MODELLING AND FORECASTING BOND RATE OF NIGERIA ECONOMY USING AUTOREGRESSIVE INTEGRATED MOVING AVERAGE (ARIMA) TECHNIQUES

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Abstract

This study is on modelling and forecasting bond rate of Nigeria economy. The bond market in Nigeria, especially for FGN bonds, facilitates government borrowing to meet monetary policy goals, infrastructure development, and budget deficits. Autoregressive Integrated Moving Average (ARIMA) modeling technique was used to analyze and forecast key indicators of the Nigerian 10-year Federal Government Bond market, specifically focusing on Total Subscription, Total Successful Bids, and Bond Rate from 2013 to 2023. Using time series analysis in R software package, the data was tested for stationarity and ARIMA models were then fitted, with ARIMA(0,1,2)(0,0,2)[12] selected for Total Subscription, ARIMA(0,1,1) for Total Successful Bids, and ARIMA(1,1,0) for Bond Rate. Model diagnostics, including the Ljung-Box test, confirmed the adequacy of the fitted models. The resulting forecasts indicated stable future bond rates around 14.1%, while subscription and bid values showed fluctuations consistent with market dynamics. The models captured trends, seasonality and short term dependencies effectively. The study demonstrates that ARIMA modeling offers a robust framework for forecasting bond market behavior in Nigeria, providing valuable insights for policymakers, investors, and financial analysts in planning, risk management, and policy

Keywords: ARIMA, Bond Market, Stationarity, Monetary Policy and Ljung-Box test

1. Introduction

A bond is a type of fixed-income financial instrument that represents a loan given to a borrower, typically a business or government agency, by an investor. Through promoting liquidity, stabilizing economies, and directing capital towards profitable industries. The bond market plays one of the important key roles in national finance. The bond market in Nigeria, especially for FGN bonds, facilitates government borrowing to meet monetary policy goals, infrastructure development, and budget deficits (Central Bank of Nigeria, 2019). It represents investor confidence in the economy and acts as a standard for pricing other financial instruments. The bond market is a gauge of the state of the economy since it also affects inflation, interest rates, and currency rates. An effective bond market increases fiscal sovereignty for emerging nations like Nigeria by promoting financial inclusion and lowering dependency on outside borrowing (Adelegan & Radzewicz-Bak, 2020). The national government issues government bonds as debt instruments to pay down the nation's debt and fund public spending. These financial products, which frequently include regular interest payments (coupons) for a certain period, provide people, institutions, and international investors with a safe and comparatively predictable investment channel (Brigo & Mercurio, 2018).
The ability of government bonds, like FGN bonds, to control the money supply, manage fiscal deficits, and stabilize financial institutions makes them essential to economic policy (Brigo & Mercurio, 2018).. The Nigerian government finances social programs, debt servicing, and essential infrastructure by issuing bonds, and the yields on these bonds reflect market sentiments regarding

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risk and economic stability (Okereke & Ndubuisi, 2019). Since the Central Bank of Nigeria (CBN) utilizes bond yields to assess investor sentiment and modify policy rates, they have an impact on monetary policy decisions. Additionally, government bonds assist the stability of the financial system by providing banks and institutional investors with a safe asset. Thus, it is essential to comprehend bond yield dynamics in order to formulate policies and plan the economy effectively (Umar & Abdulhakeem, 2021). The Debt Management Office (DMO), acting on behalf of the Federal Government, issues Federal Government of Nigeria (FGN) Bonds, which are long-term debt instruments. These bonds are crucial for boosting investment, preventing budget deficits, and advancing macroeconomic stability. FGN bonds have grown to be a significant part of the capital market and a national standard for risk-free interest rates over time (Oladipupo & Omodero, 2021). Therefore, it is of note for monetary authorities and fiscal planners as well as investors to comprehend and predict their yields.

For investors and governments to predict market movements and make wise decisions, bond yield modelling is essential. Bond yield forecasting that is accurate aids investors in managing risks, optimizing portfolio returns, and evaluating the effects of macroeconomic shifts like inflation or currency rate swings (Adebayo & Ojo, 2022). Yield projections help policymakers with fiscal planning, interest rate policy, and debt management. Predictive models offer insights into future yield movements, allowing for proactive actions in Nigeria, where bond markets are impacted by exogenous shocks (such as changes in the price of oil) and economic volatility. This emphasizes the necessity of strong statistical techniques for efficient bond rate analysis and forecasting (Eke & Okoye, 2023).

A bond is a debt security issued by a borrower, typically a government or corporation, to raise funds from investors, who in return receive periodic interest payments and the repayment of the principal at maturity (Mishkin, 2019). Bonds represent a contractual obligation, where the issuer borrows capital and commits to fulfilling payment terms over a specified period. Nigeria's financial environment depends heavily on government bonds, especially FGN bonds, which are the main source of borrowing for the government to fund infrastructure projects and budget deficits. These bonds offer liquidity and investment opportunities since they are issued by the Debt Management Office (DMO) and are traded on the bond market. Short-term (like Treasury bills), medium-term (like 5-year bonds), and long-term (like 20-year bonds) are the three maturity categories for government bonds. They are a benchmark for pricing other financial instruments in Nigeria because of their relative safety and the government's power to tax or generate money (CBN, 2019).

One important measure of the state of the economy and investor sentiment is the yield on government bonds, which is the return an investor receives from owning a bond. According to Umar and Abdulhakeem (2021), yields and bond prices are negatively correlated; when demand for bonds rises, prices rise and yields fall. A number of factors, including macroeconomic ones like inflation, interest rates, and currency rate volatility affect FGN bond yields in Nigeria. High inflation, for example, reduces the real return on bonds and forces yields higher to make up for the loss to investors. Political stability, investor risk tolerance, and market liquidity are further factors that affect FGN bond yields. Because big transactions by institutional investors considerably influence prices, low market liquidity in Nigeria, which is caused by limited involvement from regular investors, might result in yield volatility (Adelegan & Radzewicz-Bak, 2020).

