A SINGLE ACCEPTANCE SAMPLING PLAN FOR THE TRUNCATED LIFE TESTS BASED ON THE PERCENTILES OF ZECH DISTRIBUTION AND APPLICATIONS

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ABSTRACT

This study proposes a Single Acceptance Sampling Plan (SASP) for the truncated life test based on the percentiles of Zech distribution. The methodology is designed to address quality assessment in both industrial and biomedical contexts, thereby demonstrating the distribution's dual applicability across the two domains. The use of percentiles, over the conventional median, provides a more flexible and informative approach in evaluating product reliability with the lower and upper percentiles accounting for the early and late failures in improving decision-making accuracy. This research constructs an efficient sampling scheme that minimizes the required sample size while simultaneously satisfying consumer's and producer's risk. This is achieved by formulating and solving constrained optimization problems tailored towards the proposed plan. The performance of the SASP is examined through simulated datasets, arbitrarily chosen parameter values, and real-life data. To validate its practical utility, the proposed plan is benchmarked against SASPs derived from other lifetime distributions. Furthermore, the adoption of multiple percentiles enhances the robustness of the plan, offering a significant improvement over earlier models that focus solely on the median. Comparative study indicates that the proposed SASP based on the Zech distribution consistently yields lower sample sizes and fewer inspection cycles, thereby reducing both inspection time and cost.

Key Words: Acceptance Sampling Plans, Consumer's Risk, Lots, Operating Characteristic Function, Probability distributions, Producer's Risk

1. Introduction

Acceptance Sampling Plans (ASPs) constitute a fundamental tool in statistical quality control, widely employed to determine whether a batch of products should be accepted or rejected based on inspection of sample taken randomly from it. They are particularly useful in situations where full inspection is impractical due to cost or time constraints. In the case of destructive testing, acceptance sampling plans come to the rescue. By providing a balance between inspection effort and decision accuracy, ASPs help maintain required quality standards while optimizing resource utilization.

In life-testing experiments, it is a common practice to terminate tests at a pre-assigned time and record the number of failures observed within that period (Balakrishnan et al., 2007). In such contexts, product's quality is typically assessed using lifetime characteristics such as the mean, median, or selected percentiles. While the mean is suitable for symmetric distributions, the median is often preferred for skewed lifetime data. However, reliance on a single summary measure,

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particularly the median, may not adequately capture the variability and structural features of lifetime distributions, especially in the presence of skewness or extreme observations. This limitation motivates the use of multiple percentile-based measures, which provide a more comprehensive and informative basis for quality assessments.

Recent studies have proposed ASPs under various lifetime distributions. For instance, Alomani and Al-Omari (2022) developed sampling plans based on the two-parameter Xgamma distribution, while Jayalakshmi and Vijilamery (2022) introduced percentile-based plans using the Gompertz–Fréchet distribution. More recently, Al-Momani and Al-Masri (2025) proposed ASPs based on the beta power-transformed Birnbaum–Saunders distribution. While these studies have contributed significantly to the development of acceptance sampling theory, their approaches remain largely distribution-specific and typically rely on a single quality parameter, most commonly the median.

In contrast to these existing methods, the present study introduces several key distinctions. First, instead of relying solely on the median, the proposed sampling plan incorporates multiple lifetime percentiles, allowing for a more comprehensive characterization of product quality, particularly for skewed distributions. Second, existing approaches are often developed within narrowly defined application domains, while the proposed plan is formulated to be applicable across both industrial quality control and medical (healthcare) quality assessment contexts.

This study therefore aims to address these limitations by developing a single acceptance sampling plan based on multiple lifetime percentiles while employing a flexible lifetime distribution. Specifically, the objectives are to: (i) develop an ASP that incorporates lower and upper percentile measures in addition to conventional metrics; (ii) employ a single flexible lifetime distribution as the underlying basis for the design of the sampling plan, thereby ensuring a unified and coherent framework; (iii) establish the applicability of the proposed sampling plan to both industrial quality control and medical (healthcare) quality assessment contexts; and (iv) assess the performance of the proposed plan through comparative analysis with existing methods.

To achieve these objectives, the Zech distribution is adopted due to its flexibility in modeling skewed lifetime data. The proposed sampling plan is constructed using simulated data, selected parameter configurations, and real datasets to ensure both theoretical soundness and practical relevance. The design is guided by two specified points on the operating characteristic (OC) curve, enabling effective control of both producer's and consumer's risks.

By integrating multiple percentile-based measures within a unified distributional framework, this study advances existing acceptance sampling methodologies and provides a practical and versatile tool for quality assessment across diverse application domains.

2. Zech Distribution

The Zech distribution, introduced by Adeyeye et al. (2022), is a flexible three-parameter distribution exhibiting positive skewness. It is constructed through the inversion of the cumulative distribution function (CDF) of Gompertz inverse exponential distribution, originally proposed by Oguntunde et al. (2018), via the inverse transformation method.

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The mathematical formulation of the Zech distribution is defined through its cumulative distribution function (CDF), probability density function (PDF), and quantile function, which are presented in Equations (1), (2), and (3), respectively.

$$G(t) = e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}} \;;\; t > 0, \theta > 0, \gamma > 0, \delta > 0 \tag{1}$$

$$g(t) = \gamma\theta e^{-\theta t}\left[1-e^{-\theta t}\right]^{-\delta-1} e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t}\right]^{-\delta}\right\}}; \quad t > 0, \theta > 0, \gamma > 0, \delta > 0 \tag{2}$$

Where γ and δ are the shape parameters and θ is the scale parameter.

The q^{th} quantile, t_q, of Zech distribution is

$$t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\} \tag{3}$$

Where $q \sim Uniform\ (0,1)$.

2.1 Failure Probability of Zech distribution.

The failure probability p is the probability that an item fails before the experiment time, t_0. The knowledge of failure probabilities enables quality control managers to make informed decisions regarding the acceptance or rejection of a batch, based on a predetermined level of acceptable risk. Under the Zech distribution, the probability of failure of products prior to a specified inspection time is derived as follows:

$$p = e^{\frac{\gamma}{\delta}\left\{1-\left[1-e^{-\theta t_0}\right]^{-\delta}\right\}} \tag{4}$$

From equation (3), by setting

$$\xi_q = -\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\} \tag{5}$$

$$t_q = \frac{\xi_q}{\theta} \tag{6}$$

The scale parameter θ can be expressed from equation (6) as

$$\theta = \frac{\xi_q}{t_q} \tag{7}$$

The experiment is stopped at the time t_0 indicated by

$$t_0 = a t_q^0 \tag{8}$$

where t_0 is the termination time, a is the termination ratio, t_q is the true quantile life of the product and t_q^0 is the specified quantile life of the product.

By substituting $\frac{\xi_q}{t_q}$ for θ in equation (4),

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(-\frac{\xi_q}{t_q} t_0\right)\right]^{-\delta}\right\}\right) \quad (9)$$

Substituting at_q^0 for t_0 in equation (9),

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(-\frac{\xi_q}{t_q} at_q^0\right)\right]^{-\delta}\right\}\right) \quad (10)$$

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(-a\xi_q \frac{t_q^0}{t_q}\right)\right]^{-\delta}\right\}\right) \quad (11)$$

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(-a\xi_q \left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (12)$$

Substituting $-\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$ for ξ_q in equation (12)

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(a\left\{\ln\left[1 - \left(1 - \frac{\delta}{\gamma} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}\left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (13)$$

The final form of the failure probability of Zech distribution is given in equation (14)

$$p = exp\left(\frac{\gamma}{\delta}\left\{1 - \left[1 - exp\left(aln\left[1 - \left(1 - \frac{\delta}{\gamma} lnq\right)^{-\frac{1}{\delta}}\right]\left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (14)$$

2.2 Procedures for Carrying out Single Acceptance Sampling Plans

Given the confidence level P^* and assuming that the lot size is large enough such that $\frac{n}{N} \leq 0.10$, then the probability of accepting a lot $P_a(p)$ can be obtained based on the cumulative binomial distribution function up to an acceptance number, c and a smallest sample size, n.

