ENHANCING VOLATILITY FORECASTING IN THE NIGERIAN STOCK MARKET USING GARCH MODELS WITH ADVANCED INNOVATION DENSITIES

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ABSTRACT
This study investigates the role of advanced innovation densities in improving volatility forecasting accuracy in the Nigerian stock market. Using daily All Share Index (ASI) data spanning 2012 to 2023, the study applies GARCH-type models, including GARCH (1,1), EGARCH (1,1), and APARCH (1,1), under multiple error distributions such as Normal, Student-t, Generalized Error Distribution (GED), and their skewed variants. Preliminary analyses confirm the presence of volatility clustering, non-normality, and ARCH effects in the return series. Model parameters are estimated using Maximum Likelihood Estimation (MLE), while forecasting performance was evaluated using Root Mean Square Error (RMSE). Empirical findings reveal that models incorporating heavy-tailed and skewed innovation densities significantly outperform the conventional normal distribution in forecasting volatility. In particular, the APARCH (1,1) model with GED innovation density demonstrates superior predictive performance, capturing extreme market fluctuations more effectively. This results once again underscore the importance of selecting appropriate innovation densities in volatility modelling, especially in emerging markets characterized by structural instability and frequent shocks. The study provides valuable insights for investors, risk managers, and policymakers seeking to improve forecasting accuracy and enhance financial decision making.

Keywords: APARCH, Volatility clustering, GARCH-type models, Innovation densities, Nigerian Stock Exchange, Volatility forecasting

1.0 Introduction

The global financial landscape is characterized by persistent uncertainty, where volatility defined as the degree of variation in asset prices over time has become the single most critical metric for risk assessment, asset pricing, and portfolio allocation. Across developed economies such as the United States, the United Kingdom, and Japan, extensive research has demonstrated that volatility clustering, leverage effects, and heavy tailed return distributions are stylized facts that must be accommodated by any credible forecasting model (Ahmed & Zaidi, 2009; Enow, 2023). Studies examining global stock market indices have shown that during crisis periods, the inclusion of skewed and heavy tailed distributions such as the skewed Student-t considerably improves one day ahead Value-at-Risk forecasts regardless of the underlying volatility specification (Miron & Tudor, 2010). This finding underscores a universal truth in financial econometrics: the assumption of normally distributed returns is not merely inadequate but potentially dangerous for risk management (Bollerslev, 1987). On the African continent, the volatility dynamics of emerging markets present additional layers of complexity. Research on the BRVM10 index, the leading stock market benchmark in West Africa, has confirmed the

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presence of nonlinear dynamics, structural breaks, and regime shifts that necessitate regimes witching volatility models capable of accommodating heavy tails and asymmetric shocks (N'Dri et al., 2025). Within the Nigerian context specifically, the stock market has experienced periods of heightened volatility driven by the nation's heavy reliance on oil revenues, frequent macroeconomic shocks, political uncertainties, and most recently, unprecedented policy changes including fuel subsidy removal and the floating of the naira (Olowe, 2009; Emenyonu et al., 2023). These factors create a challenging environment where accurate volatility forecasting is not merely an academic exercise but a practical necessity for investors, portfolio managers, and regulatory bodies seeking to navigate an increasingly turbulent financial ecosystem.

The evolution of volatility modeling since Engle's (1982) introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model and Bollerslev's (1986) subsequent generalization (GARCH) has provided researchers with a robust framework for capturing time-varying volatility. However, a persistent challenge has been the distributional assumption placed on the innovation process. The original GARCH model assumed normally distributed errors, yet financial returns consistently exhibit excess kurtosis (fat-tailed) and skewness, leading to systematic underestimation or overestimation of true volatility (Gujarati, 2003; Adubisi et al., 2022). This misspecification has profound consequences: under forecasting volatility exposes investors to unanticipated losses, while over forecasting leads to inefficient capital allocation and reduced returns. Recognizing this limitation, researchers have introduced alternative error distributions including the Student-t distribution, which Bollerslev (1987) advocated for its heavier tails, and the Generalized Error Distribution (GED), which Nelson (1991) proposed as a flexible alternative capable of accommodating various tail thicknesses. More recently, the incorporation of skewness parameters has given rise to skewed versions of these distributions, including the Skewed Normal (SNORM), Skewed Student-t (SSTD), and Skewed Generalized Error Distribution (SGED), which simultaneously account for asymmetry and leptokurtosis (Agboola et al., 2018; Samson et al., 2020). Empirical evidence from multiple financial markets confirms that these skewed and heavy tailed distributions consistently outperform their symmetric counterparts in both in sample fit and out-of-sample forecasting accuracy, as measured by Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and root mean square error (RMSE) (Musa et al., 2020; Akanbi et al., 2025). The theoretical justification rests on the recognition that financial markets respond asymmetrically to positive and negative shocks—a phenomenon known as the leverage effect, where negative news tends to increase volatility more than positive news of equivalent magnitude (Nelson, 1991; Glosten et al., 1993).