The Autoregressive Integrated Moving Average (ARIMA) model is a robust time-series forecasting tool that combines three components: autoregression (AR), integration (I), and moving average (MA). The AR component models the relationship between a variable and its past values, the MA

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component accounts for past forecast errors, and the I component addresses non-stationarity through differencing (Afolabi & Oladipupo, 2021). Its flexibility in handling trends and volatility makes it ideal for modeling financial variables like FGN bond yields, which exhibit time-varying patterns influenced by economic shocks and policy changes. ARIMA's application in financial forecasting, including bond yields, is well-documented due to its ability to capture linear dependencies in time-series data. In Nigeria, where bond yields are subject to volatility from macroeconomic and external factors, ARIMA provides a practical approach to generate reliable forecasts (Adebayo & Ojo, 2022). Several international studies have explored the application of ARIMA models in forecasting bond yields. For instance, Alqaralleh (2019) used ARIMA to forecast U.S. Treasury bond yields and concluded that the model performed well in short-term predictions but struggled with long-term volatility. Similarly, Gyamerah and Agalega (2020) applied ARIMA to model Ghanaian government bond yields and found the ARIMA(1,1,1) model to be the best fit, emphasizing the suitability of ARIMA in African debt markets where data is often volatile and non-linear. In another study, Dinh and Huong (2021) used ARIMA and Exponential Smoothing methods to forecast Vietnam's bond rates. Their findings supported the use of ARIMA in capturing autocorrelations in financial time series. Meanwhile, Rahman and Sultana (2020) modeled Bangladeshi government bond rates and concluded that ARIMA outperformed machine learning models in accuracy, especially when data size is limited. These studies confirm that ARIMA is often a practical choice in both developed and emerging markets due to its interpretability and data efficiency. A popular time-series forecasting method that uses patterns in past data to predict future values is the Autoregressive Integrated Moving Average (ARIMA) model. Bond yields, which frequently show patterns and seasonality, can be analyzed using ARIMA since it models stationary or non-stationary time series by combining autoregressive (AR), differencing (I), and moving average (MA) components (Oladapo & Adebayo, 2020). It is a favored instrument in financial econometrics for predicting economic variables such as interest rates and bond yields because of its adaptability and resilience.

2. Methodology

Source of Data

The Federal Government of Nigeria's monthly bond yield rates, which were taken from open financial documents, made up the dataset used in this investigation. The Central Bank of Nigeria (CBN), the main organization in charge of monetary policy and debt issuance in the nation, provided these rates.

Stationarity Testing (ADF Test)

A stationary time series is a time series whose statistical properties do not change over time. A stationary time series is one whose statistical properties such as mean, variance, autocorrelation, etc. are all constant over time. A time series must be stationary for ARIMA modeling. The Augmented Dickey-Fuller (ADF) test is employed to test this condition.

$$\Delta Y_t = \alpha_1 + \alpha_{2t} + \beta \gamma_{t-1} \sum_{i=1}^{n} \gamma_i \, \Delta Y_{t-i} + \epsilon_i \quad (1)$$

ΔY_t: First difference of Y_t, Y_{t-1}: Lagged value of Y_t
β: Test coefficient α_1: Constant term
α_2: Coefficient of the variable
ε_i: Gaussian white noise error term

The ADF test is a one-sided test used to determine whether a time series is stationary or contains a unit root. The null hypothesis (H_0: $\delta = 0$) suggests that the series has a unit root, indicating non-

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stationarity (i.e., it is integrated of order one or higher). Conversely, the alternative hypothesis (H_1: $\delta < 0$) implies that the series is stationary. The decision rule states that if the computed ADF test statistic is lower than the MacKinnon critical values, the null hypothesis is rejected, confirming that the series is stationary. However, if the null hypothesis is accepted, it indicates that the series is non-stationary, and differencing is required to achieve stationarity.

The Mean and Covariance Function

The mean of a stationary stochastic process will be denoted by μ and is given by

$$\mu = E(Y_t) = \int_{-\infty}^{\infty} yt\, f(yt) dyt \quad (2)$$

This definition implies that a stationary stochastic process has a constant mean or level about which the process fluctuates. The variance of a stationary stochastic process denoted by σ^2 is given by:

$$\sigma^2 = E[y_t - \mu]^2 \int_{-\infty}^{\infty} yt\, f(yt) dyt \quad (3)$$

for any time t. The variance is a measure of the fluctuation of the stochastic process about its level. If the process is stationary, the means and similarly the variances are all the same.

The Auto Covariance Function

The autocovariance of a random variable y_t and y_{t+k} separated by a constant time interval or lag k is denoted by c_k. The autocovariance is a measure of the linear dependence between two random variables separated by a fixed number of time periods or lag k.

The Auto Correlation Function

Autocorrelation, also known as serial correlation, is the correlation of a signal with a delayed copy of itself as a function of delay. Informally, it is the similarity between observations as a function of the time lag between them. The sample autocorrelation function (ACF) for a series gives correlations between the series x_t and lagged values of the series for lags of 1, 2, 3, and so on. The lagged values can be written as x_{t-1}, x_{t-2}, x_{t-3}, and so on. The ACF gives correlations between xt and x_{t-1}, x_t and x_{t-2}, and so on. The ACF can be used to identify the possible structure of time series data. That can be tricky going as there often isn't a single clear-cut interpretation of a sample autocorrelation function. The ACF of the residuals for a model are also useful. In a sample ACF of residuals, there shouldn't be any significant correlations for any lag.

The Partial Auto Correlation Function

In time series analysis, the partial autocorrelation function (PACF) gives the partial correlation of a time series with its own lagged values, controlling for the values of the time series at all shorter lags. It contrasts with the autocorrelation function, which does not control for other lags.

Akaike Information Criterion

Akaike Information Criterion (AIC) is an estimator of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection. The formula for AIC is given as:

$$AIC = n + n\, log\, 2\pi + n\, (RSS/n) + 2(p+1)$$

Bayesian Information Criterion

Bayesian Information Criterion (BIC) or Schwarz Criterion (SBIC) is a criterion for model selection among a finite set of models; the model with the lowest BIC is preferred. It is based, in part, on the likelihood function and it is closely related to the Akaike Information Criterion (AIC).