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$$\mathrm{P_a}(p) = \sum_{d=0}^{c} \binom{n}{d} p^d (1-p)^{n-d} \tag{15}$$

In a single acceptance sampling plan, $\mathrm{P_a}(p)$ denotes the probability of accepting a lot, where p represents the probability of failure of individual item, as defined in Equation (14). The primary objective is to determine the minimum sample size, $n,$ required to ensure that the true percentile life of a product exceeds the specified percentile life. This determination can be formulated as a non-linear optimization problem subject to relevant constraints. To protect both consumers and producers, the probability of accepting a lot is constrained such that it does not exceed the complement of the consumer's confidence level p^*, while simultaneously satisfying conditions that limit the producer's risk. The optimization problem thus seeks the smallest sample size,$n,$ that satisfies these risk constraints, ensuring that the lot's acceptance criterion reliably reflects the specified product reliability. The lot is accepted if the number of defectives,$d,$ does not exceed the acceptance number, $c,$ at the end of pre-determined testing time, t_0. The lot is rejected and the test terminated if $(c+1)$ failures occur before or at t_0.

To determine the design parameters n and c for a given quality level, along with specific consumer's and producer's risks, β and α respectively, the following optimization problem needs to be solved.

Minimize the sample size, n
Subject to
$P_a(P_1) \leq \beta$
$P_a(P_2) \geq 1-\alpha$
$n > 1, c \geq 0$

$$P_a\left(p_1 | \frac{t_q}{t_q^0} = r_1\right) = \sum_{d=0}^{c} \binom{n}{d} P_1^d (1-P_1)^{n-d} \leq 1-p^* \tag{16}$$

Where p^* represents the level of confidence.
Note: $1-p^* = \beta$

$$P_a\left(p_2 | \frac{t_q}{t_q^0} = r_2\right) = \sum_{d=0}^{c} \binom{n}{d} P_2^d (1-P_2)^{n-d} \geq 1-\alpha \tag{17}$$

The failure probabilities at the limiting quality level and acceptable quality level are represented by p_1 and p_2 respectively. d is the number of failed items at or before t_0.

Equations (16) and (17) represent the probability of accepting the lot at the limiting and acceptable quality levels respectively.

The optimal sampling plan parameters (n, c) were obtained using a numerical search procedure. For fixed values of the design parameters (α, β, r_1, r_2 and a), all feasible pairs of (n, c) were systematically evaluated. For each pair, the acceptance probabilities $P_a(p_1)$ and $P_a(p_2)$ were computed. Pairs that satisfied the constraints were considered feasible, and the pair with the smallest sample size, n was selected as the optimal solution. The analysis was performed using R statistical software, and the code is available upon request to ensure full reproducibility.

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In the context of the proposed single acceptance sampling plan, the ratios of the quantiles corresponding to the consumer's risk and the producer's risk are denoted by r_1 and r_2, respectively. These quantile ratios help define the points of interest on the distribution of items' lifetimes or failure probabilities. The specific values of the failure probabilities used in calculating the acceptance probabilities, given in Equations (16) and (17) are explicitly defined in Equations (18) and (19), respectively. These values are critical in designing the plan to meet both consumer's and producer's protection criteria.

$$p_1 = exp\left(\frac{\gamma}{\delta}\left\{1-\left[1-exp\left(aln\left[1-\left(1-\frac{\delta}{\gamma}lnq\right)^{-\frac{1}{\delta}}\right]\right)\right]^{-\delta}\right\}\right) \quad (18)$$

$$p_2 = exp\left(\frac{\gamma}{\delta}\left\{1-\left[1-exp\left(aln\left[1-\left(1-\frac{\delta}{\gamma}lnq\right)^{-\frac{1}{\delta}}\right]\left(\frac{t_q}{t_q^0}\right)^{-1}\right)\right]^{-\delta}\right\}\right) \quad (19)$$

The quantile ratio, $r_1 = \frac{t_q}{t_q^0} = 1$, is associated with the failure at the consumer's risk. Conversely, the quantile ratio, $r_2 = \frac{t_q}{t_q^0}$ with values 2, 4, 6, 8, and 10, is linked to the failure at the producer's risk. The termination ratio, $a = \frac{t_0}{t_q^0}$ is considered at values 0.5, 0.7, and 1.0. The consumer's risks (β) are examined at levels $0.25, 0.1, 0.05,$ and 0.01 while the producer's risk (α) is fixed at 0.05. Analyses are conducted at the 10th, 25th, 50th and 75th percentiles. The values for the the quantile ratio and termination ratios are chosen to conform to standard practice.

Setting $r_1 = 1$aligns with the Lot Tolerance Percent Defective (LTPD), reflecting the lowest quality a consumer is willing to accept. This choice ensures the plan properly controls the probability of accepting a poor-quality lot.

Setting $r_2 = 2,4,6,8,10$, aligns with the Acceptable Quality Level (AQL). The multiple values of r_2 capture typical levels of acceptable product quality, providing insight into the plan's behaviour under varying producer's risk scenarios. This also allows the ASP to be generalized across different production standards.

The termination ratios were selected to balance practical time and cost constraints with statistical reliability. Values of $0.5, 0.7,$ and 1.0 reflect common truncation scenarios in life-testing experiments and provide sensitivity analysis for different test durations.

Examining multiple levels $0.25, 0.10, 0.05$ and 0.01shows how the plan responds to different levels of risk tolerance for consumers. Multiple consumer risk levels were examined to evaluate the sensitivity of the sampling plan and to demonstrate its applicability under both lenient and stringent quality assurance standards.

The producer's risk is fixed at 0.05, consistent with conventional acceptance sampling practice, ensuring that high-quality lots are rarely rejected.

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Analyzing multiple percentile: 10th, 25th, 50th and 75th ensures that the sampling plan's performance is assessed across different portions of the lifetime distribution, capturing both early failures and long-lived items, applicable in reliability and survival analysis.

Overall, the chosen values for quantile ratios, termination ratios, risk levels and percentiles are grounded in both standard acceptance sampling practice and practical considerations. They allow us to explore the ASP's performance across a wide range of production quality scenarios, different test durations, and multiple points in the product lifetime distribution. This ensures that the resulting table is robust, informative, and applicable to both industrial and medical quality control contexts.

3.0 Results and Discussions

3.1 A Single ASP for the Percentiles of Zech Distribution Using Arbitrary Parameter Values

The arbitrary parameter values are used to design acceptance sampling plans when the true parameter values are unknown, thereby helping to illustrate how the sampling plan behaves under certain conditions. Tables 1, 2, and 3 show the behaviours of the proposed sampling plan when $\gamma = 0.5, \delta = 0.5$; $\gamma = 1.0, \delta = 0.2$; $\gamma = 1.5, \delta = 0.2$ respectively.