Despite the growing body of evidence supporting advanced innovation densities, the literature on volatility forecasting in the Nigerian Stock Exchange (NSE) reveals significant gaps. While several studies have applied GARCH family models to NSE data, many have relied on normal distribution assumptions or limited comparisons between standard Student-t and GED distributions without incorporating skewed variants (Atoi, 2014; Oyenuga et al., 2022). For instance, Atoi (2014) examined volatility using GARCH models with normal, Student-t, and GED distributions but did not explore skewed versions. Similarly, Samson et al. (2020) investigated skewed error distributions and found that the APARCH (1,1) skewed normal model performed best, yet their study period concluded before the major economic disruptions of 2023. More recent work by Adegboyo and Sarwar (2025) applied EGARCH and GJRGARCH models with Student-t distributions to NSE data spanning 2012 to 2024, revealing significant leverage effects and volatility clustering, while identifying GJRGARCH with

Student-t as the optimal forecasting model. Akanbi et al. (2025) extended this line of inquiry by comparing multiple GARCH variants with skewed error distributions across three distinct datasets Bitcoin, USD Naira exchange rates, and the NSE All Share Index finding that the skewed Student-t distribution consistently outperformed other innovations, with GJRGARCH(1,1)SSTD emerging as optimal for the all share index. Kaigama et al. (2025) introduced a sine modulated Student-t error innovation for GARCH models, demonstrating that GARCH(1,1) with sine Student-t achieved superior model fit and forecasting accuracy on NSE data, while also showing that GARCH(1,2) with sine exponentiated Student-t innovations proved most effective for capturing extreme tail events. These findings collectively establish that advanced innovation densities particularly those accommodating both skewness and fat-tailed are essential for accurate volatility modeling in the Nigerian context. However, no single study has systematically compared the full spectrum of symmetric and skewed distributions (NORM, STD, GED, SNORM, SSTD, SGED) across multiple GARCH-type specifications (GARCH, EGARCH, APARCH) while also incorporating the unique market shocks of 2023, including the fuel subsidy removal and currency floatation (Yaya et al., 2016; Kuhe, 2018). This represents a critical gap, as the predictive performance of these model distribution combinations during periods of structural change has direct implications for risk management and investment decision making in the postreform Nigerian economy (Emenogu et al., 2020; Samaila et al., 2022).

This study therefore aims at enhancing volatility forecasting in the Nigerian stock market by systematically evaluating the performance of GARCH-type models specifically GARCH(1,1), EGARCH(1,1), and APARCH(1,1) under six distinct innovation densities: Normal (NORM), Student-t (STD), Generalized Error Distribution (GED), Skewed Normal (SNORM), Skewed Student-t (SSTD), and Skewed Generalized Error Distribution (SGED). Using daily All Share Index (ASI) data from February 2, 2012, to December 7, 2023, comprising 2,934 observations with 2,702 used for in-sample estimation and 232 for out-of-sample validation, this research employs maximum likelihood estimation and evaluates model performance using loglikelihood (LL), AIC, BIC, mean squared error (MSE), RMSE, and mean absolute error (MAE) (Bollerslev, 1986; Ding et al., 1993). The primary objective is to identify the model distribution combination that minimizes forecasting error while accurately capturing volatility clustering, persistence, and asymmetry. This study makes several important contributions to the literature. First, it provides one of the few comprehensive comparative analyses of six innovation densities across three GARCH-type models within the context of the Nigerian stock market, where such systematic comparisons remain relatively limited (Olayemi & Olubiyi, 2021; Adubisi et al., 2022).. Furthermore, it extends the empirical evidence on the superiority of skewed distributions in emerging market contexts, testing whether findings from other markets generalize to Nigeria (Akanbi et al., 2025; Kaigama et al., 2025). Also, the study period captures unprecedented policy shocks of the fuel subsidy removal and naira floatation of 2023 thus providing unique insights into model performance during structural breaks (Emenyonu et al., 2023; Adegboyo & Sarwar, 2025). The rigorous out-of-sample forecasting evaluation using 232 trading days offers practical guidance for financial practitioners seeking reliable volatility predictions (Tamilselvan & Vali, 2016; Enow, 2023). Again, the identification of the optimal model combination provides a concrete, implementable tool for risk management, option pricing, and portfolio optimization in the Nigerian stock market (Ladokhin, 2009; Banumathy & Azhagaiah, 2015). This research contributes to financial econometrics in emerging economies, demonstrating that the integration of asymmetric power specifications with skewed heavy tailed distributions significantly enhances forecasting accuracy, thereby supporting more

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informed investment decisions, improved regulatory oversight, and greater market stability (Qamruzzaman, 2015; Mubarik & Javid, 2016).

2.0 Methodology

This study adopted a quantitative time series research design to evaluate the predictive performance of GARCH-type volatility models under alternative innovation density specifications for the Nigerian Stock Exchange (NSE). Volatility is an unobservable latent variable that must be estimated from historical price data, and time series econometrics provides the only rigorous framework for modeling conditional heteroskedasticity and generating out-of-sample forecasts (Engle, 1982; Bollerslev, 1986). All statistical analyses were conducted using R software version 4.2.2, utilizing the rugarch package for GARCH model estimation and the t-series package for stationarity testing.

2.1 Source of Data

Daily All Share Index (ASI) data were obtained from Investing.com (www.investing.com) spanning from February 2, 2012, to December 7, 2023, yielding 2,934 observations. Of these, 2,702 observations (February 2, 2012, to December 30, 2022) were used for in-sample model estimation, and 232 observations (January 1, 2023, to December 7, 2023) were reserved for out-of-sample forecast validation. The sample period was deliberately extended to capture the unprecedented policy shocks of 2023, including fuel subsidy removal and naira floatation, which introduced structural breaks that test the robustness of volatility models under extreme market conditions.
The daily returns were computed from daily closing price using the formula:

$$\boxed{r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) x100} \tag{1}$$

Where:

$\boxed{r_t}$ is the daily return at time t
P_t is the closing ASI value at time t
P_{t-1} is the closing ASI value at time t-1
.