The formula for BIC is given as:

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$$BIC = n + n\,log\,2\pi + n\,(RSS/n) + (log\,n)(p+1)$$

Diagnostic Checking (Residual Analysis)

After estimation, model adequacy is checked using residual analysis to ensure residuals are white noise. Techniques include plotting residuals, performing the Ljung-Box Q-test, and checking for autocorrelation in residuals.

The Ljung-Box Q-statistic is given by:

$$Q = n(n+2)\sum_{k=1}^{m}\frac{\hat{\rho}_k^2}{n-k} \tag{4}$$

Where:

n: *Sample size* , $\hat{\rho}_k$: *Autocorrelation at lag k*, m: *Number of lags to test*

Model Specification

The general ARIMA (p,d,q) model is specified as:

$$Y_t = c + \emptyset_1 Y_{t-1} + \cdots + \emptyset_p Y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{5}$$

Where:

Y_t: *Observed value at time t*, c: *Constant term*, $\emptyset_i$: *Autoregressive coefficients* (AR)

$\emptyset_j$: *Moving average coefficients* (MA), d: *Order of differencing to achieve stationarity*

ε_t: *Error term assumed to be white noise*

The values of p, d, and q were selected based on model identification using ACF, PACF, and stationarity tests. This specification provides the foundation for forecasting FGN bond yields.

3. Data Analysis

Three key financial indicators—the total number of subscriptions, the total number of successful bids, and the Bond rate (yield)—are the main focus of modelling and forecasting.

4.2 Time Series Plot and Trend Analysis

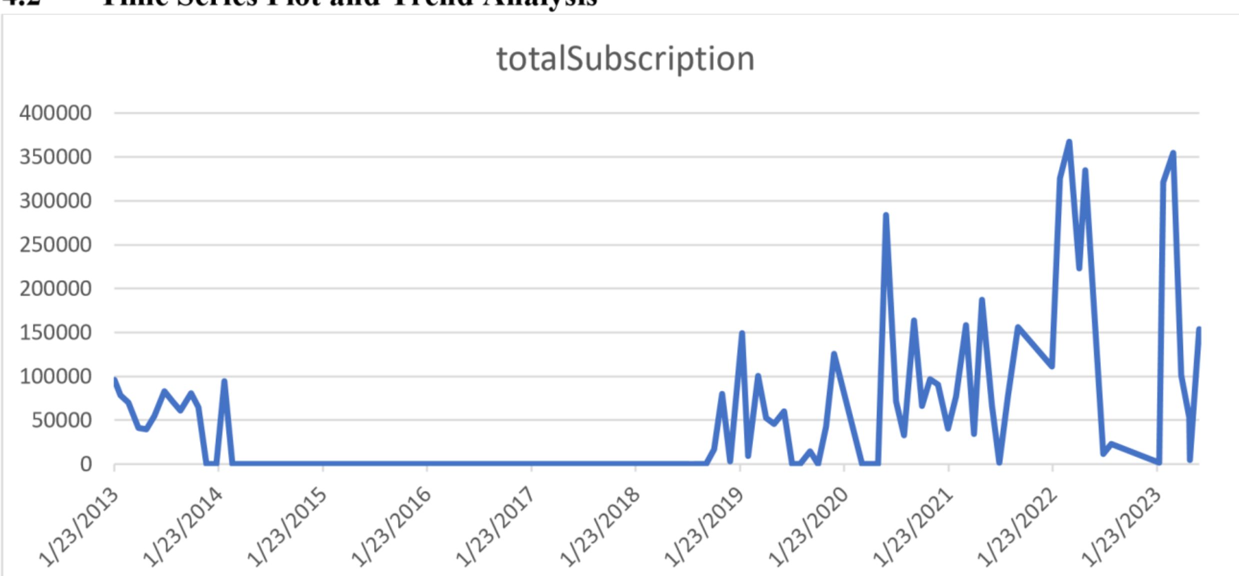

Figure 1: Time Series Plot for Total Subscription

Figure1, illustrates the aggregate investor demand for the Federal Government of Nigeria (FGN) 10-Year bonds throughout time is seen in the time series plot for total subscription. The plot initially displays a modest level of volatility, with early years (2013–2016) seeing variations in investor interest. Subscription volumes noticeably increased between 2017 and 2019, which may indicate improving macroeconomic conditions or rising investor confidence. However, after 2020, the pattern seems to be rather irregular, with some notable peaks and dips. This could be due to both local monetary policy changes and worldwide economic uncertainties, such as the COVID19 pandemic. The recent irregularity might potentially be a result of shifting expectations for inflation, risk perceptions, or financial market liquidity restrictions.