Table 1: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 0.5$ and $\delta = 0.5$) at the 25th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	50	3	0.9534	45	5	0.9659	33	6	0.9570
	4	26	1	0.9947	16	1	0.9863	10	1	0.9713
	6	14	0	0.9862	8	0	0.9735	5	0	0.9516
	8	14	0	0.9959	8	0	0.9904	5	0	0.9792
	10	14	0	0.9986	8	0	0.9961	5	0	0.9902
0.10	2	90	5	0.9638	70	7	0.9640	55	9	0.9539
	4	38	1	0.9888	23	1	0.9725	20	2	0.9834
	6	22	0	0.9784	13	0	0.9574	15	1	0.9906
	8	22	0	0.9935	13	0	0.9845	9	0	0.9629
	10	22	0	0.9978	13	0	0.9937	9	0	0.9824
0.05	2	115	6	0.9622	86	8	0.9558	70	11	0.9541
	4	46	1	0.9839	28	1	0.9605	23	2	0.9764
	6	29	0	0.9716	28	1	0.9960	18	1	0.9866
	8	29	0	0.9915	17	0	0.9798	11	0	0.9549
	10	29	0	0.9971	17	0	0.9918	11	0	0.9786
0.01	2	169	8	0.9578	127	11	0.9556	101	15	0.9547
	4	64	1	0.9702	49	2	0.9821	37	3	0.9824
	6	44	0	0.9572	38	1	0.9927	24	1	0.9767
	8	44	0	0.9871	26	0	0.9693	24	1	0.9954
	10	44	0	0.9956	26	0	0.9874	17	0	0.9671

Table 1 shows the optimal parameters of the single ASP when $\gamma = 0.5$ and $\delta = 0.5$

Table 2: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 1.0$ and $\delta = 0.2$) at the 50th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	35	7	0.9524	28	8	0.9501	25	10	0.9544
	4	14	2	0.9785	10	2	0.9689	7	2	0.9602
	6	10	1	0.9794	7	1	0.9714	5	1	0.9602
	8	10	1	0.9922	7	1	0.9882	5	1	0.9823
	10	5	0	0.9556	3	0	0.9505	5	1	0.9911
0.10	2	60	11	0.9543	45	12	0.9521	37	14	0.9542
	4	19	2	0.9514	17	3	0.9693	12	3	0.9560
	6	14	1	0.9608	9	1	0.9534	9	2	0.9809
	8	14	1	0.9847	9	1	0.9805	7	1	0.9650
	10	14	1	0.9931	9	1	0.9906	7	1	0.9820
0.05	2	74	13	0.9505	58	15	0.9549	49	18	0.9576
	4	27	3	0.9708	19	3	0.9555	16	4	0.9674
	6	22	2	0.9869	15	2	0.9804	11	2	0.9662
	8	16	1	0.9802	11	1	0.9711	8	1	0.9547
	10	16	1	0.9910	11	1	0.9860	8	1	0.9765
0.01	2	113	19	0.9584	-	-	-	-	-	-
	4	40	4	0.9700	28	4	0.9514	22	5	0.9622
	6	29	2	0.9724	20	2	0.9576	17	3	0.9756
	8	22	1	0.9639	20	2	0.9875	14	2	0.9785
	10	22	1	0.9832	15	1	0.9744	11	1	0.9566

Table 2 shows the optimal parameters of the single ASP when $\gamma = 1.0$ and $\delta = 0.2$

Table 3: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 1.5$ and $\delta = 0.2$) at the 75th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	19	6	0.9597	15	7	0.9632	16	10	0.9665
	4	6	1	0.9587	6	2	0.9831	5	2	0.9567
	6	6	1	0.9909	4	1	0.9861	3	1	0.9750
	8	3	0	0.9593	4	1	0.9954	3	1	0.9907
	10	3	0	0.9755	2	0	0.9650	3	1	0.9960
0.10	2	31	9	0.9607	26	11	0.9590	22	13	0.9594
	4	11	2	0.9787	8	2	0.9594	7	3	0.9772
	6	8	1	0.9835	5	1	0.9776	4	1	0.9530
	8	8	1	0.9950	5	1	0.9924	4	1	0.9822
	10	5	0	0.9594	5	0	0.9971	4	1	0.9922
0.05	2	36	10	0.9541	29	12	0.9563	26	15	0.9569
	4	13	2	0.9661	11	3	0.9799	8	3	0.9608
	6	10	1	0.9744	6	1	0.9675	6	2	0.9865
	8	10	1	0.9921	6	1	0.9889	5	1	0.9714
	10	6	0	0.9515	6	1	0.9955	5	1	0.9873
0.01	2	53	14	0.9577	41	16	0.9500	36	20	0.9551
	4	20	3	0.9760	14	3	0.9524	11	4	0.9671
	6	13	1	0.9578	11	2	0.9850	8	2	0.9673
	8	13	1	0.9667	9	1	0.9748	6	1	0.9587
	10	13	1	0.9950	9	1	0.9897	6	1	0.9815

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Table 3 shows the optimal parameters of the proposed single ASP when $\gamma = 1.5$ and $\delta = 0.2$

Observations from Tables 1 – 3
i. As the consumer's risk decreases, the sample size increases, ensuring more items are inspected so that the chance of accepting poor-quality lots is minimized, thereby protecting the consumer.
ii. The probability of acceptance of the lot is high across the quantile ratios. This makes it almost impossible to reject a product of good quality, thereby protecting the producer.
Iii. There is inverse relationship between the termination ratios and the sample sizes. This allows quicker decisions with fewer inspections, which reduces cost and time.

3.2 A Single ASP for the Percentiles of Zech Distribution Using Simulated Data

Data sets were generated from Zech distribution with a replication number $m = 10000$; random samples of size 50 were further randomly selected. The selected true parameter values are $\gamma = 0.5, \delta = 0.5, \theta = 1.5$.
The simulated data are presented in the Appendix.
The estimates of the shape parameters of Zech distribution, fitted on the simulated data are $\hat{\gamma} = 0.2876$ and $\hat{\delta} = 0.7260$
Table 4 shows the optimal parameters of the proposed single ASP for the simulated data at 25th percentile.

Table 4: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 0.2876$ and $\delta = 0.7260$) at the 25th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	46	2	0.9759	33	3	0.9683	24	4	0.9547
	4	16	0	0.9896	9	0	0.9696	10	1	0.9924
	6	16	0	0.9992	9	0	0.9961	5	0	0.9862
	8	16	0	0.9999	9	0	0.9994	5	0	0.9967
	10	16	0	1.0000	9	0	0.9999	5	0	0.9992
0.10	2	79	3	0.9777	51	4	0.9614	40	6	0.9540
	4	27	0	0.9826	14	0	0.9530	15	1	0.9830
	6	27	0	0.9987	14	0	0.9939	9	0	0.9754
	8	27	0	0.9999	14	0	0.9991	9	0	0.9942
	10	27	0	1.0000	14	0	0.9998	9	0	0.9985
0.05	2	91	3	0.9650	67	5	0.9638	55	8	0.9624
	4	35	0	0.9774	30	1	0.9952	18	1	0.9759
	6	35	0	0.9984	19	0	0.9917	11	0	0.9700
	8	35	0	0.9998	19	0	0.9987	11	0	0.9929
	10	35	0	1.0000	19	0	0.9998	11	0	0.9981
0.01	2	135	4	0.9633	101	7	0.9674	81	11	0.9635
	4	53	0	0.9661	41	1	0.9911	24	1	0.9588
	6	53	0	0.9975	28	0	0.9878	17	0	0.9541
	8	53	0	0.9998	28	0	0.9981	17	0	0.9890
	10	53	0	1.0000	28	0	0.9997	17	0	0.9971

Table 4 shows the optimal parameters of the proposed single ASP when $\gamma = 0.2876$ and $\delta = 0.7260$

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Observations from Table 4
i. There is an inverse relationship between the consumer's risk and sample size.
ii. There is an inverse relationship between the termination ratio and sample size.
iii. There is a direct relationship between the quantile ratio and probability of acceptance. As the quantile ratio increases, the probability of acceptance approaches 1.

3.3 Application of the Proposed Plan to Small Electric Cart Data

In this section, a single acceptance sampling plan for a truncated life test is developed based on the percentiles of Zech distribution. The analysis utilizes lifetime data (in months) from 20 small electric carts employed by a manufacturing company for internal transport and delivery within a large industrial facility. This dataset was previously used by Amjad and Ahsan ul Haq (2021) in the design of acceptance sampling plans based on the Power Lomax distribution. The observations are as follows:

0.9, 1.5, 2.3, 3.2, 3.9, 5.0, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15.0, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, and 53.0

The estimates of the parameters of Zech distribution, fitted on the lifetime data, as well as the goodness-of-fit criteria and P-value are presented in table 5 below.

