\right)$ represents the indicator function that is equals to 1 at the stationary region and zero otherwise. $\underline{V} = \left(X^{*'}X^{*}\right)^{-1}$ the prior covariance matrix of θ, and $\underline{\theta}$ is the prior mean of θ.

2.1.3 Posterior Distribution of Dynamic Panel Data Model

Mathematically, the posterior is primarily denoted as $p(\theta, h, \Phi|y)$, where:

$$p(\theta, h, \Phi|y) \propto p(y|\theta, h, \Phi) \times p(\theta) \times p(h) \times p(\Phi)$$

Note:

$$p(\theta, h, \Phi|y) \neq p(y|\theta, h, \Phi) \times p(h|y, \theta, \Phi) \times p(\Phi|\theta, h, y)$$

Hence, multiplying equations (16), (17), (18) and (19) together gives the posterior resulted to equation (20)

$$p(\theta, h, \Phi|y) \propto p(\Phi) \times \left\{exp\left[-\frac{1}{2}h(y^* - X^*\theta)'(y^* - X^*\theta) + (\theta-\underline{\theta})'\underline{V}^{-1}(\theta-\underline{\theta})\right]\right\} \times h^{\frac{N+\underline{v}-2}{2}} exp\left[-\frac{h\underline{v}}{2\underline{s}^{-2}}\right] \quad (20)$$

This joint posterior density for β, h, Φ does not resemble any familiar distribution form; so, it cannot be solved analytically but only through a posterior simulation procedure.

2.1.4 Matrix Multiplication to Simplify the Posterior Distribution

From equation (20), we have

$$h(y^* - X^*\theta)'(y^* - X^*\theta) + (\theta-\underline{\theta})'\underline{V}^{-1}(\theta-\underline{\theta})$$

$$= h\left(y^{*\prime}y^* - 2\theta' X^{*\prime}y + \theta X^{*\prime}X^*\theta\right) + (\theta-\underline{\theta})'\underline{V}^{-1}(\theta-\underline{\theta})$$

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$$= h^*y^{*\prime}y^* - 2h^*\theta'X^{*\prime}y^* + h^*\theta'X^{*\prime}X^*\theta + \theta'\underline{V}^{-1}\theta - 2\theta'\underline{V}^{-1}\underline{\theta} + \underline{\theta}'\underline{V}^{-1}\underline{\theta}$$

$$= h^*y^{*\prime}y^* + \underline{\theta}'\underline{V}^{-1}\underline{\theta}' + \theta'(\underline{V}^{-1} + h^*X^{*\prime}X^*)\theta - 2\theta'(h^*X^{*\prime}y^* + \underline{V}^{-1}\underline{\theta}) \quad (21)$$

Let $\overline{V} = (\underline{V}^{-1} + hX^{*\prime}X^*)^{-1}$ (22)

$$\Rightarrow \overline{V}^{-1} = (\underline{V}^{-1} + hX^{*\prime}X^*) \quad (23)$$

so let, $\overline{\theta} = \overline{V}(h^*X^{*\prime}y^* + \underline{V}^{-1}\underline{\theta})$ (24)

Hence, substituting equations (21), (22) and (23) into (24) to gives equation (25);

$$hy^{*\prime}y^* + \underline{\theta}'\underline{V}^{-1}\underline{\theta} + \theta'\overline{V}^{-1}\theta - 2\theta'\overline{V}^{-1}\overline{\theta}. \quad (25)$$

Applying a simple mathematical assumption by including $-\overline{\theta}'\overline{V}^{-1}\overline{\theta}$ and $+\overline{\theta}'\overline{V}^{-1}\overline{\theta}$ into equation (25) helps to achieve our desired result and still leaving the equation unchanged.

$$= hy^{*\prime}y^* + \underline{\theta}'\underline{V}^{-1}\underline{\theta} + \theta'\overline{V}^{-1}\theta - 2\theta'\overline{V}^{-1}\overline{\theta} - \overline{\theta}'\overline{V}^{-1}\overline{\theta} + \overline{\theta}'\overline{V}^{-1}\theta$$

$$= hy^{*\prime}y^* + \underline{\theta}'\underline{V}^{-1}\underline{\theta} - \overline{\theta}'\overline{V}^{-1}\overline{\theta} + \theta'\overline{V}^{-1}\theta - 2\theta'\overline{V}^{-1}\overline{\theta} + \overline{\theta}'\overline{V}^{-1}\overline{\theta}.$$