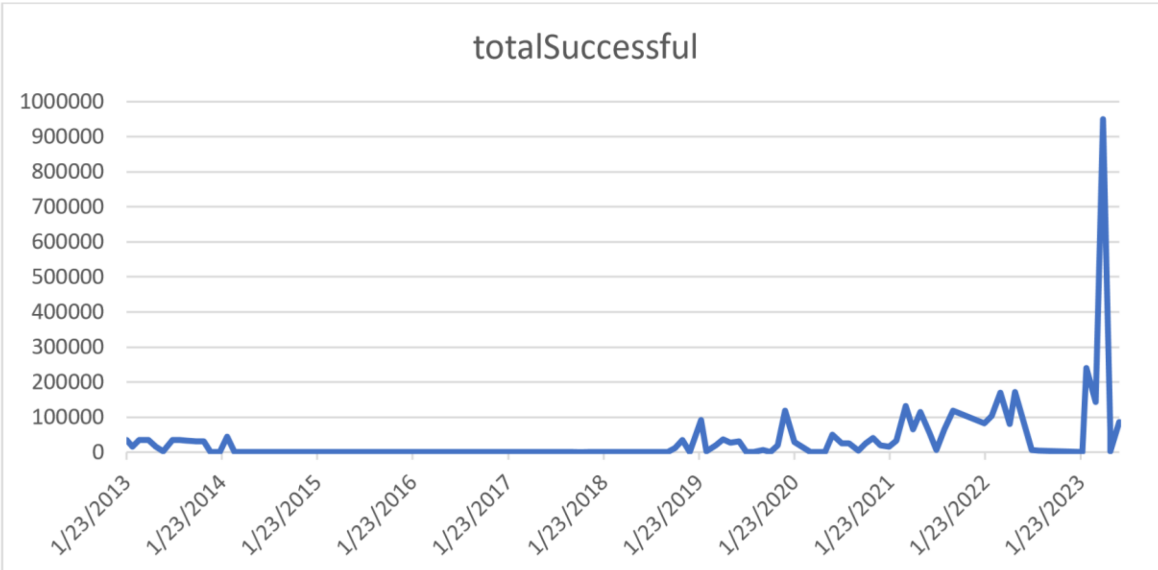

Figure 2: Time Series Plot for Total Successful Bids

The time series plot for total successful bids reveals how much of the subscribed amount was accepted by the government in each auction. The pattern closely resembles that of total subscription but at a consistently lower magnitude, indicating partial fulfillment of investor demand. This is expected, as the Debt Management Office (DMO) often accepts only the volume that aligns with fiscal planning. From 2016 to 2019, we see periods of increased acceptance, followed by more selective acceptance from 2020 onwards. This selective trend may reflect tightening public debt management strategies or cautious borrowing amidst economic instability.

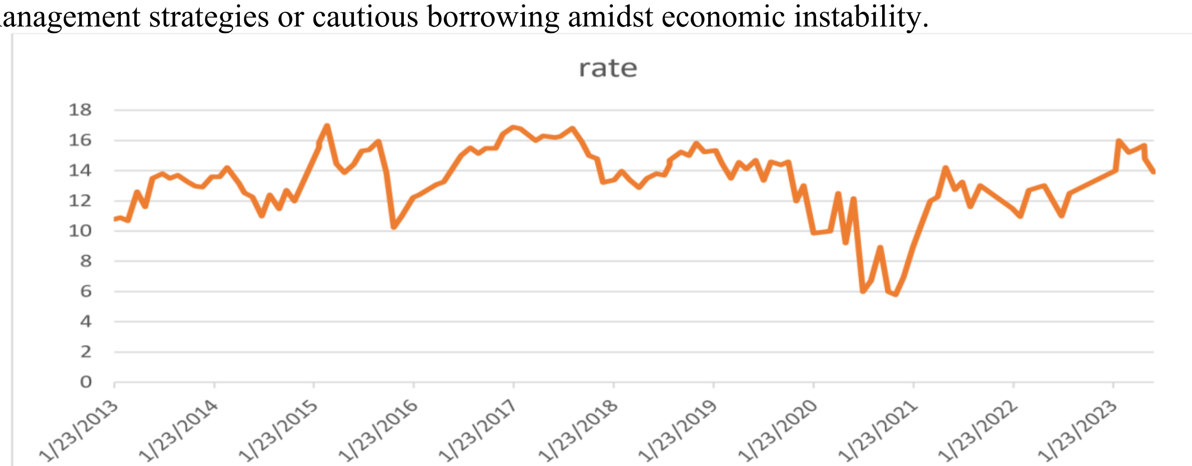

Figure 3: Time Series Plot for Bond Rate

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The bond rate (yield) plot reflects the cost of borrowing for the government. Between 2013 and 2015, rates generally trended upward, peaking around 2016–2017, suggesting higher inflation expectations or reduced investor confidence. From 2018 through 2020, there is a declining trend, reaching lows around 2020–2021, which aligns with monetary easing during the pandemic to stimulate borrowing. Post-2021, however, the yield shows a resurgence, indicating rising inflationary pressures, increased risk premium, or a response to tighter global financial conditions. The bond rate's cyclical behavior highlights the interplay between fiscal needs and market expectations.

Table 1: Shapiro-Wilk normality test

Variable	W Statistic	p-value	Interpretation
Total Subscription	0.6449	2.785e-15	Not Normally Distributed ($p < 0.05$)
Total Successful	0.31576	< 2.2e-16	Not Normally Distributed ($p < 0.05$)
Bond Rate	0.92449	6.723e-06	Not Normally Distributed ($p < 0.05$)

Source: Computation with R software package

Interpretation: The results of the Shapiro-Wilk normality test show significant departures from normalcy, with p-values significantly below the 0.05 level for the variables total subscription ($W = 0.6449$, $p < 0.001$), total successful bids ($W = 0.31576$, $p < 0.001$), and bond rate ($W = 0.92449$, $p < 0.001$). This suggests that these variables' distributions are not normal, which could be because of skewness, outliers, or structural alterations with time. Therefore, time series modelling methods such as ARIMA are suitable for predicting these variables since they require normally distributed residuals rather than a rigid assumption of normality in the raw data.

Stationarity Test

Table 2: ADF Test Results at Level and First Difference

Variable	Test Level	Dickey-Fuller Statistic	Lag Order	p-value	Stationary (at 5%)
Total Subscription	Level	-3.123	4	0.1105	No
	First Difference	-8.933	4	< 0.01	Yes
Total Successful	Level	-2.6202	4	0.3192	No
	First Difference	-13.507	4	< 0.01	Yes
Bond Rate	Level	-2.7053	4	0.2838	No
	First Difference	-4.7291	4	< 0.01	Yes

Source: Computation with R software package

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Interpretation: The Augmented Dickey-Fuller (ADF) test findings for the three variables (bond rate, total subscription, and total successful bids) at both their initial levels and first-differenced forms are shown in Table 2. According to their respective p-values (0.1105, 0.3192, and 0.2838), which are higher than the 5% significance level, all three variables exhibit non-stationarity at the level form. This suggests that the data contains unit roots, indicating that the variance and mean do not remain constant over time. Additionally, the series display stochastic patterns or trends that defy the assumptions necessary for time series modelling with ARIMA in its unprocessed form. The ADF test findings, however, show that all three variables become stationary once initial differencing is applied to each series, with p-values less than 0.01.