TABLE 5: MLE Estimates, SE, AIC, BIC, AD, with the P-Value for the lifetime data

γ	δ	θ	AIC	BIC	AD	P-Value
0.8474 (0.5699)	0.2856 (0.3964)	0.0652 (0.0280)	152.5674	155.5546	0.0767	0.9772

The results presented in Table 5 indicate that the Zech distribution provides a good fit to the lifetime data, as evidenced by a p-value greater than 0.05. The standard errors (SE) are shown in parentheses.

Figure 1 displays the empirical and theoretical cumulative distribution functions (CDFs), histogram with theoretical density curves, as well as the Q–Q and P–P plots for the electric cart data fitted to the Zech distribution.

Figure 1: Empirical and theoretical cumulative distribution functions (CDFs), histogram with theoretical densities, Q–Q plot, and P–P plot of the electric cart data fitted to the Zech distribution.

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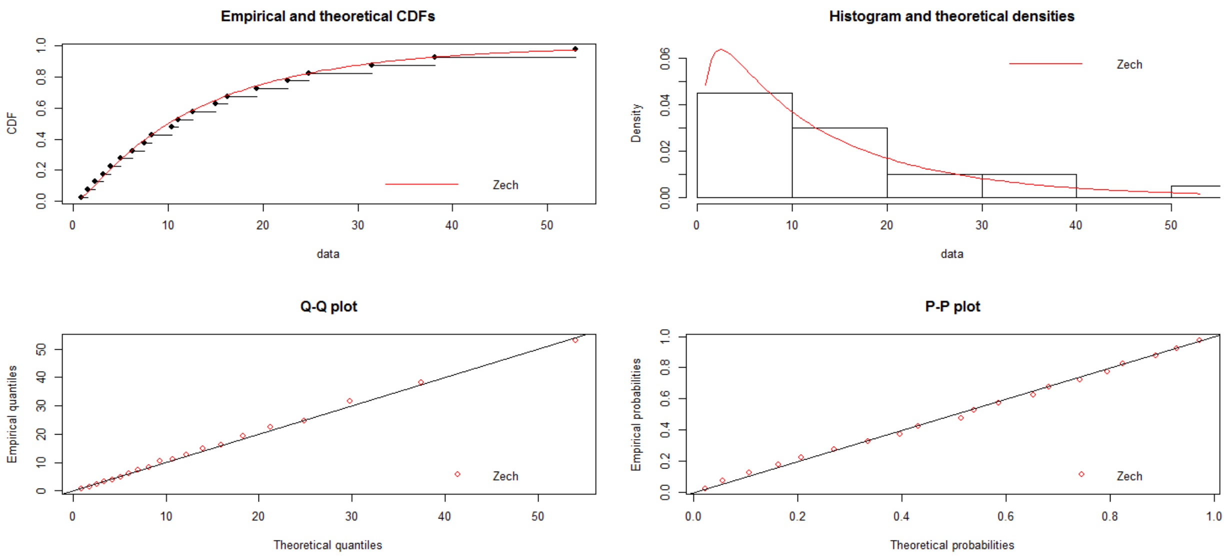

Figure 1 presents the empirical and theoretical cumulative distribution functions (CDFs), histogram with theoretical density curves, as well as the Q–Q and P–P plots for the electric cart data fitted to the Zech distribution.

From Figure 1, it is evident that the Zech distribution provides a good fit to the empirical data. This is supported by the alignment in the P–P and Q–Q plots, the comparison between the histogram and the theoretical density curve, and the close match between the empirical and theoretical CDFs. The proximity of the plotted lines to the data points further reinforces the conclusion that the Zech distribution fits the dataset well.

The optimal parameters for the proposed sampling plan at the 50th percentile, based on the electric cart data, are provided in Table 6.

Table 6: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 0.8474$ and $\delta = 0.2856$) at the 50th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
	2	35	7	0.9501	31	9	0.9568	27	11	0.9610
	4	14	2	0.9811	10	2	0.9699	7	2	0.9592
0.25	6	10	1	0.9835	7	1	0.9743	5	1	0.9613
	8	10	1	0.9945	7	1	0.9904	5	1	0.9839
	10	5	0	0.9652	3	0	0.9575	5	1	0.9924
	2	59	11	0.9567	48	13	0.9567	39	15	0.9589
	4	18	2	0.9626	16	3	0.9761	12	3	0.9547
0.10	6	13	1	0.9725	9	1	0.9580	9	2	0.9818
	8	13	1	0.9907	9	1	0.9840	7	1	0.9680
	10	13	1	0.9963	9	1	0.9930	7	1	0.9846
	2	73	13	0.9522	61	16	0.9580	49	18	0.9506
	4	27	3	0.9750	19	3	0.9571	16	4	0.9662
0.05	6	16	1	0.9593	15	2	0.9832	11	2	0.9678
	8	16	1	0.9860	11	1	0.9763	8	1	0.9585
	10	16	1	0.9944	11	1	0.9896	8	1	0.9799
	2	107	18	0.9515	-	-	-	-	-	-
	4	27	4	0.9595	57	3	0.9759	22	5	0.9606
0.01	6	20	2	0.9633	37	1	0.9614	17	3	0.9769
	8	15	1	0.9573	37	1	0.9885	14	2	0.9811
	10	22	1	0.9895	15	1	0.9808	11	1	0.9626

Table 6 shows the optimal parameters of the proposed sampling plan at 50th percentile using the electric cart data.

From Table 6, it is observed that there is an inverse relationship between the consumer risks and the number of samples

.

3.3.1 Estimation of the true 50th percentile lifetime of the Small Electric Carts Data Using Zech Distribution.

The estimates of the parameters of Zech distribution fitted on Electric Cart data are: $\hat{\theta} = 0.0652, \hat{\gamma} = 0.8474, and\ \hat{\delta} = 0.2856$

The true qth quantile of Zech distribution is given as $t_q = -\frac{1}{\theta}\left\{\ln\left[1-\left(1-\frac{\delta}{\gamma}\ln q\right)^{-\frac{1}{\delta}}\right]\right\}$

Setting $q = 0.5$, $t_{0.5} = -\frac{1}{0.0652}\left\{\ln\left[1-\left(1-\frac{0.2856}{0.8474}\ln 0.5\right)^{-\frac{1}{0.2856}}\right]\right\}$

$$= -15.3374\{\ln[1-(1-0.3370\times -0.6931)^{-3.5014}]\}$$
$$= -15.3374\{\ln[1-(1.2335747)^{-3.5014}]\}$$
$$= -15.3374\{\ln 0.5205\}$$
$$= -15.3374\times -0.6530$$
$$= 10.0153 \cong 10 \text{ months}$$

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Therefore, the estimated true 50th percentile lifetime of the electric cart is approximately 10 months.

As an illustration, consider a scenario where the manufacturer of small electric carts intends to apply the developed single acceptance sampling plan to determine whether the true 50th percentile lifetime of the carts, assumed to follow the Zech distribution with estimated shape parameters $\hat{\gamma} = 0.8474$ and $\hat{\delta} = 0.2856$, exceeds a specified benchmark. The estimated true 50th percentile lifetime is approximately 10 months ($t_q = 10$ months), while the specified 50th percentile lifetime is set at 2.5 months (i.e., $t_q^0 = 2.5$ months). The quantile ratio, or quality level, is thus $r_2 = \frac{t_q}{t_q^0} = \frac{10}{2.5} = 4$, indicating that the estmated actual median lifetime of the electric carts is four times the specified threshold.

Assuming a producer's risk ($\alpha = 0.05$), a consumer's risk ($\beta = 0.25$), a termination ratio ($a = 1.0$), and a quality level ($r_2 = 4$), the optimal parameters selected from Table 6 are: sample size, $n = 7$ and acceptance number, $c = 2$.