Let,

$$Q = hy^{*\prime}y^* + \underline{\theta}'\underline{V}^{-1}\underline{\theta} - \overline{\theta}'\overline{V}^{-1}\overline{\theta}$$

and

$$(\theta - \overline{\theta})'\overline{V}^{-1}(\theta - \overline{\theta}) = \theta'\overline{V}^{-1}\theta - 2\theta'\overline{V}^{-1}\overline{\theta} + \overline{\theta}'\overline{V}^{-1}\overline{\theta}$$

Which then gives equation (26)

$$\Rightarrow h(y^* - X^*\theta)'(y^* - X^*\theta) + (\theta - \underline{\theta})'\underline{V}^{-1}(\theta - \underline{\theta}) = (\theta - \overline{\theta})'\overline{V}^{-1}(\theta - \overline{\theta}) + Q \quad (26)$$

Replacing the expression in equation (21) with the expression in equation (26), gives equation (27):

$$P(\beta, h, \Phi|y) \propto p(\Phi)\, exp\left[-\tfrac{1}{2}\{\left(\theta^* - \overline{\theta}^*\right)'\overline{V}^{-1}\left(\theta^* - \overline{\theta}^*\right)\}\right] \times exp[-\tfrac{1}{2}Q]h^{*\frac{N+\underline{v}-2}{2}} exp\left(\frac{-h^*\underline{v}}{2\underline{s}^{-2}}\right) \quad (27)$$

2.1.5 Posterior for Model Parameter θ

It can be verified that the posterior of θ conditional on all the other parameters in the model is multivariate normal, disregarding all terms not involving θ in equation (11) we obtain equation (28):

$$P(\theta|y, h, \Phi) \propto exp\left[-\tfrac{1}{2}\{\left(\theta - \overline{\theta}\right)'\overline{V}^{-1}\left(\theta - \overline{\theta}\right)\}\right] \quad (28)$$

This looks like the kernel of a multivariate normal density, this implies that in equation (29):

$$\theta|y, h, \Phi \sim N(\bar{\theta}, \bar{V}) \quad (29)$$

where

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$$\bar{V} = \left(\underline{V}^{-1} + hX^{'}\Phi^{-1}X\right)^{-1}$$

and

$$\bar{\theta} = \bar{V}\left(\underline{V}^{-1}\underline{\theta} + hX^{'}\Phi^{-1}X\hat{\theta}(\Phi)\right)$$

2.1.6 Posterior for Error Precision ($\boldsymbol{h}$)

The posterior of "h" conditional on all other key parameters in the model is Gamma. Similarly, treating equation (11) as a function of "h" and disregarding quantities not involving h we have equation (30)

$$P(h|y,\theta,\Phi) \propto h^{\frac{N+\underline{v}-2}{2}} exp[-\frac{h}{2}\{(y^* - X^*\theta)'(y^* - X^*\theta) + \underline{vs}^2\}]. \tag{30}$$

Hence, we have equation (31)

$$h|y,\theta,\Phi \sim G(\bar{s}^{-2},\bar{v}). \tag{31}$$

$$\bar{s}^2 = \frac{(y-X\beta)'\Phi^{-1}(y-X\beta)+\underline{v}\underline{s}^2}{\bar{v}}.$$

2.1.6 Posterior for Φ

The posterior of **Φ**conditional on all other key parameters in the model is multivariate Normal; by ignoring other terms in the model, gives equation (32):

$$p(\Phi \mid y,\theta,h) \propto f_N\left(\rho \mid \bar{\rho},\bar{V}_\rho\right)1(\rho \in \varphi), \tag{32}$$

where

$$\bar{V}_\rho = \left(\underline{V}_\rho^{-1} + E^{'}E\right)^{-1} \text{ and } \bar{\rho} = \bar{V}_\rho\left(\underline{V}_\rho^{-1}\underline{\rho} + hE^{'}E\right)$$

Also, E is a $(T-p)\times k$ matrix that has t^{th} row given by $\left(\varepsilon_{t-1},\ldots,\varepsilon_{t-p}\right)$.

The formulae of equations (28), (30) and (32) resemble those of the conjugate normal-gamma priors but do not directly relate to the posteriors of interest as equation (33)

$$\Rightarrow p(\theta,h,\Phi \mid y) \neq p(y \mid \theta,h,\Phi)\times p(h \mid y,\theta,\Phi)\times p(\Phi \mid \theta,h,y). \tag{33}$$

Hence, the conditional posteriors in equations (28) to (32) do not directly express everything about the posterior, $p(\theta,h,\Phi \mid y)$.