3.1 Modelling and Forecasting

The general ARIMA (p, d, q) model is specified as:

$$Y_t = c + \emptyset_1 Y_{t-1} + \cdots + \emptyset_p Y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (6)$$

Table 3: ARIMA Estimation Output for Total Subscription

Coefficient	**Estimate**	**Std. Error**
AR1	0.5721	0.0945
AR2	-0.3187	0.0872
MA1	-0.6445	0.0929
MA2	-0.1875	0.0886
SMA1	-0.2290	0.1088
SMA2	0.3608	0.1377
Model	**ARIMA(0,1,2)(0,0,2)[12]**	
Log Likelihood	-1424.7	
AIC	2859.4	
AICc	2859.95	
BIC	2873.08	
Sigma² (Residual Var)	4.166×10^9	

Source: Computation with R software package

Interpretation: The results of the ARIMA estimation for the total number of subscriptions for Nigerian 10-Year Federal Government Bonds are shown in Table 4.3. ARIMA(0,1,2)(0,0,2)[12] is the model that was found using the automatic selection technique. This means that the initial time series was differenced once in order to achieve stationarity. Both short-term variations and seasonal

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dynamics in the data are captured by the model's two seasonal moving average (MA) terms and two non-seasonal MA terms. The model's ability to account for monthly seasonality is confirmed by the existence of seasonal components with a frequency of 12.

According to the computed coefficients, the one non-seasonal moving average term (MA1) is -0.6445, and the other (MA2) is -0.1875. These warning indications imply that the current subscription values are inversely correlated with previous forecast errors. SMA1 and SMA2 for the seasonal moving average terms are -0.2290 and 0.3608, respectively. While SMA1 suggests a minor dampening seasonal adjustment, SMA2's sign suggest a moderate and positive seasonal influence from historical errors. The estimations appear to be statistically trustworthy based on the very tiny standard errors for these coefficients.

The ARIMA model's applicability is further confirmed by the model fit statistics. The information criteria—AIC (2859.4), AICc (2859.95), and BIC (2873.08)—are all rather low, suggesting a reasonable model fit, and the log-likelihood is -1424.7. The scale of the subscription data is reflected in the estimated residual variance (sigma2), which is 4.166×10^9. Even though this deviation is significant in absolute terms, it is acceptable as long as the residuals are white noise, which is typically confirmed in later diagnostics.

All things considered, both seasonal and non-seasonal trends in the entire subscription data are successfully captured by the ARIMA(0,1,2)(0,0,2)[12] model. According to the model's structure and fit statistics, it is suitable for short-term forecasting and offers a solid foundation for comprehending how investors' interest in government bonds changes over time.

Table 4: Residual Diagnostics for Total Subscription Model

Test	**Value**
Model	ARIMA(0,1,2)(0,0,2)[12]
Ljung-Box Q*	22.095
Degrees of Freedom (df)	19
p-value	0.2795
Model Degrees of Freedom	4
Total Lags Used	23

Source: Computation with R software package

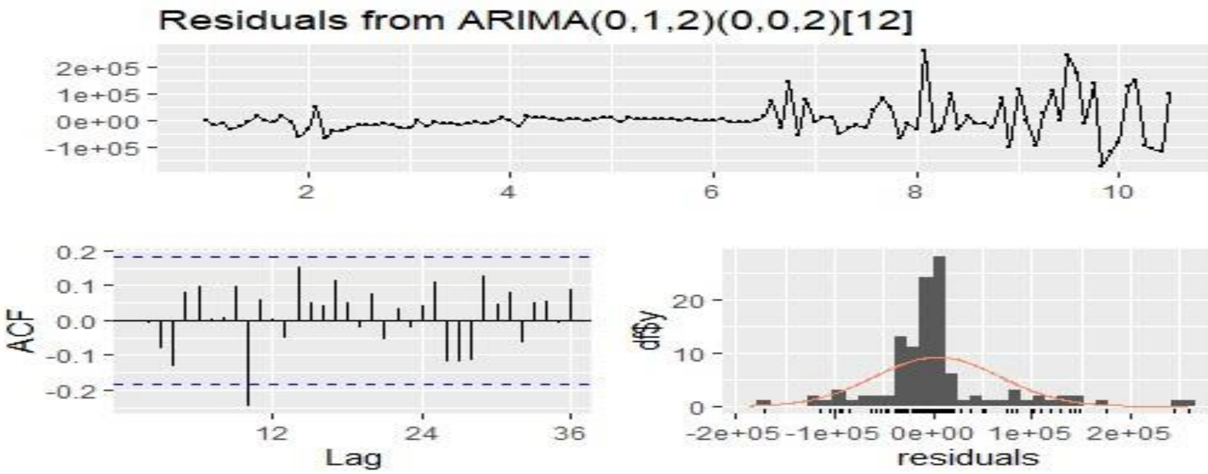

Figure 4: Residual from ARIMA(0,1,2)(0,0,2)[12]

Interpretation: The Ljung-Box test is used to check for the presence of autocorrelation in the residuals of the fitted ARIMA model. In this case, the test yields a Q-statistic of 22.095 with 19 degrees of freedom and a p-value of 0.2795. Since the p-value is greater than the conventional significance levels (0.05), we fail to reject the null hypothesis that the residuals are independently distributed. This implies that there is no significant autocorrelation left in the residuals, and hence, the model has adequately captured the structure in the data.