This implies that a sample of seven small electric carts should be randomly selected from the lot and subjected to a life test. If the number of failures observed by the end of the specified experiment time is not more than two, it can be statistically concluded that the true 50th percentile lifetime of the carts exceeds the specified benchmark, and the lot will be accepted. Otherwise, the lot will be rejected.

For the purpose of this practical illustration, three out of the seven tested carts failed before reaching 2.5 months. In this case, irrespective of the actual distribution of lifetimes in the overall population, the experimenter would advise the company that the median lifetime of the small electric carts does not meet the desired quality standard.

3.4 Application of the Proposed Plan to Asthmatic Patients' Data

This section presents the development and application of the proposed single acceptance sampling plan to asthma data obtained from 214 asthmatic patients, representing their lengths of stay at State Hospital, Ilaro. The observed data are presented as follows:

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3,
3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5,
5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 7, 7,
7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,
8, 9, 9, 10, 10, 10, 10, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13,
14, 15, 15, 16, 17, 19, 21, 21, 21, 21, 21, 21, 22, 22, 22, 22, 22, 22, 22, 23,
23, 23, 23, 23, 23, 23, 23, 23, 24, 24, 24, 27, 28, 28, 29, 30, 31, 31, 32, 32,
32, 32, 33, 33, 33, 33, 33, 33, 33, 33, 33, 34, 34, 34, 35, 37, 38, 41, 41, 42,
42, 42, 42, 42, 43, 44, 44, 45, 45, 45, 45, 48, 49, 51, 51, 51, 52, 52, 53, 55.
55, 56, 56, 56, 61, 63, 65, 66, 66, 66, 67, 71, 72, 78

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Table 7 presents the parameter estimates for the Zech distribution fitted to the asthma data, along with the selection criteria (Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)) and goodness-of-fit statistics (Anderson-Darling Statistic (AD)) and the corresponding p-value. The standard errors (SE) are provided in parentheses.

TABLE 7: MLE Estimates, SE, AIC, BIC, AD, with the P-Value for the lifetime data

γ	δ	θ	AIC	BIC	AD	P-Value
0.5174 (0.1091)	0.3512 (0.0936)	0.0375 (0.0057)	1682.368	1692.466	2.6408	0.8665

The results presented in Table **7** demonstrate that the Zech distribution provides a strong fit to the Asthma data, with a notable p-value of 0.8665.

Figure 2 illustrates the empirical and theoretical cumulative distribution functions (CDFs), histogram with theoretical densities, Q–Q plot, and P–P plot for the Zech distribution applied to the asthma data.

Figure 2: Plots of empirical and theoretical cdfs, histogram and theoretical densities, Q-Q plot and P-P plot

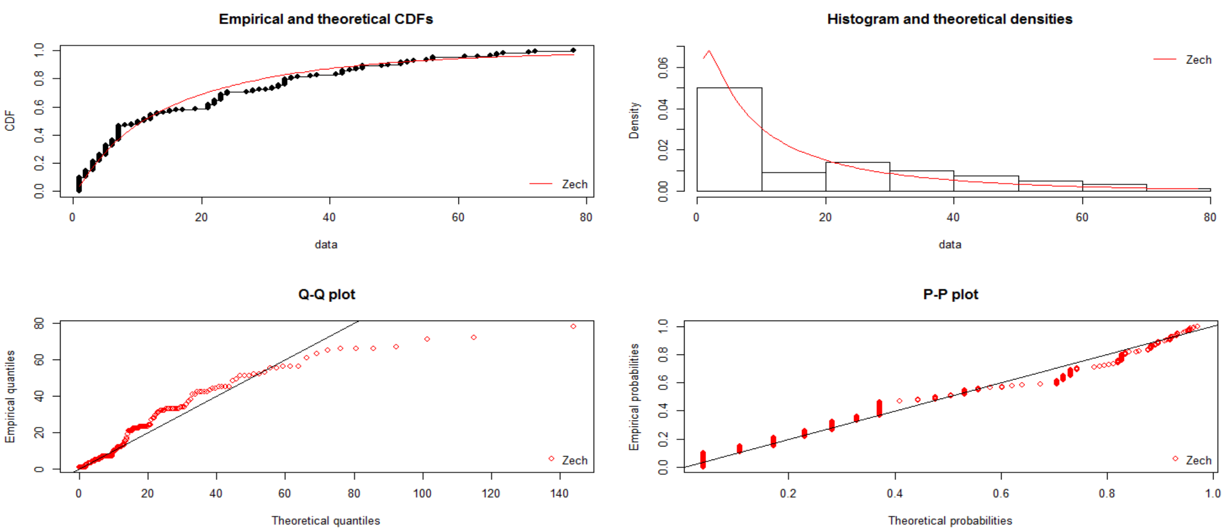

Figure 2 shows the curves of empirical and theoretical cdfs, histogram and theoretical densities, O-Q plot and P-P plot of the asthma data fitted on Zech distribution.

Table 8 presents the optimal parameters of the proposed single acceptance plan on the asthma data at 25th percentile.

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Table 8: Optimal Parameters of the Single ASP Based on Zech Distribution ($\gamma = 0.5174$and $\delta = 0.3512$) at the 25th Percentile

		$a = 0.5$			$a = 0.7$			$a = 1.0$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	71	6	0.9559	54	7	0.9518	47	9	0.9514
	4	22	1	0.9655	22	2	0.9827	15	2	0.9710
	6	22	1	0.9934	15	1	0.9872	10	1	0.9787
	8	11	0	0.9702	8	0	0.9514	10	1	0.9925
	10	11	0	0.9835	8	0	0.9712	5	0	0.9589
0.10	2	117	9	0.9569	92	11	0.9565	73	13	0.9506
	4	43	2	0.9803	29	2	0.9639	25	3	0.9741
	6	32	1	0.9863	21	1	0.9756	15	1	0.9540
	8	32	1	0.9965	21	1	0.9925	15	1	0.9833
	10	19	0	0.9717	12	0	0.9571	15	1	0.9932
0.05	2	149	11	0.9577	114	13	0.9521	93	16	0.9510
	4	51	2	0.9692	42	3	0.9788	29	3	0.9579
	6	38	1	0.9811	26	1	0.9637	23	2	0.9845
	8	38	1	0.9951	26	1	0.9887	18	1	0.9764
	10	24	0	0.9644	26	1	0.9959	18	1	0.9902
0.01	2	218	15	0.9553	168	18	0.9501	-	-	-
	4	81	3	0.9766	54	3	0.9526	43	4	0.9540
	6	53	1	0.9648	45	2	0.9846	31	2	0.9656
	8	53	1	0.9906	35	1	0.9800	24	1	0.9595
	10	53	1	0.9970	35	1	0.9927	24	1	0.9829

Table 8 shows the optimal parameters of the proposed single acceptance sampling plan at 25th percentile, using the asthma data.

Excerpts from Table 8

i. As the consumer's risk decreases, the sample size increases.
ii. As the termination ratio increases, the sample size decreases.
Iii. The probability of acceptance of the lot is high across the quantile ratios.

3.4.1 Estimation of the True 25th Percentile Length of Stay of Asthmatic Patients Using the Zech Distribution

The estimates of the parameters of Zech distribution fitted on the asthma data are: $\hat{\theta} = 0.0375, \hat{\gamma} = 0.5174, \hat{\delta} = 0.3512$

The true qth quantile is given as $t_q = -\frac{1}{\theta}\left\{\ln\left[1 - \left(1 - \frac{\delta}{\gamma} ln\, q\right)^{-\frac{1}{\delta}}\right]\right\}$

Setting $q = 0.25, t_{0.25} = -\frac{1}{0.0375}\left\{\ln\left[1 - \left(1 - \frac{0.3512}{0.5174} ln\, 0.25\right)^{-\frac{1}{0.3512}}\right]\right\}$

$$= -26.6667\{\ln[1 - (1 - (0.6788 \times -1.3863))^{-2.8474}]\}$$

$$= -26.6667\{\ln[1 - (1.9410)^{-2.8474}]\}$$

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$$= -26.6667\{\ln 0.8487\}$$

$$= -26.6667 \times -0.1640$$

$$= 4.3733 \cong 4 \text{ days}$$

Therefore, the estimated true 25th percentile length of stay of the asthmatic patients is approximately 4 days.