2.2 Data Generation Process (DGP)

Using error specifications to estimate the dynamic panel data model's parameters. In this work, Markov Chain Monte Carlo (MCMC) experiments are used. The model in equation (34) would be used to produce data through a Monte-Carlo simulation process.

$$y_{it} = \delta y_{i,t-i} + \beta_i x_{it} + \varepsilon_{it}, \tag{34}$$

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where x = x^2

$$i = 1, \ldots, N \text{ and } t = 1, \ldots, T$$

where δ is the parameter of lagged exogenous variable and μ_{it} is the random error.

The following are specified:

(i) The lagged variable is generated from Normal distribution with zero mean and variance 0.25 that is $\delta \sim N(0, 0.25)$.

(ii) The explanatory variables X_{it} are generated from uniform $U[0, 0.5]$ distribution i.e $X_{it} \sim U(0, 0.5)$.

(iii) The error terms μ_{it} and u_i are generated using Normal distribution with mean zero and variance 0.25 that is $\mu_{it}, u_i \sim N(0, 0.25)$.

(iv) The values for the regression coefficients β_i are $\beta_1 = 2$ and $\beta_2 = 3$

(v) The cross-section units specified as $N = 10, 50, 100$ and the time periods as $T = 5, 10, 20$.

(vi) The different combinations considered for the individual (N) and time (T) were N<T, N=T, N>T. The Markov Chain Monte Carlo with 1000 iterations were made for the three scenarios.

The parameters of the model are obtained using Bayesian approach and the sensitivity of the estimators were assessed with the following techniques,

(i) Posterior Mean

(ii) Posterior Standard Deviation

(iii) Numerical Standard Error (NSE).

To estimate the parameters of Dynamic panel data model with error specifications. The study adopts Markov Chain Monte Carlo (MCMC) experiments. The model in equation

$y_{it} = \delta y_{i,t-i} + \beta_i x_{it} + \varepsilon_{it}$, where x = x^2

$i = 1, \ldots, N$ and $t = 1, \ldots, T$

Where δ is the parameter of lagged exogenous variable and μ_{it} is the random error.

3. Results

The precision and numerical standard error criteria are used to evaluate the effectiveness of hierarchical Bayesian estimators on dynamic panel data models when the parameters vary among people. The three cross-sectional (N) and time-period (T) possibilities are also given careful consideration.

Firstly, when N<T: (10, 15), (15, 20), (20, 50)

N=T: (10, 10), (20, 20), (50, 50)

N>T: (20, 5), (50, 10), (100,15)

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Using reasonably informative priors, the impact of prior knowledge on the posterior estimates of heterogeneous dynamic panel data models will also be examined. The Markov Chain Monte Carlo (MCMC) method was used to simulate the data and generate 10,000 iterations of posterior estimations.
In this section, the results for the model efficiency and the efficiency hierarchical Bayesian Estimators are displayed.

Examine the model efficiency of hierarchical Bayesian at various panel data structure Table 1: Efficiency result for the models (N<T)

(N, T)	**Precision**	**NSE**
(10, 15)	22.3076	0.0328
(15, 20)	24.5384	0.0361
(20, 50)	26.9922	0.0397