This logarithmic transformation stabilizes variance, which is standard in financial econometrics for modeling volatility dynamics (Tsay, 2010).

2.2 Stationarity and ARCH Effect Testing

The stationarity of the return series were verified using both the Augmented Dickey-Fuller (ADF) test, with the null hypothesis of a unit root, and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, with the null hypothesis (H_0) of stationarity. The use of dual-testing approach was necessary because the ADF test has low power against near-unit-root alternatives, while the KPSS test complements it by testing the opposite null hypothesis (H_0), to validate stationarity claim. The presence of autoregressive conditional heteroskedasticity (ARCH) effects was examined using the Lagrange Multiplier (LM) test of Engle (1982) and the Ljung-Box Q-statistic applied to squared residuals. Detection of ARCH effects is a prerequisite for applying any GARCH-family model; failure to reject the null of no ARCH effects would render volatility modeling unnecessary (Gujarati, 2003).

ISSN NUMBER: 1116-249X

2.3 GARCH-Type Volatility Models

Prior to estimating the conditional variance models, the conditional mean equation was specified using an autoregressive moving average ARMA(r,s) process. The optimal ARMA order was selected based on the Akaike Information Criterion (AIC) and examination of the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the return series. The ARMA(1,1) specification was found to adequately remove serial dependence from the mean equation before estimating the GARCH-type volatility models. This procedure is important because misspecification of the mean equation may lead to biased volatility parameter estimates (Tsay, 2010)

2.3.1 GARCH(1,1)

The standard GARCH(1,1) model proposed by Bollerslev (1986) is specified as:

$$r_t = \mu_t + \varepsilon_t \tag{2}$$

and

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3}$$

where $\omega > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$,

2.3.2 EGARCH (1,1)

The EGARCH(1,1) model of Nelson (1991) is specified as:

$$\log(\sigma_t^2) = \omega + \beta_1 \log(\sigma_{t-1}^2) + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \tag{4}$$

Where γ_1 captures asymmetric volatility responses to shocks.

2.3.3 APARCH(1,1)

The APARCH(1,1) model of Ding et al. (1993) is specified as:

$$\sigma_t^\delta = \omega + \alpha_1 \left(|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1} \right)^\delta + \beta_1 \sigma_{t-1}^\delta \tag{5}$$

Where δ is the power parameter and γ_1 measure asymmetry

This model was chosen because it nests several other models, allows the power parameter δ to be estimated rather than fixed, and captures both asymmetric responses and long memory simultaneously, offering the greatest flexibility for emerging market data where volatility dynamics are complex (Samson et al., 2020). The selection of the GARCH-type of order (1,1) for all models follows the overwhelming empirical evidence that parsimonious first-order lag structures are sufficient to capture volatility dynamics in daily financial returns, and higher-order models typically lead to over-parameterization without significant improvement in fit (Bollerslev, 1986; Hansen & Lunde, 2005).

2.4 Conditional Innovation Densities

The study employed six innovation densities specified for each volatility model, representing a progression from simple symmetric distributions to complex skewed and heavy-tailed alternatives. The Normal (NORM) distribution assumes zero skewness and excess kurtosis of zero, serving as the baseline model against which improvements can be measured. The Student-t (STD) distribution introduces a shape parameter (degrees of freedom) that accommodates excess kurtosis when the estimated degrees of freedom are less than infinity, with values < 5 indicates extremely fat-tailed (Bollerslev, 1987). The Generalized Error Distribution (GED) introduces a shape parameter where values < 2 indicate heavier tails than the normal, values equal to 2 recover normality, and values greater than 2 indicate thinner tails, providing a flexible alternative to the Student-t (Nelson, 1991). The Skewed Normal (SNORM), Skewed Student-t (SSTD), and Skewed Generalized Error Distribution (SGED) extend their symmetric counterparts by incorporating a skewness parameter that captures the asymmetric shape of return distributions, where values different from unity indicate departure from symmetry (Fernandez & Steel, 1998; Lambert & Laurent, 2001). The inclusion of all six densities is justified because financial returns in emerging markets exhibit both excess kurtosis and negative skewness simultaneously, and models that ignore either feature will produce biased volatility forecasts (Adubisi et al., 2022; Akanbi et al., 2025).

2.5 Model Estimation, Selection, and Forecast Evaluation

All models were estimated via maximum likelihood estimation (MLE) under their respective distributions. Competing specifications were compared using the Akaike Information Criterion (AIC) (Akaike, 1974), defined as

$$\boxed{AIC = -2In(\hat{L}) + 2k} \tag{6}$$

where L is the maximized log-likelihood and k is the no. of parameters; the Bayesian Information Criterion (BIC) (Schwarz, 1978), defined as

$$\boxed{BIC = 2In(\hat{L}) + kIn(n)} \tag{7}$$

where n is the no. of observations; and the Log-Likelihood (LL) value. Lower AIC and BIC values indicate better model fit.