Table 5: forecast total Subscription

Month	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Aug 10, 2023	102,248.19	19,525.29	184,971.10	-24,265.57	228,762.00
Sep 10 2023	119,826.63	32,032.62	207,620.60	-14,442.73	254,096.00
Oct 10, 2023	87,409.56	-1,477.68	176,296.80	-48,531.74	223,350.90
Nov 10, 2023	184,744.47	94,777.29	274,711.70	47,151.54	322,337.40
Dec 10, 2023	137,689.52	46,655.18	228,723.90	-1,535.49	276,914.50
Jan 11, 2024	193,208.31	101,119.29	285,297.30	52,370.32	334,046.30
Feb 11, 2024	125,979.11	32,847.29	219,110.90	-16,453.72	268,411.90
Mar 11, 2024	65,522.57	-28,640.52	159,685.70	-78,487.45	209,532.60
Apr 11, 2024	145,837.14	50,653.96	241,020.30	267.03	291,407.30
May 11, 2024	201,187.20	104,994.75	297,379.70	54,073.54	348,300.90

Source: Computation with R software package

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Interpretation: Table 5 presents the forecast summary for total subscription to the Nigerian 10 Year Federal Government Bonds for the 10-month period spanning August 2023 to May 2024. The point forecasts represent the expected subscription values for each month based on the fitted ARIMA model. Notably, subscription levels are projected to vary substantially over the forecast horizon, with the lowest expected value in March 2024 (₦65,522.57) and the highest in May 2024 (₦201,187.20). This suggests fluctuating investor interest across the months, possibly influenced by macroeconomic conditions, fiscal policies, or seasonal demand patterns.

The forecast intervals (Lo 80/Hi 80 and Lo 95/Hi 95) provide a measure of uncertainty around each point estimate. For instance, in November 2023, the 95% confidence interval ranges from ₦47,151.54 to ₦322,337.40, indicating a high degree of uncertainty despite the relatively strong point forecast of ₦184,744.47. The presence of negative lower bounds in some months (e.g., August, October, March) further reflects this uncertainty and should not be interpreted literally but as a statistical indication of variability in the forecast. These wide intervals suggest that while the model captures historical trends well, future bond subscriptions remain highly volatile.

Table 6: ARIMA Estimation Output for Total Successful

Coefficient	Estimate	Std. Error
MA1	-0.8868	0.0386
Model	**ARIMA(0,1,1)**	
Log Likelihood	-1463.17	
AIC	2930.34	
AICc	2930.45	
BIC	2935.81	
Sigma² (Residual Var)	8.197×10^9	

Source: Computation with R software package

Interpretation: The results of the ARIMA estimation for the total number of successful bids on Nigerian 10-year FGN bonds are shown in Table 6. The detected model is ARIMA(0,1,1), which includes one non-seasonal moving average (MA) term and indicates that the series was differenced once to ensure stationarity. The series does not show autoregressive or seasonal components, and its straightforward structure indicates that recent past errors have a major impact on the current prices of all successful bids.

At -0.8868 with a standard error of 0.0386, the predicted MA1 coefficient is statistically significant and reasonably near -1. This suggests that an overestimation in the prior forecast will probably be followed by an underestimation in the subsequent one, and vice versa, as it shows a high negative autocorrelation in the model's error term. This coefficient effectively represents short-term interdependence, indicating that historical forecasting errors have a significant corrective influence on current results.

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A decent model fit is shown by the log-likelihood value of -1463.17 and the model selection criteria, which include AIC (2930.34), AICc (2930.45), and BIC (2935.81), all of which fall within acceptable limits. When evaluating competing models, lower AIC and BIC values are desirable; in this instance, they validate that the ARIMA(0,1,1) structure is efficient and parsimonious for this dataset.

Lastly, the spread of the forecast errors is reflected in the residual variance (σ^2), which is predicted to be 8.197×10^9. This degree of variation makes sense considering the size of the overall number of successful bids. The ARIMA(0,1,1) model captures crucial short-term dynamics without overfitting and offers a statistically sound and comprehensible method of predicting the total number of successful bids.

Table 7: Residual Diagnostics Table: Total Successful

Test	Value
Model	ARIMA(0,1,1)
Ljung-Box Q*	14.104
Degrees of Freedom (df)	22
p-value	0.8978
Model Degrees of Freedom	1
Total Lags Used	23

Source: Computation with R software package

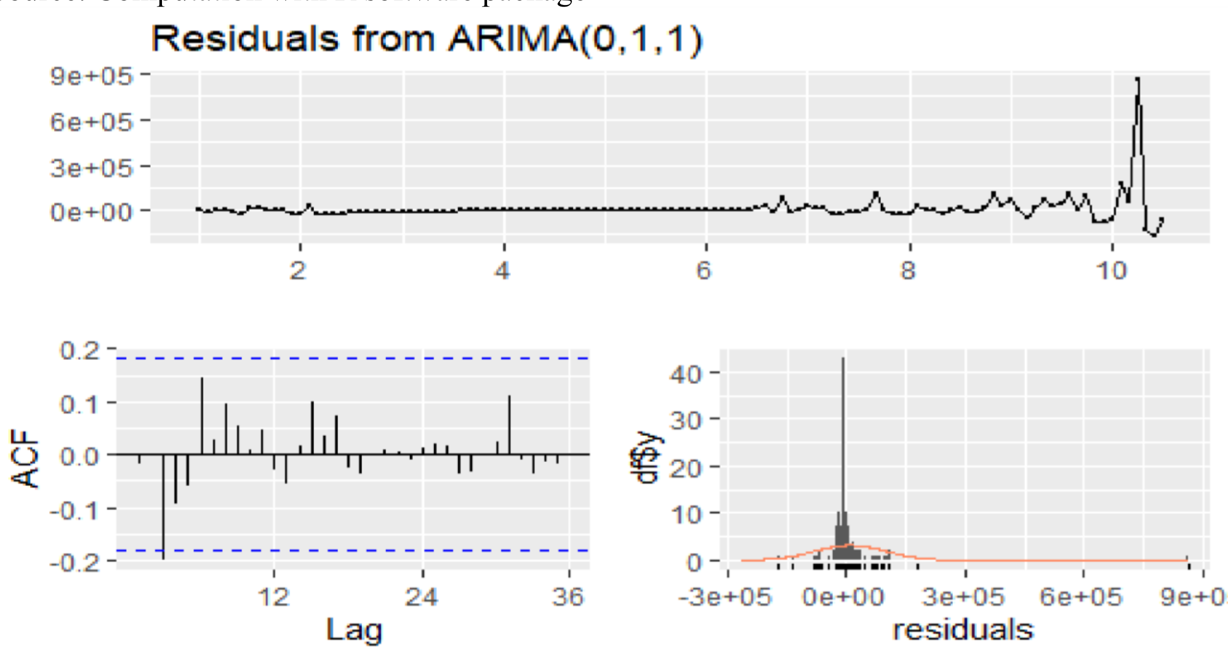

Figure 5: Residual from ARIMA (0,1,1)