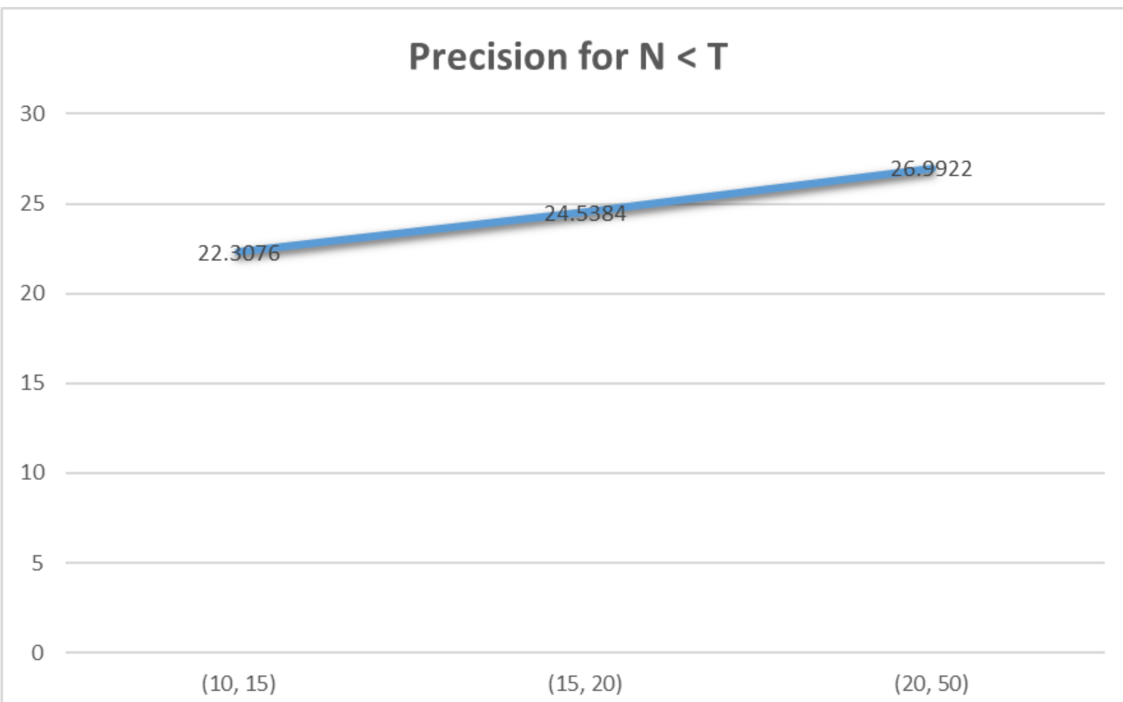

Fig 1: Line plot for the Model Precision at N<T

The precision estimates for the model efficiency results at N<T is considered. Table 1 shows the efficiency results for N<T. The result shows increase in precision as the sample size increases. Also, the numerical standard error increases consistently as the sample size increases. Fig 1 shows the line plot

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representation for the Model Precision at N<T.

Table 2: Efficiency result for the models (N = T)

	Precision	NSE
(10, 10)	25.3626	0.0529
(20, 20)	25.6405	0.0529
(50, 50)	25.8525	0.0437

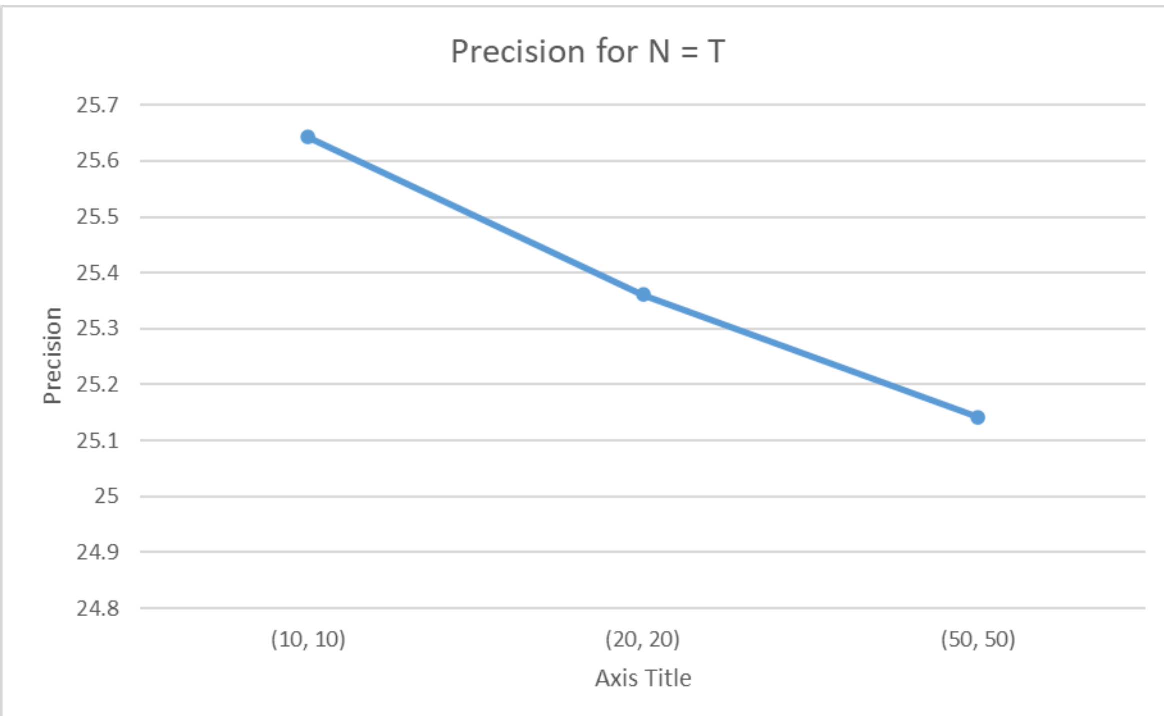

Fig 2: Line plot for the Model Precision at N=T

The precision estimates for the model efficiency results at N=T is considered. Table 2 shows the efficiency results for N=T. The result shows decrease in precision as the sample size increases. Also, the numerical standard error increases consistently as the sample size increases. Fig 2 shows the line plot representation for the Model Precision at N=T.