Out-of-sample forecast performance was evaluated using three metrics: Root Mean Squared Error (RMSE), Mean Squared Error (MSE), and Mean Absolute Error (MAE). These are defined as:

$$\boxed{MSE = \frac{1}{n}\sum_{t=1}^{n}\left(y_t - \widehat{y}_t\right)^2} \tag{8}$$

$$\boxed{RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_t - \widehat{y}_t\right)^2}} \tag{9}$$

$$\boxed{MAE = \frac{1}{n}\sum_{t=1}^{n}\left|y_t - \widehat{y}_t\right|^2} \tag{10}$$

RMSE was used for the primary evaluation metric because it penalizes large forecast errors more heavily than MAE, which is desirable for risk management applications where large prediction mistakes have disproportionately severe consequences (Hansen & Lunde, 2005). Lower values of all three metrics which include Bias, MSE and MAE indicate superior predictive accuracy. This integrated methodological framework enables a rigorous comparative assessment of GARCH-type models and innovation densities in forecasting volatility of the Nigerian stock market.

ISSN NUMBER: 1116-249X

2.6 Forecast performance

Forecast bias was also evaluated to determine whether the models systematically overestimated or underestimated volatility. The Bias was computed as:

$$Bias = \frac{1}{n}\sum_{t=1}^{n}\left(\hat{\sigma}_t - \sigma_t\right) \tag{11}$$

Where: $\hat{\sigma}_t$ denotes forecast volatility and σ_t represents realized volatility.

3.0 Results and Discussion

3.1 Descriptive Statistics of Daily Returns

The descriptive statistics as shown in table 1 for daily NSE All Share Index returns from February 2, 2012, to December 30, 2022 (n = 2,702). The mean return (0.033%) is near zero, while the minimum (-5.033%) and maximum (7.985%) values indicate notable variability. The standard deviation (0.969%) reflects moderate volatility typical of emerging markets. Positive skewness (0.303) suggests that large positive returns occur more often than large negative returns, consistent with prior NSE studies. Kurtosis (5.631) exceeds the normal benchmark, indicating fat-tailed, while the Jarque–Bera statistic (3618, $p < 0.0001$) confirms non-normality. These findings justify the use of heavy-tailed and skewed distributions in volatility modeling.

Table 1: Descriptive Statistics for Daily ASI Returns (02/02/2012 to 30/12/2022)

	Mean	Median	Min	Max	Std. Dev.	Skewness	Kutosis	JB(p-value)
Return series	0.03335	-0.001	-5.033	7.985	0.9694	0.3026	5.631	3618(<0.0001)

3.2 Graphical Examination of Daily Stock Prices and Returns

The first step in time series analysis involves plotting the original and return series to assess non-stationarity and volatility. Figures 1 present the graphical characteristics of the data over time. Figure 1 shows the daily All Share Index prices from February 2, 2012, to December 30, 2022, revealing time-varying mean and variance with a visible trend. These features indicate that the price series is not covariance stationary which is a common property of financial data. Hence, transformation into returns is necessary for further analysis.

ISSN NUMBER: 1116-249X

Figure 1: Time Plot of the monthly All Share Index from 01/02/2012 to 30/12/2022

Figure 2 shows the daily return series, squared returns, absolute returns, and their respective autocorrelation functions (ACF). The return series fluctuates around a constant mean of approximately zero with no discernible trend, suggesting that the return series is weakly stationary. More importantly, the plot reveals volatility clustering: periods of large changes in stock returns tend to be followed by large changes, and small changes tend to be followed by small changes. This phenomenon is called Volatility Clustering and is a stylized fact of financial time series and provides visual evidence supporting the use of GARCH-family models. The ACF plots of squared and absolute returns show significant autocorrelation that decays slowly, further confirming the presence of volatility clustering and ARCH effects.

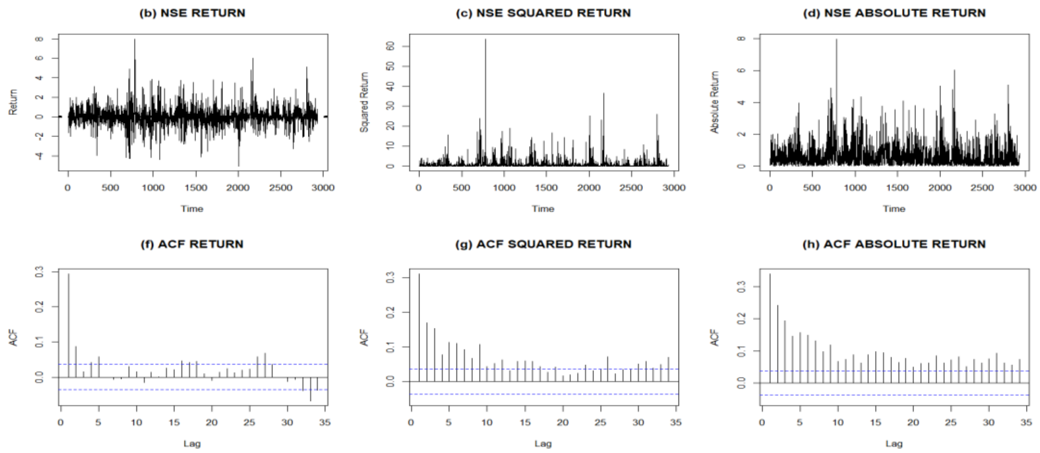

Figure 2: Daily All Share Index Returns, Squared Returns, Absolute Returns and their ACF’s

These plots of daily return, squared returns and Absolute return series presented in Figure 2 suggests that the series has a constant mean and variance with absence of trend indicating that it is generated by a random walk and is thus weakly stationary. The plot in Figure 2 also indicates that some periods are more clustered than others as large changes in stock returns tend to be followed by large changes and small changes are followed by small changes. This phenomenon is described as volatility clustering. Also the ACF showed that the series is stationary and confirms volatility clustering.