To illustrate the application of the proposed Acceptance Sampling Plan (ASP) presented in Table 8, consider a medical practitioner seeking to evaluate whether the 25th percentile length of stay in the hospital of asthmatic patients exceeds a specified threshold. The estimated true 25th percentile length of stay of the patients is 4.0 days. Assuming the specified benchmark is 2.0 days, this results in a quantile ratio $r_2 = \frac{4.0}{2.0} = 2$, indicating that the estimated actual length of stay is twice the specified value.

Given a consumer's risk ($\beta = 0.1$), a producer's risk ($\alpha = 0.05$), a termination ratio($a = 0.7$), and a quantile ratio ($r_2 = 2$), the optimal plan parameters obtained from Table 8 are: sample size, $n = 92$ and acceptance number, $c = 11$.

According to this plan, a random sample of 92 asthmatic patients should be selected. Each patient is observed up to the termination time, calculated as $a \times$ (specified 25th percentile length of stay), i.e., $0.7 \times 2 = 1.4$ days. Any patient who stays beyond this termination time is considered a failure in the context of the sampling plan. If the number of failures observrd is not more than 11, it can be statistically concluded within the defined risk levels that the true 25th percentile length of stay exceeds the specified threshold. If more than 11 failures occur, the lot is rejected.

For the purpose of this practical illustration, it is observed that 22 individuals in the sampled community fail before the test termination time of 1.4 days. Based on this outcome, medical professionals may advise the government that the 25th percentile length of stay of asthmatic patients is below the acceptable standard, suggesting the need for further investigation or public health intervention.

3.5 Comparative Study

The efficiency of the proposed acceptance sampling plan is compared with the single acceptance sampling plans for the truncated life tests based on New Weibull-Pareto distribution proposed by Nasiru & Luguterah; and Gompertz distribution previously used by Tripathi, Al-Omari, & Alomani (2022), using both industrial and medical epidemiology datasets.

In Table 9, the estimates of the parameters, selection and goodness of fit criteria and the performance ratings of the competing distributions are given.

3.5.1 Application of the Competing Distributions to the Small Electric Cart Data

The parameter estimates, selection and goodness-of-fit criteria, and performance ratings for the models fitted to the electric cart data are presented in Table 9.

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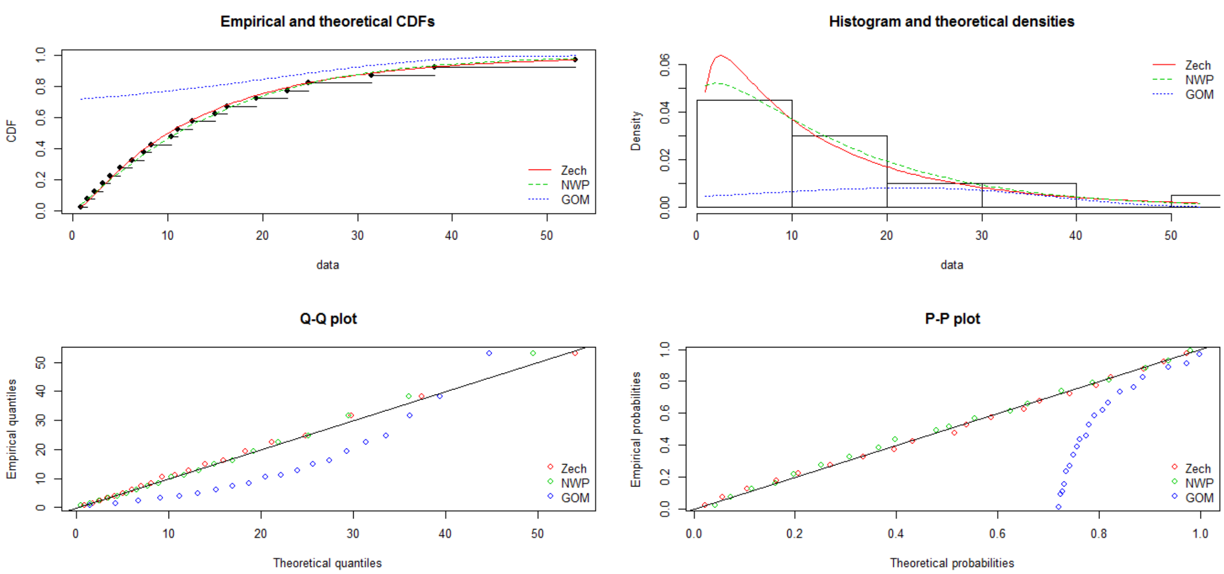
Table 9: Estimates of parameters, selection and goodness-of-fit criteria, and performance ratings for the fitted models using the electric cart data.

Distributions	Estimates	-LL	AIC	BIC	AD	Performance Rating
Zech	$\hat{\gamma} = 0.8474$ $\hat{\delta} = 0.2856$ $\hat{\theta} = 0.0652$	73.2837	152.5674	155.5546	0.0767	I
NWP	$\hat{\gamma} = 1.9992$ $\hat{\delta} = 0.0499$ $\hat{\theta} = 0.0164$	73.5528	153.1055	156.0927	0.0800	II
GOM-D	$\hat{\gamma} = 0.2625$ $\hat{\theta} = 16.5075$	105.0494	214.0978	216.0902	12.9505	III

Table 9 presents the parameter estimates, model selection and goodness-of-fit criteria, as well as performance ratings for the competing models fitted to the electric cart data. Among the models evaluated, the Zech distribution demonstrates the best performance, exhibiting the lowest values across the selection and goodness-of-fit criteria.

Figure 3 displays the empirical and theoretical cumulative distribution functions (CDFs), histograms with theoretical densities, Q–Q plots, and P–P plots for the competing distributions applied to the electric cart data.

Figure 3: The empirical and theoretical cdfs, histogram and theoretical densities, Q-Q plot and P-P plots of the competing distributions on the electric cart data.



From figure 3, it can be observed that Zech distribution provides the best fit to the electric cart data than the Gompertz and New Weibull-Pareto distributions.

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In Table 10, the optimal parameters of the proposed sampling plan are compared with the optimal parameters of the single acceptance sampling plans based on the New Weibull-Pareto and Gompertz distributions at 10th percentile when the termination ratio is 0.5, using the electric cart data.

Table 10: Comparison of the Optimal Single ASP's Parameters under Zech, NWPD, and Gompertz Distributions at the 10th Percentile for the electric cart Data ($a = 0.5$)

		$Zech$			$NWPD$			$Gompertz$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	151	5	0.9563	196	3	0.9587	-	-	-
	4	80	1	0.9921	103	1	0.9872	108	3	0.9658
	6	41	0	0.9787	53	0	0.9619	83	2	0.9755
	8	41	0	0.9912	53	0	0.9784	57	1	0.9594
	10	41	0	0.9958	53	0	0.9861	57	1	0.9730
0.10	2	274	5	0.9660	-	-	-	-	-	-
	4	115	1	0.9843	149	1	0.9745	168	4	0.9577
	6	68	0	0.9649	149	1	0.9945	140	3	0.9787
	8	68	0	0.9855	88	0	0.9644	112	2	0.9748
	10	68	0	0.9931	88	0	0.9770	112	2	0.9859
0.05	2	-	-	-	-	-	-	-	-	-
	4	140	1	0.9773	181	1	0.9636	221	5	0.9599
	6	88	0	0.9548	181	1	0.9920	163	3	0.9654
	8	88	0	0.9813	114	0	0.9541	132	2	0.9617
	10	88	0	0.9911	114	0	0.9703	132	2	0.9783
0.01	2	-	-	-	-	-	-	-	-	-
	4	195	1	0.9584	-	-	-	-	-	-
	6	195	1	0.9951	253	1	0.9848	243	4	0.9633
	8	135	0	0.9714	253	1	0.9949	210	3	0.9691
	10	135	0	0.9864	175	0	0.9548	176	2	0.9552

Table 10 presents a comparison of the optimal parameters of the proposed single acceptance sampling plan with those of corresponding plans based on the Gompertz and New Weibull-Pareto distributions using the elctric cart data.