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Table 3: Efficiency result for the models (N > T)

	Precision	NSE
(20, 5)	24.2996	0.0582
(50,5)	25.4508	0.0774
(50, 10)	25.5146	0.0840
(100, 5)	26.1783	0.0851
(100, 10)	26.5961	0.0936
(100, 15)	26.7903	0.0944
(200, 10)	26.9365	0.1030
(200, 20)	27.4578	0.1133

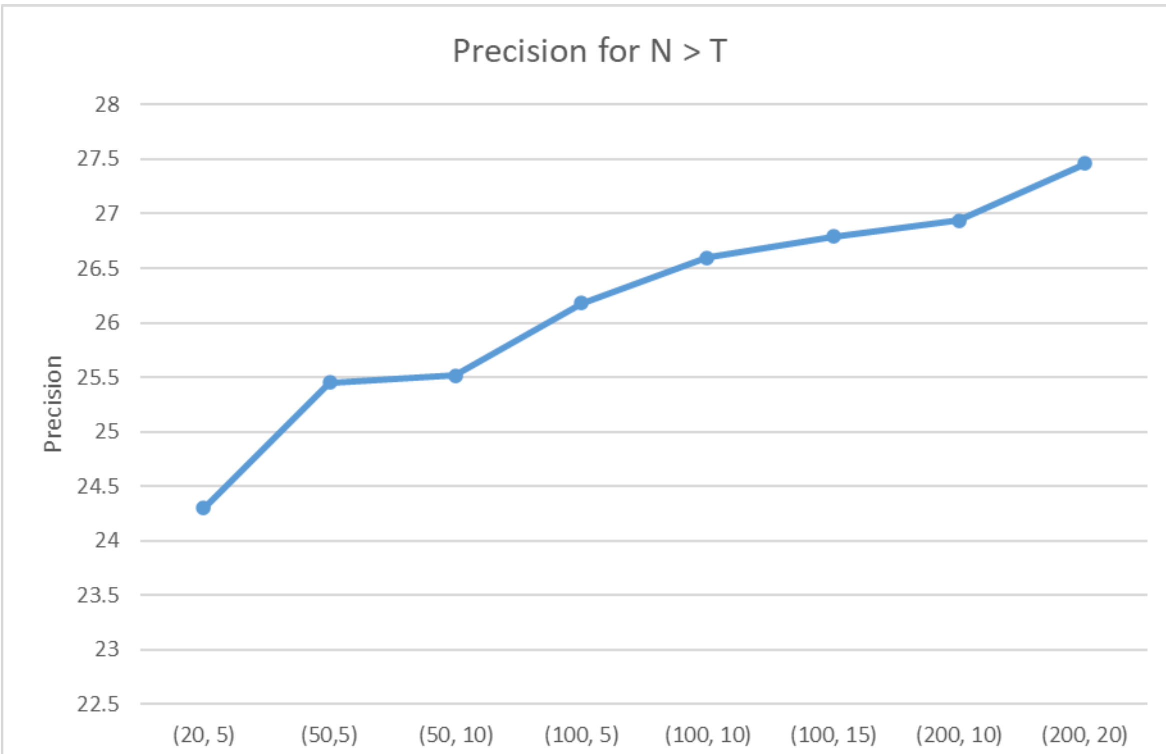

Fig 3: Line plot for the Model Precision at N>T

The precision estimates for the model efficiency results at N=T is considered. Table 3 shows the efficiency results for N=T. The result shows increase in precision as the sample size increases. Also, the numerical standard error increases consistently as the sample size increases. Fig 3 shows the line plot representation for the Model Precision at N>T.