ISSN NUMBER: 1116-249X

3.3 Unit Root and Heteroskedasticity Tests Results

A stationarity test was conducted using the KPSS test and the Augmented Dickey-Fuller test. Based on the ASI of NSE index returns at all levels, it was determined that all return series were stationary.

Table 2: KPSS Test for Stationarity & ADF Test for Stationarity

H_0: Stationary H_0: Non Stationary

H_1: Non Stationary H_1: Stationary

	Stationarity test		
	Test Statistics	p-value	Comment
ADF	-12.829	0.01	Stationary at all lag
KPSS	0.15628	0.1	Stationary at all lag

Table 2 presents the results of stationarity tests for the daily return series. The Augmented Dickey-Fuller (ADF) test statistic of -12.829 with a p-value of 0.01 indicates rejection of the null hypothesis of a unit root at the 5% significance level, confirming that the return series is stationary. The KPSS test statistic of 0.15628 with a p-value of 0.1 fails to reject the null hypothesis of stationarity, providing complementary evidence that the series is stationary. The consistency of results from both tests confirms that the return series is suitable for GARCH modeling without further differencing.

3.4 ARCH effect tests for the daily returns

The Lagrange multiplier (LM) test statistic for the daily returns

H_0: There is no ARCH effect H_1: There is ARCH effect

Table 3: ARCH effect

	Df	Chi-squared	p-value
ARCH LM-test	10	203	< 0.0001
	15	213	< 0.0001
	20	214	< 0.0001

Table 3 presents the ARCH effect test results using Engle's Lagrange Multiplier test. The test statistics for lags 10, 15, and 20 are all highly significant ($p < 0.0001$), providing overwhelming evidence of autoregressive conditional heteroskedasticity in the return series. The rejection of the null hypothesis of no ARCH effects confirms that GARCH-type models are appropriate for modeling the volatility dynamics of NSE returns. Since all three conditions for a GARCH process stationarity of the data, volatility clustering, and evidence of ARCH effects—have been satisfied, we proceed to estimate the model parameters.

3.5 Parameter Estimates of the Volatility Models

Table 4 shows that For the GARCH(1,1)-GED model, $\alpha_1 = 0.290$ and $\beta_1 = 0.639$, with a sum of 0.929 satisfying stationarity, indicate that shocks decay slowly with a half-life of about 10 trading days; the EGARCH leverage parameter (γ_1) was positive and significant (0.398 to 0.513), indicating that positive shocks increase volatility more than negative shocks during policy reforms and economic recovery (Kuhe, 2018; Musa et al., For the GARCH(1,1)-GED model, shocks fade slowly with a half-life of roughly 10 trading days, as indicated by $\alpha_1 = 0.290$ and $\beta_1 = 0.639$, with a sum of 0.929 meeting stationarity. In contrast to traditional leverage effects, the EGARCH leverage parameter (γ_1) was positive and significant (0.398 to 0.513), suggesting that positive shocks increase volatility more than negative shocks. This is consistent with previous NSE research showing positive shock dominance during policy reforms and economic recovery (Kuhe, 2018; Musa et al., 2020). For the APARCH model, the power parameter (δ) approximated 1.0 (0.958 to 1.112), suggesting the conditional standard deviation is the natural scale for NSE volatility modeling. Shape parameters for Student-t (≈3.07-3.18) and GED (≈0.97) confirmed fat-tailed beyond normality, while skew parameters exceeded unity (SNORM: 1.036-1.049; SGED: 1.006-1.020), confirming positive skewness. These findings collectively justify using skewed, heavy-tailed distributions for modeling NSE returns.