The sampling plan requiring the fewest number of samples is deemed the most efficient. Based on this criterion, the plan developed using the Zech distribution is identified as the most effective among the alternatives.

3.5.2 Application of the Competing Distributions to the Asthmatic Patients' Data

The parameter estimates, model selection and goodness-of-fit criteria, as well as performance ratings for the competing models fitted to the Asthma data are presented in Table 11.

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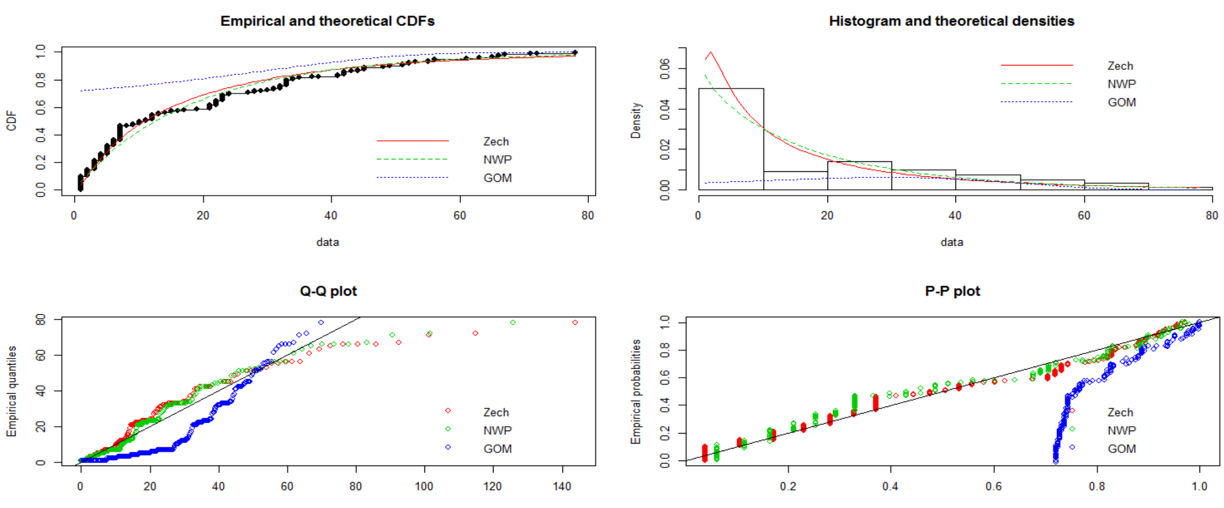
Table 11: Estimates of parameters, selection and goodness-of-fit criteria, and performance ratings for the fitted models using Asthma data.

Distributions	Estimates	-LL	AIC	BIC	AD	Performance Rating
Zech	$\hat{\gamma} = 0.5174$ $\hat{\delta} = 0.3512$ $\hat{\theta} = 0.0375$	838.184	1682.368	1692.466	2.6408	I
NWP	$\hat{\gamma} = 0.0210$ $\hat{\delta} = 0.9411$ $\hat{\theta} = 0.3051$	844.0439	1694.088	1704.186	2.9566	II
GOM-D	$\hat{\gamma} = 0.2637$ $\hat{\theta} = 22.0066$	1189.593	2383.185	2389.917	135.7093	III

As shown in Table 11, the Zech distribution demonstrates superior performance, yielding the lowest values for the selection and goodness-of-fit metrics, and is therefore considered the best among the competing models.

Figure 4 illustrates the empirical and theoretical cumulative distribution functions (CDFs), histogram with theoretical densities, Q–Q plot, and P–P plot for the competing distributions fitted to the asthma data.

Figure 4: The empirical and theoretical cdfs, histogram and theoretical densities, Q-Q plot and P-P plot.



The close alignment of the Zech distribution's theoretical curves with the empirical data indicates that it provides the best fit to the asthma dataset.

Table 12 presents a comparison of the optimal parameters of the proposed sampling plan with those of single acceptance sampling plans based on the New Weibull-Pareto and Gompertz distributions. The comparison is conducted at the 25th percentile under a termination ratio of 0.5, using asthma data.

Table 12: Comparison of the Optimal Single ASP's Parameters under Zech, NWPD, and Gompertz Distributions at the 25th Percentile for the Asthma Data ($a = 0.5$)

		$Zech$			$NWPD$			$Gompertz$		
β	r_2	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$	n	c	$P_a(p_2)$
0.25	2	71	6	0.9559	-	-	-	107	10	0.9503
	4	22	1	0.9655	76	11	0.9509	42	3	0.9671
	6	22	1	0.9934	52	7	0.9572	32	2	0.9767
	8	11	0	0.9702	40	5	0.9534	22	1	0.9605
	10	11	0	0.9835	34	4	0.9516	22	1	0.9738
0.10	2	117	9	0.9569	-	-	-	184	16	0.9563
	4	43	2	0.9803	127	17	0.9509	65	4	0.9597
	6	32	1	0.9863	82	10	0.9524	43	2	0.9503
	8	32	1	0.9965	69	8	0.9620	43	2	0.9760
	10	19	0	0.9717	56	6	0.9533	31	1	0.9507
0.05	2	149	11	0.9577	-	-	-	228	19	0.9515
	4	51	2	0.9692	-	-	-	85	5	0.9628
	6	38	1	0.9811	110	13	0.9588	63	3	0.9667
	8	38	1	0.9951	83	9	0.9525	51	2	0.9628
	10	24	0	0.9644	76	8	0.9645	51	2	0.9790
0.01	2	218	15	0.9553	-	-	-	-	-	-
	4	81	3	0.9766	-	-	-	129	7	0.9653
	6	53	1	0.9648	162	18	0.9584	93	4	0.9659
	8	53	1	0.9906	127	13	0.9578	80	3	0.9714
	10	53	1	0.9970	106	10	0.9503	67	2	0.9577

Table 12 shows the comparison of the optimal parameters of the proposed single acceptance sampling plan with the same plans based on Gompertz and New Weibull-Pareto distributions.

The sampling plan that requires the minimum number of samples for inspection is regarded as the most efficient. Accordingly, the single acceptance sampling plan based on the Zech distribution is identified as the most effective among the compared models. The operating characteristic (OC) curves of the competing sampling plans are presented in Figure 5.