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Interpretation: The Ljung-Box test is a statistical test used to determine whether there is significant autocorrelation in the residuals of a time series model. In this case, the test statistic (Q*) is 14.104 with 22 degrees of freedom and a p-value of 0.8978. Since the p-value is significantly higher than 0.05, we fail to reject the null hypothesis, meaning that there is no significant autocorrelation in the residuals. This indicates that the ARIMA(0,1,1) model has successfully captured the underlying structure in the data, and the residuals behave like white noise. The results confirm that the model is well specified and suitable for forecasting, as no systematic pattern remains in the errors.

Table 8: Forecast Total Successful

Month	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Aug 10, 2023	144,358.1	28,329.55	260,386.6	-33,092.24	321,808.4
Sep 10 2023	144,358.1	27,588.43	261,127.7	-34,225.69	322,941.9
Oct 10, 2023	144,358.1	26,851.98	261,864.2	-35,351.99	324,068.2
Nov 10, 2023	144,358.1	26,120.12	262,596.0	-36,471.28	325,187.4
Dec 10, 2023	144,358.1	25,392.76	263,323.4	-37,583.68	326,299.8
Jan 11, 2024	144,358.1	24,669.82	264,046.3	-38,689.32	327,405.5
Feb 11, 2024	144,358.1	23,951.22	264,764.9	-39,788.32	328,504.5
Mar 11, 2024	144,358.1	23,236.88	265,479.3	-40,880.81	329,597.0
Apr 11, 2024	144,358.1	22,526.74	266,189.4	-41,966.88	330,683.0
May 11, 2024	144,358.1	21,820.70	266,895.5	-43,046.66	331,762.8

Source: Computation with R software package

Interpretation: Table 8 presents the 10-month forecast for Total Successful Bids on Nigerian 10Year Federal Government Bonds, covering the period from August 2023 to May 2024. The point forecast remains constant at ₦144,358.10 for each month, which indicates that the ARIMA(0,1,1) model has projected a steady expected value based on recent historical behavior. This flat forecast pattern is characteristic of simple differenced ARIMA models when the trend is weak or when differencing has removed most of the signal. The 80% and 95% confidence intervals (Lo 80–Hi 80, Lo 95–Hi 95) widen progressively over time, reflecting increasing uncertainty the further out the forecast extends. For instance, in August 2023, the 95% interval spans from ₦-33,092.24 to ₦321,808.40, while by May 2024, it expands to ₦-43,046.66 to ₦331,762.80. This increasing range indicates that although the model predicts a constant mean value, its confidence in the precision of that forecast diminishes as time progresses.

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The negative lower bounds of the prediction intervals (e.g., -₦33,092.24 in August 2023 and ₦43,046.66 in May 2024) are not meaningful in a practical financial context since the number of successful bids cannot be negative. Rather, they highlight the statistical nature of the uncertainty and suggest that while the point forecast is stable, actual outcomes could deviate significantly due to market volatility, investor behavior, or external economic shocks.

Bond Rate

Table 9: ARIMA Estimation Output for Bond Rate

Coefficient	Estimate	Std. Error
AR1	-0.2711	0.0898
Model	**ARIMA(1,1,0)**	
Log Likelihood	-194.07	
AIC	392.13	
AICc	392.24	
BIC	397.61	
Sigma² (Residual Var)	1.777	

Source: Computation with R software package

Interpretation: Table 9 stated the selected model is ARIMA(1,1,0), indicating that the series was differenced once to achieve stationarity and includes one autoregressive (AR) term without any moving average (MA) or seasonal components. This suggests that the current bond rate is primarily influenced by its immediate past value and the underlying trend over time.

The estimated AR(1) coefficient is -0.2711 with a standard error of 0.0898, which is statistically significant and negative. This implies a mild inverse relationship between the current change in bond rate and its previous value—meaning that an increase in the previous period is likely followed by a slight decrease in the current period, and vice versa. Such a pattern reflects mean-reverting behavior, which is typical in financial rate data.

The model's goodness-of-fit indicators support its adequacy. The log-likelihood is -194.07, and model selection criteria are AIC = 392.13, AICc = 392.24, and BIC = 397.61. These values are relatively low, suggesting that the model balances simplicity and explanatory power effectively. The relatively small residual variance ($\sigma^2 = 1.777$) indicates a good fit to the observed data, with minimal unexplained fluctuations.

Overall, the ARIMA(1,1,0) model provides a statistically sound approach to modeling the bond rate. It captures short-term dependencies and trend behavior effectively, making it suitable for forecasting bond yields. However, given the relatively small AR coefficient, the influence of past

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bond rates on current changes is moderate, and other exogenous variables may also be important in explaining rate movements.