Table 4: Parameter estimates of volatility models for Nigerian stock market

Model	Distribution	μ (p-value)	ω (p-value)	α_1 (p-value)	β_1 (p-value)	γ_1 (p-value)	δ (p-value)	Skew (p-value)	Shape (p-value)
GARCH(1,1)	NORM	-	0.1329 (0.0248) [0.0001]	0.2517 (0.0316) [0.0001]	0.6127 (0.0504) [0.0001]	-	-	-	-
	STD	-	0.1099 (0.0273) [0.0001]	0.3718 (0.0650) [0.0001]	0.6272 (0.0498) [0.0001]	-	-	-	3.185 (0.2146) [0.0001]
	GED	- 0.0127 (0.0043) [0.0031]	0.0974 (0.0240) [0.0001]	0.2903 (0.0456) [0.0001]	0.6386 (0.0538) [0.0001]	-	-	-	0.9668 (0.0340) [0.0001]
	SNORM	-	0.1247 (0.0232) [0.0001]	0.2413 (0.0301) [0.0001]	0.6302 (0.0477) [0.0001]	-	-	1.0488 (0.0208) [0.0001]	-
	SSTD	-	0.1096 (0.0273) [0.0001]	0.3711 (0.0585) [0.0001]	0.6279 (0.0499) [0.0001]	-	-	1.0045 (0.0251) [0.0001]	3.1847 (0.2157) [0.0001]
	SGED	-	0.0965 (0.0223) [0.0001]	0.2853 (0.0433) [0.0001]	0.6422 (0.0502) [0.0001]	-	-	1.0142 (0.0156) [0.0001]	0.9692 (0.0341) [0.0001]
EGARCH(1,1)	NORM	-	-	0.0283 (0.0153) [0.07]	0.8509 (0.0223) [0.0001]	0.3981 (0.0332) [0.0001]	-	-	-
	STD	-	-	-	0.8995 (0.0202) [0.0001]	0.5126 (0.0545) [0.0001]	-	-	3.0732 (0.2219) [0.0001]
	GED	- 0.0112 (0.0043) [0.01]	-0.0362 (0.0130) [0.0053]	-	0.8872 (0.0232) [0.0001]	0.4320 (0.0446) [0.0001]	-	-	-
	SNORM	-	-	-	0.8581 (0.0168) [0.0001]	0.3945 (0.0324) [0.0001]	-	1.0474 (0.1649) [0.0001]	-
	SSTD	-	-	-	0.8995 (0.0207) [0.0001]	0.5124 (0.0609) [0.0001]	-	1.0005 (0.0574) [0.0001]	3.0737 (0.2294) [0.0001]
	SGED	-	-0.0360 (0.0127) [0.0046]	-	0.8891 (0.0225) [0.0001]	0.4271 (0.0429) [0.0001]	-	1.0203 (0.0148) [0.0001]	0.9731 (0.0339) [0.0001]
APARCH(1,1)	NORM	-	0.1491 (0.0253) [0.0001]	0.2448 (0.0252) [0.0001]	0.6561 (0.0428) [0.0001]	-0.0921 (0.0405) [0.0228]	1.0983 (0.1531) [0.0001]	-	-

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STD	-	0.1047 (0.0229) [0.0001]	0.3176 (0.0429) [0.0001]	0.6985 (0.0384) [0.0001]	-	0.9623 (0.1505) [0.0001]	-	3.0715 (0.2195) [0.0001]
GED	-0.0119 (0.0041) [0.0035]	0.1082 (0.0240) [0.0001]	0.2626 (0.0336) [0.0001]	0.6965 (0.0435) [0.0001]	-	0.9912 (0.1686) [0.0001]	-	0.9709 (0.0340) [0.0001]
SNORM	-	0.1422 (0.0246) [0.0001]	0.2410 (0.0247) [0.0001]	0.6660 (0.0418) [0.0001]	-	1.1125 (0.1544) [0.0001]	1.0361 (0.0207) [0.0001]	-
SSTD	-	0.1053 (0.0231) [0.0001]	0.3191 (0.0438) [0.0001]	0.6974 (0.0388) [0.0001]	-	0.9589 (0.1516) [0.0001]	0.9929 (0.0305) [0.0001]	3.0652 (0.2212) [0.0001]
SGED	-0.0090 (0.0035) [0.0111]	0.1076 (0.0233) [0.0001]	0.2614 (0.0328) [0.0001]	0.6979 (0.0422) [0.0001]	-	0.9922 (0.1671) [0.0001]	1.0058 (0.0172) [0.0001]	0.9720 (0.0340) [0.0001]

3.6 Model Selection Based on In-Sample Fit

Table 5 shows that among all volatility model specifications, the APARCH(1,1) model with GED distribution achieved the best in-sample fit, with the highest log-likelihood (-3182.333) and lowest AIC (2.3616) and BIC (2.3769), outperforming both GARCH and EGARCH specifications, which can be attributed to the APARCH model's ability to estimate the power parameter (δ) and capture asymmetric responses through the leverage parameter (γ). Among GARCH(1,1) specifications, the GED distribution outperformed normal and Student-t distributions (log-likelihood = -3191.45, AIC = 2.3669), while for EGARCH(1,1), the GED distribution again produced superior fit (log-likelihood = -3183.79, AIC = 2.3619), indicating that incorporating asymmetric responses improves model fit for NSE returns. The skewed versions (SSTD and SGED) performed similarly to their symmetric counterparts, and these findings are consistent with prior research where Akanbi et al. (2025) and Kaigama et al. (2025) demonstrated that flexible error distributions improve GARCH model fit, while Samson et al. (2020) documented that asymmetric power models effectively capture NSE volatility dynamics.

Table 5: In-Sample Model Selection Criteria

Model	Distribution	LL	AIC	BIC
GARCH(1,1)	NORM	-3415.891	2.5323	2.5411
	STD	-3208.87	2.3798	2.3907
	GED	**-3191.45**	**2.3669**	**2.3778**
	SNORM	-3413.013	2.5309	2.5418
	SSTD	-3208.854	2.3805	2.3936
	SGED	**-3191.419**	2.3676	2.3807
EGARCH(1,1)	NORM	-3408.751	2.5278	2.5387
	STD	-3198.789	2.3730	2.3861
	GED	**-3183.79**	**2.3619**	**2.3750**
	SNORM	-3406.353	2.5267	2.5398
	SSTD	-3198.789	2.3738	2.3891
	SGED	**-3183.603**	2.3625	2.3778
APARCH(1,1)	NORM	-3404.65	2.5255	2.5386
	STD	-3196.72	2.3722	2.3875
	GED	**-3182.333**	**2.3616**	**2.3769**
	SNORM	-3403.181	2.5251	2.5404
	SSTD	-3196.682	2.3730	2.3904
	SGED	**-3182.329**	2.3623	2.3798

ISSN NUMBER: 1116-249X

Note: Bold values indicate the best-performing model for each specification. Lower AIC and BIC values indicate better fit.