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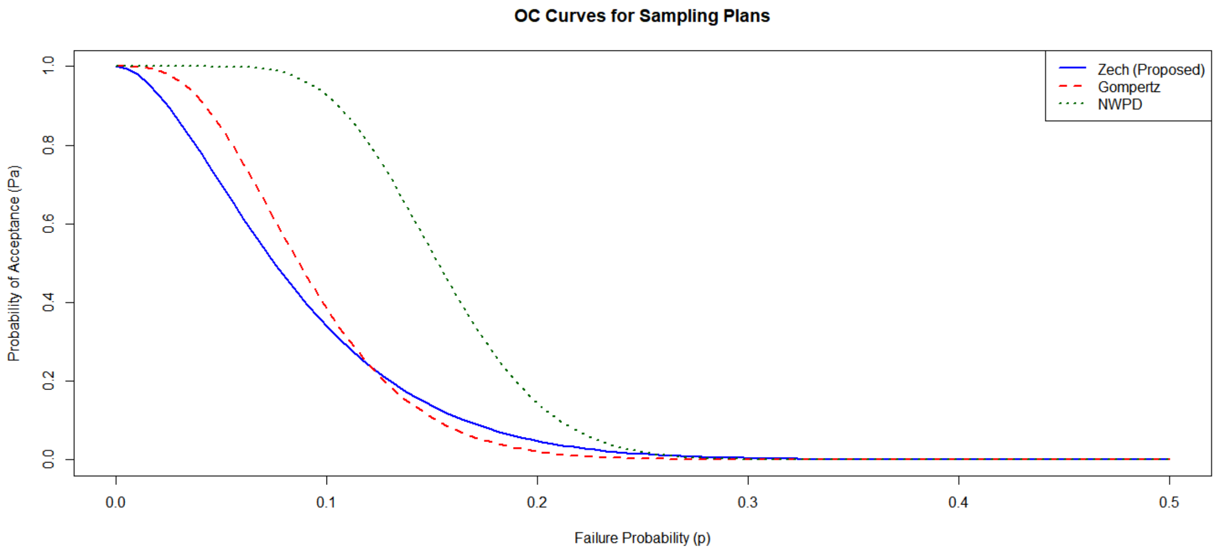

Figure 5: The overlapping OC curves of the competing single acceptance sampling plans

The OC curve of the proposed sampling plan exhibits a steeper decline compared to the Gompertz and NWPD-based plans, indicating superior discriminatory power between good and bad lots.

4.0 Discussion of Results

This study presents the development of a single acceptance sampling plan based on a truncated life test under the Zech distribution. Key findings are summarized as follows: The design results, based on arbitrary parameter values across the 25th, 50th, and 75th percentiles (as shown in Tables 1–3), demonstrate that a decrease in the consumer's risk leads to an increase in the required sample size. Additionally, an inverse relationship is observed between termination ratios and sample sizes. Table 4 illustrates the proposed plan constructed using simulated data from the Zech distribution. The outcomes reinforces the reliability of the proposed method. Parameter estimates, along with selection and goodness-of-fit criteria and the p-value for the Zech distribution fitted to electric cart data, are reported in Table 5. A high p-value of 0.9772 indicates a strong fit to the dataset. Table 6 demonstrates the practical application of the proposed plan to electric cart data, providing a comprehensive explanation of its implementation. In Table 7, maximum likelihood estimates (MLEs) of the Zech distribution parameters for the asthma data are provided, accompanied by a p-value of 0.8665, further confirming the distribution's suitability. Table 8 reports the optimal parameters of the proposed sampling plan at the 25th percentile, based on the asthma data under specified constraints. Table 9 presents parameter estimates, model selection, goodness-of-fit criteria, and performance rankings for three competing distributions using electric cart data. The Zech distribution emerged as the best-fitting model, exhibiting the lowest selection and goodness-of-fit metrics. A comparison of the optimal sampling plan parameters for the Zech distribution with those based on Gompertz and New Weibull-Pareto distributions at 10th percentile (Table 10), using electric cart data reveals that Zech distribution requires the smallest sample size, thus offering the most efficient solution. Table 11 provides a similar comparative analysis using asthma data at 25th percentile, while Table 12 reiterates that the Zech-based plan is the most effective and cost-efficient among the alternatives. Figure 1 displays the empirical and theoretical cumulative distribution functions (CDFs), histogram with fitted densities, Q–Q plot, and P–P plot of the industrial data fitted to the Zech distribution. The close alignment of theoretical and empirical values confirms the model's adequacy. Similar results are shown in Figure 2 for the asthma data. Figures 3 and 4 present comparative plots (Q–Q, P–P, CDFs, and histograms) for the three competing models across both datasets, supporting the Zech distribution as the best fit overall. The operating characteristic curves displayed in Figure 5 shows that Zech-based single ASP has the steepest slope, indicating a superior discriminatory power between good and bad lots.

4.1: Limitations, Computational Complexity and Transition from Theory to Application.

4.1.1: Limitations

The present study is based on a single lifetime distribution rather than a comprehensive survey of multiple lifetime distributions. While this allows for a focused and in-depth analysis, it may not fully capture the diversity of models available in the acceptance sampling plan (ASP) literature. Future studies could extend this framework to a broader class of lifetime distributions to enhance general applicability.

In addition, the application of the proposed sampling plan was limited to industrial and medical datasets. Although these domains provide relevant and practical validation, the performance of the plan in other fields, such as education, agriculture, and related areas, has not been investigated. Further research in these domains would help to establish the wider applicability and robustness of the proposed approach.

4.1.2: Computational Complexity

The optimization of the sampling plan was carried out using a brute-force enumeration approach, which offers several important advantages. Notably, this method guarantees the identification of optimal parameter combinations within the defined search space, ensuring accuracy and reproducibility of results. In addition, it is straightforward to implement and does not rely on tuning parameters or convergence criteria, making it particularly suitable for structured optimization problems such as acceptance sampling plan design.

However, as with most exhaustive search techniques, the computational effort increases with the size of the parameter space. The inclusion of multiple design parameters such as sample size, acceptance number, termination ratio, percentiles, and quantile ratios can lead to higher computational requirements, especially when finer search resolutions are considered. This is more pronounced when working with highly skewed lifetime distributions, where careful estimation of percentiles may necessitate additional computational effort.

Despite this, the approach remains practical for the problem sizes considered in this study, and the resulting optimal plans justify the computational cost through improved efficiency and reduced sample sizes. For larger-scale problems, future research may consider alternative optimization strategies, such as heuristic or metaheuristic methods, to further enhance computational efficiency.

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4.1.3: From Theory to Application

With the theoretical framework established, the proposed plan is applied to industrial and medical datasets to evaluate practical performance. The following results highlight the plan's efficiency, adaptability to highly skewed lifetime distributions, and superiority in minimizing required sample sizes compared to existing approaches.

5.0 Conclusion

In this study, a single acceptance sampling plan based on truncated life tests and constructed using the percentiles of the Zech distribution is developed. The plan's performance is evaluated under various scenarios using arbitrary parameter values, simulated data, industrial data, and medical data. Practical applications of the proposed plan are demonstrated in both industrial and medical contexts. A comparative analysis with single acceptance sampling plans based on the New Weibull-Pareto and Gompertz distributions reveals that the plan developed using the Zech distribution consistently outperforms the others, establishing it as the most effective approach among the models considered.

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APPENDIX

The Simulated Data

0.02778655, 0.03317616, 0.04735757, 0.06004381, 0.06114307, 0.06284239, 0.06826930, 0.06904844, 0.07298172, 0.09098504, 0.10769026, 0.11148058, 0.13567743, 0.14090141, 0.14185104, 0.15214113, 0.16586403, 0.17557878, 0.17835086, 0.19288063, 0.20177095, 0.20434227, 0.22674940, 0.22976317, 0.23249269, 0.26384114, 0.27018619, 0.29298173, 0.30543376, 0.30790196, 0.31092006, 0.34486852, 0.35415989, 0.37646817, 0.41019261, 0.56431109, 0.58716907, 0.58789835, 0.58904138, 0.68091964, 0.68175724, 0.78287974, 0.81122984, 0.83194921, 1.04402310, 1.27194805, 1.45457358, 1.53540965, 2.34016071, 3.04635964

Notations	Meaning
$\mathrm{P_a}(p)$	Probability of accepting a lot
d	Number of defective items in a lot
q	Quantiles
t_q	True quantile life of a product
t_q^0	Specified quantile life of a product
a	Time multiplier constant
t_0	Experiment's termination time
P^*	Consumer's confidence level
n	Sample size
N	Lot size
c	Acceptance Number
β	Consumer's risk
α	Producer's risk
r_1	Quantile ratio associated with the consumer's risk
r_2	Quantile ratio associated with the producer's risk
p_1	Failure probabilities at the limiting quality level
p_2	Failure probabilities at the acceptable quality level
$P_a(p_1)$	Probability of acceptance at the limiting quality level
$P_a(p_2)$	Probability of acceptance at the acceptable quality level

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