Table 10: Residual Diagnostics Table

Test	Value
Model	ARIMA(1,1,0)
Ljung-Box Q*	26.668
Degrees of Freedom	22
p-value	0.2241
Model df	1
Total Lags Used	23

Source: Computation with R software package

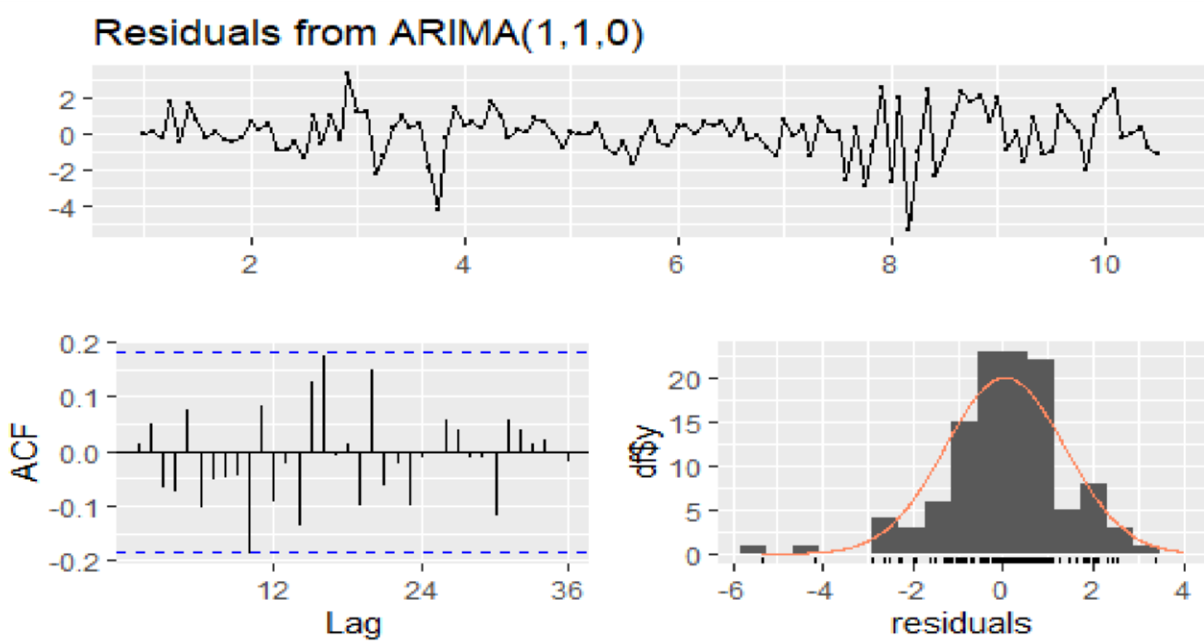

Figure 6: Residual from ARIMA (1,1,0)

Interpretation: The Ljung-Box test looks for autocorrelation in the ARIMA model's residuals. The test produces a p-value of 0.2241 and a Q* statistic of 26.668 with 22 degrees of freedom for the bond rate model (ARIMA(1,1,0)).

We are unable to rule out the null hypothesis that the residuals are distributed independently because the p-value is higher than 0.05. This indicates that the model adequately reflects the time series structure and that there is no discernible autocorrelation left in the residuals. As a result, the residuals approximate white noise and the ARIMA(1,1,0) model is suitable.

Table 11: Forecast Bond rate

Month	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Aug 10, 2023	14.14397	12.43559	15.85234	11.53123	16.75670
Sep 10 2023	14.07783	11.96377	16.19190	10.84465	17.31102
Oct 10, 2023	14.09576	11.57616	16.61537	10.24236	17.94916
Nov 10, 2023	14.09090	11.23864	16.94316	9.72874	18.45306
Dec 10, 2023	14.09222	10.93831	17.24613	9.26873	18.91571
Jan 11, 2024	14.09186	10.66372	17.52000	8.84897	19.33475
Feb 11, 2024	14.09196	10.40970	17.77421	8.46043	19.72348
Mar 11, 2024	14.09193	10.17207	18.01180	8.09701	20.08685
Apr 11, 2024	14.09194	9.94805	18.23583	7.75440	20.42947
May 11, 2024	14.09194	9.73553	18.44834	7.42939	20.75448

Source: Computation with R software package

Interpretation: Table 11 presents the 10-step ahead forecast for the bond rate based on the fitted ARIMA(1,1,0) model. The Point Forecast represents the expected bond rate for each future month, while the Lo 80 / Hi 80 and Lo 95 / Hi 95 columns provide 80% and 95% confidence intervals, respectively. These intervals offer a range of plausible values the actual bond rate might fall within, assuming the model holds.

In the short term (August to October 2023), the bond rate is expected to hover around 14.1%, with relatively wide confidence intervals. For example, the forecast for August 2023 is 14.14%, with a 95% confidence interval from 11.53% to 16.76%. This suggests a moderately high level of uncertainty in rate movements, possibly due to historical volatility or limited model memory.

From November 2023 to February 2024, the forecast continues to center around 14.09%, with a gradually increasing spread in the confidence intervals. This widening interval reflects growing forecast uncertainty as we move further from the training data. Nonetheless, the point forecasts remain fairly stable, indicating rate stationarity post-differencing, which is consistent with the model's assumption.

By March to May 2024, the point forecast still stabilizes at 14.09194%, while the 95% confidence intervals widen further (e.g., from 7.43% to 20.75% in May). This reflects the natural limitation of ARIMA models: while they can predict trends and autocorrelation patterns, they become

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increasingly imprecise as the forecast horizon extends. ARIMA(1,1,0) model predicts a stable bond rate around 14.09% over the forecast horizon. However, the expanding confidence intervals suggest caution, as real-world economic or policy shocks—absent in the model—could cause significant deviations. The forecast provides a reliable short-term outlook, with longer-term projections requiring further model enhancements or external regressors for increased accuracy.

Conclusion

All three ARIMA models performed well in terms of residual diagnostics and forecast plausibility, according to the overall forecasting results. Without overfitting, each model successfully represented the underlying structure in the data, and the predictions offered reasonable short-term expectations. An understanding of the risk and uncertainty associated with the point estimates was also made possible by the addition of confidence intervals. These results show how effective ARIMA is at simulating market variables for government bonds, particularly when the data exhibits seasonality and autocorrelated patterns. The findings imply that while interest rates show slow movement and slight autocorrelation, subscription and bid behaviours show both trend and seasonal variations. These trends reveal information about the operational consistency of government bond issuance as well as investment behaviour. Financial analysts, debt managers, and legislators who depend on predictive insights to inform choices about bond issuance tactics and market regulation will find value in the findings. The study revealed that major bond market characteristics in Nigeria may be analyzed and forecasted using ARIMA models.

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