3.7 Theoretical Properties of the APARCH-SGED Model

The APARCH(1,1)-SGED specification combines asymmetric power volatility dynamics with skewed heavy-tailed innovation densities, making it particularly suitable for modeling financial return series characterized by volatility clustering, asymmetry, and excess kurtosis. For the APARCH(1,1) process to remain covariance stationary, the persistence parameters must satisfy:

$$\boxed{\alpha_1 E(|z_t| - \gamma z_t)^{\delta} + \beta_1 < 1} \tag{12}$$

Where:

z_t represents independently and identically distributed innovations from the SGED distribution,
α_1 measures the short-run impact of shocks,
β_1 captures volatility persistence,
γ represents asymmetry, and
δ denotes the power parameter (Ding et al., 1993).

The SGED innovation density extends the generalized error distribution by incorporating a skewness parameter that allows the conditional distribution to accommodate both asymmetry and leptokurtosis simultaneously. This flexibility improves the ability of the model to capture extreme market movements commonly observed in emerging financial markets such as Nigeria. The log-likelihood function for the APARCH-SGED model is estimated through maximum likelihood estimation (MLE), where parameter estimates are obtained by maximizing the conditional log-likelihood:

$$\boxed{L(\theta) = \sum_{t=1}^{T} \log f(r_t | F_{t-1}; \theta)} \tag{13}$$

where $\boxed{f(r_t | F_{t-1}; \theta)}$ denotes the conditional SGED density of returns given past information.

Under standard regularity conditions, the MLE estimators are consistent and asymptotically normal (Bollerslev & Wooldridge, 1992). The incorporation of skewness and heavy-tail parameters enhances robustness during periods of structural instability and large market shocks.

3.8 Out-of-Sample Forecast Performance

Table 6 presents the out-of-sample forecast performance of the competing GARCH-type models using four evaluation metrics, namely Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Bias. In addition to RMSE, MSE, and MAE, forecast bias was incorporated to assess the extent of systematic overestimation or underestimation in volatility forecasts, computed as Bias = MSE − MAE^2 (Patton, 2011). The results indicate substantial differences in predictive accuracy across the innovation densities and volatility specifications. Among the competing models, the GARCH(1,1)-NORM model recorded the highest bias value (0.7496), indicating relatively poor forecasting performance and greater systematic forecast deviation, whereas the APARCH(1,1)-SGED model achieved the lowest bias value (0.4880), alongside the lowest RMSE, MSE, and MAE values. This finding suggests that the APARCH(1,1)-SGED specification provided the most accurate and stable

ISSN NUMBER: 1116-249X

volatility forecasts for the Nigerian stock market by effectively capturing asymmetry, volatility persistence, skewness, and heavy-tailed behavior inherent in financial return series. Summarily, lower Bias, RMSE, MSE, and MAE values indicate superior forecast accuracy and better model adequacy consistent with findings by Akanbi et al. (2025) and Adegboyo and Sarwar (2025).

Table 6: Out-of-Sample Forecast Performance (232 Trading Days, 2023)

Model	Innovation Dist.	BIAS	MSE	RMSE	MAE
GARCH(1,1)	NORM	**0.7496**	0.868746	0.434373	0.34524
	STD		0.697623	0.348811	0.33627
	GED	**0.7530**	**0.682917**	**0.341459**	**0.33156**
	SNORM		0.68747	0.343735	0.33387
	SSTD		0.697623	0.348811	0.33537
	SGED		0.687698	0.343849	0.33274
EGARCH(1,1)	NORM		0.74877	0.374385	0.31901
	STD		0.745114	0.372557	0.31679
	GED	**0.5725**	**0.662542**	**0.331271**	**0.30015**
	SNORM		0.68077	0.340385	0.32873
	SSTD		0.679084	0.339542	0.32501
	SGED		0.670254	0.335127	0.30113
APARCH(1,1)	NORM		0.602683	0.301341	0.30016
	STD		0.601558	0.300779	0.29857
	GED		0.589488	0.294744	0.29361
	SNORM		0.582683	0.291341	0.29024
	SSTD		0.579356	0.289678	0.27656
	SGED	**0.4880**	**0.562848**	**0.281424**	**0.27362**

Note: Bold values indicate the best-performing model for each specification. Lower Bias, RMSE, MSE, and MAE values indicate superior forecast accuracy.

APARCH-SGED model produced forecasts that tracked realized volatility more closely during periods of heightened market turbulence in 2023, further confirming its superior predictive performance relative to competing specifications.