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Examine the efficiency of hierarchical Bayesian Estimator at various panel data structure

Table 4: Efficiency result for the hierarchical Bayesian Estimators (N<T)

Std dev	(10, 15)	(15, 20)	(20, 50)
δ	0.0144	0.0014	0.0001
β_1	0.0676	0.0531	0.0503
β_2	0.0569	0.0559	0.0548
β_3	0.0859	0.0791	0.0770

Table 4 shows the efficiency results for the hierarchical Bayesian estimators at N<T. To measure efficiency of an estimator, as the sample size increases, there should be reduction in the value of the estimator's variance. The table shows the result for the standard deviation of the parameters and this is used to measure the efficiency of the estimators. At N<T, the result shows decrease in precision as the sample size increases.

Table 5: Efficiency result for the hierarchical Bayesian Estimators (N=T)

Std dev	(10, 10)	(20, 20)	(50, 50)
δ	0.0001	0.0001	0.0001
β_1	0.4779	0.4337	4.3011
β_2	0.0517	0.0507	0.0497
β_3	0.0957	0.0583	0.0580

Table 5 shows the efficiency results for the hierarchical Bayesian estimators at N=T. To measure efficiency of an estimator, as the sample size increases, there should be reduction in the value of the estimator's variance. Table 5 shows the result for the standard deviation of the parameters and this is used to measure the efficiency of the estimators. At N=T, the result shows decrease in precision as the sample size increases.

Table 6: Efficiency result for the hierarchical Bayesian Estimators (N>T)

Std dev	(20, 5)	(50, 5)	(50, 10)	(100, 5)	(100, 10)	(100, 15)
δ	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
β_1	0.0733	0.0613	0.0599	0.0513	0.0461	0.0397
β_2	0.0487	0.0458	0.0437	0.0429	0.0440	0.0367
β_3	0.0874	0.0695	0.0601	0.0565	0.0520	0.0511

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Table 6 shows the efficiency results for the hierarchical Bayesian estimators at N>T. To measure efficiency of an estimator, as the sample size increases, there should be reduction in the value of the estimator's variance. Table 6 shows the result for the standard deviation of the parameters and this is used to measure the efficiency of the estimators. At N>T, the result shows decrease in precision as the sample size increases.

4. Discussion of results

This study assesses the performance of hierarchical Bayesian estimators and the model efficiency for three different panel data structures: NT. Regardless of the panel structure, the results from Tables 1 through 3 and Figures 1 through 3 consistently demonstrate that model precision increases with sample size. This suggests that more accurate estimations are typically produced by larger datasets. Nevertheless, a rise in Numerical Standard Error (NSE) is also noted as sample size increases. This could be because of the increased computational complexity and difficulties with model convergence as data dimension increases. The standard deviation of parameter estimations is used to evaluate the efficiency of the hierarchical Bayesian estimators, as indicated in Tables 4 to 6. The standard deviation generally decreases as sample size increases at NT structure, where most standard deviations decrease as N and T rise, suggesting improved estimator performance.

Overall, the findings support the claim that hierarchical Bayesian estimators provide improved precision and reduced variance in large panel data, particularly when N>T. In balanced panels, however, where estimator variance may become unpredictable as panel size increases, care must be taken.

5. Conclusion

This work used Markov Chain Monte Carlo (MCMC) techniques and a Monte Carlo data creation procedure to investigate the effectiveness of hierarchical Bayesian estimators in dynamic panel data models. The study examined the effects of sample size and panel balance on the accuracy and variance of Bayesian parameter estimates by simulating three different panel structures: N<T, N=T and N>T.

Important results show that regardless of panel structure, model precision continuously improves with increasing sample size. However, due to the additional processing cost and possible convergence issues of Bayesian estimation in high-dimensional data, this gain in precision is accompanied by an increase in numerical standard error (NSE). The diminishing standard deviations of parameter estimations across the various panel configurations further demonstrate the hierarchical Bayesian estimators' improving efficiency with sample size.

The findings, which clearly demonstrate a decrease in estimator variance as cross-sectional units increase, are especially noteworthy in situations where N>T. This demonstrates how reliable hierarchical Bayesian techniques are in situations with a lot of distinct units and short time periods. Balanced panels (N=T), on the other hand, displayed less consistent behavior, indicating that the interplay between time and cross-sectional dimensions may cause estimator performance to plateau or even fluctuate.

All things considered, the results offer validity to the use of hierarchical Bayesian techniques in dynamic panel data settings, particularly when dealing with heterogeneous data and large sample sizes. To improve comprehension of estimator behavior in dynamic situations, future research can investigate non-linear model extensions, include different prior distributions, or contrast hierarchical Bayesian estimators with other Bayesian or frequentist approaches.

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