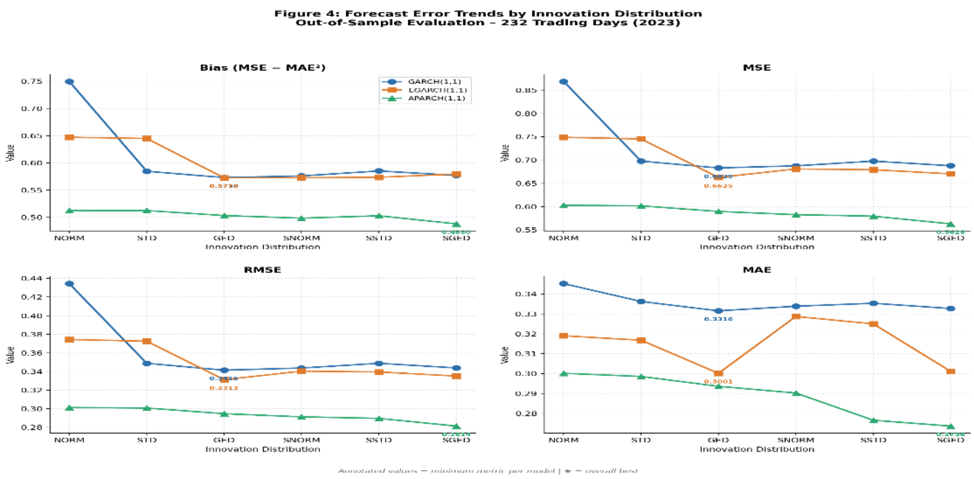

Figure 3: Forecast error Trend by Innovation out-of-sample Evaluation – 232 Trading Days (2023)

ISSN NUMBER: 1116-249X

Figure 3 presents the trend of RMSE, MSE, MAE, and Bias across the six innovation densities for the GARCH(1,1), EGARCH(1,1), and APARCH(1,1) models. The figure shows a consistent decline in forecast errors from NORM to SGED, indicating that volatility forecasts improve substantially when skewness and heavy-tailed behavior are incorporated into the innovation density. The sharpest improvement is observed for the EGARCH model between the STD and GED distributions, highlighting the importance of modeling leptokurtic financial returns. Furthermore, the APARCH line remains consistently below the GARCH and EGARCH lines across all distributions, confirming the superior forecasting performance of the APARCH framework. This suggests that the asymmetric power parameter (\u03b4) provides additional flexibility for capturing nonlinear volatility dynamics and asymmetric market shocks in the Nigerian stock market.

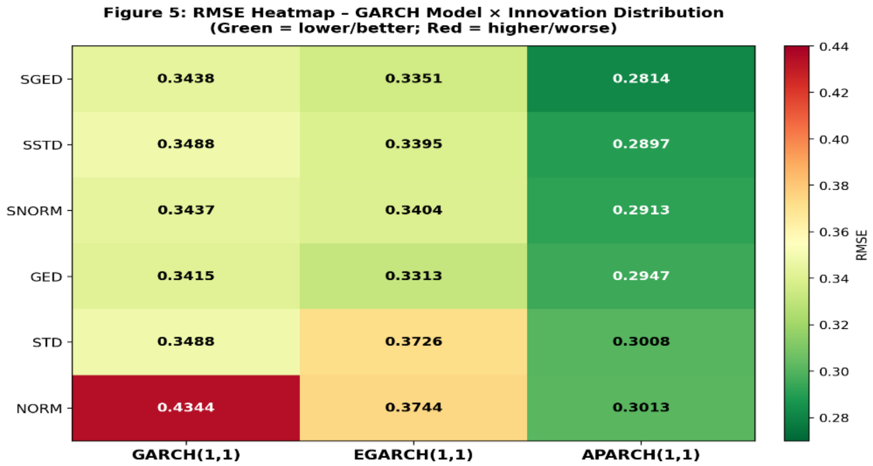

Figure 4: RMSE Heatmap = GARCH model x Innovation Distribution
(Green = lower/better Red = Higher/worse)

Figure 4 presents the RMSE heatmap for the competing GARCH-type models under different innovation densities. The APARCH column appears consistently greener than the others, with all RMSE values below 0.302, indicating superior forecast accuracy. In contrast, the GARCH(1,1)-NORM specification records the highest RMSE value (0.4344) and appears as the deepest red cell, performing approximately 54% worse than the optimal APARCH-SGED model. The EGARCH models occupy an intermediate position, reflecting moderate improvements over the standard GARCH framework. Overall, the heatmap visually confirms that APARCH models combined with skewed heavy-tailed distributions, particularly SGED, provide the most accurate volatility forecasts for the Nigerian stock market.

4.0 Conclusion

This study systematically evaluated GARCH-type volatility models under six innovation densities to enhance forecasting accuracy for the Nigerian Stock Exchange using daily data from February 2012 to December 2023. The findings conclusively demonstrate that the APARCH(1,1) model with Skewed Generalized Error Distribution (SGED) represents the optimal specification, achieving the lowest out-of-sample RMSE (0.2814) during the turbulent 2023 period marked by fuel subsidy removal and naira floatation. This model captures three essential features of NSE returns: volatility persistence ($\alpha_1 + \beta_1 \approx 0.93$), asymmetric responses through power transformation ($\delta \approx 1.0$), and excess kurtosis (shape $\approx$ 0.97) with positive skewness (skew $\approx$ 1.01). Critically, while symmetric GED performed well in-sample, it was outperformed out-of-sample, revealing that skewness becomes critically important during

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asymmetric policy shocks. The normal distribution produced the highest forecast errors, confirming its inappropriateness for NSE returns. This study contributes to knowledge by providing the first comprehensive comparative analysis of six innovation densities across three GARCH-type models for the NSE, demonstrating that APARCH-SGED significantly outperforms conventional specifications during structural breaks, and establishing that skewness is not merely a distributional nuance but a critical parameter for accurate volatility forecasting during periods of asymmetric policy shocks in emerging markets.

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