COVARIANCE MONITORING OF HIGH DIMENSIONAL TIME SERIES HEALTH DATA UNDER VIOLATION OF SELECTED ASSUMPTIONS

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ABSTRACT
Monitoring covariance structures in high-dimensional time series is crucial for understanding evolving dependencies in complex systems such as healthcare, finance, and industrial processes. Traditional covariance estimators often fail when the number of variables exceeds the sample size or when data contain outliers, non-stationarity, and structural changes. This study examines robust and sparse covariance estimation techniques for high-dimensional time series, with application to healthcare data in Nigeria. A simulation study based on multivariate autoregressive models was conducted under stationary and non-stationary conditions. Sample sizes (n = 100, 300), dimensions (p = 10, 15, 150), contamination levels (0%, 5%, 10%), and mean shifts were considered. Data were generated from normal and heavy-tailed distributions. Performance was evaluated using Mean Squared Error, Frobenius Norm Error, Condition Number, and change-detection metrics. Three estimators were compared: Minimum Covariance Determinant (MCD), Ridge, and Graphical LASSO, with covariance changes detected using the CUSUM procedure. Results show that MCD consistently provides superior robustness across contamination levels and dimensions. Ridge and LASSO perform well under clean normal data but deteriorate with outliers and structural shifts, especially in small samples. Robust covariance estimation, particularly MCD, offers a reliable framework for monitoring high-dimensional healthcare time series.
Key words: MCD, Ridge, Graphical LASSO outliers, non-stationarity, and structural shifts

1. INTRODUCTION
Covariance monitoring involves tracking the covariance matrix of a multivariate time series data over time. The covariance matrix represents the variance and covariance between different variables in the data. Monitoring covariance structures in high-dimensional time series data is crucial for various applications, including health, finance, industrial process control, and climate analysis and network security. In many real-world systems, dependencies between multiple variables evolve over time, making it essential to detect and analyze changes in their relationships. High-dimensional time series data is increasingly common in healthcare, with applications in disease surveillance, patient monitoring and personalized medicine (Chen et al., 2017; Li et al., 2020). Covariance monitoring is a crucial task in analyzing such data, as it enables the detection of changes or anomalies in the relationships between variables.
Traditional statistical methods often struggle with high-dimensional settings due to the "curse of dimensionality," where the number of variables exceeds the available observations. This challenge necessitates specialized techniques, such as regularized estimators, dimensionality reduction and advanced change-point detection methods. However, estimating and tracking covariance matrices in time series is challenging due to high dimensionality, non-stationarity, and limited observations relative to the number of assets. Traditional sample covariance estimation is unreliable in high dimensions, leading to issues such as spurious correlations and

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unstable risk estimates. To address these challenges, modern approaches incorporate regularization techniques such as shrinkage (Benoit &, Alexandre 2023), sparse graphical models and dynamic factor models (Luke et.al, 2024) to improve robustness and adaptability.

Furthermore, methods have been increasingly used to identify abrupt shifts in covariance structures, enabling early detection of market crashes or liquidity crises. Machine learning techniques, such as deep learning-based covariance forecasting, offer additional improvements by capturing nonlinear dependencies and regime changes more effectively. The covariance structure of high-dimensional time series data is crucial for understanding complex dependencies and detecting structural changes across various domains, including health finance, industrial process control, healthcare, and environmental monitoring. Unlike independent observations, time series data exhibit temporal dependencies, making the estimation and tracking of covariance matrices more challenging, particularly in high-dimensional settings where the number of variables exceeds the available observations (Jiang et al., 2023).

Covariance monitoring in time series data is essential for identifying changes in relationships between variables over time. For instance, in financial markets, shifts in asset correlations can indicate regime changes or systemic risk, while in industrial systems, variations in sensor dependencies may signal equipment failures. However, traditional covariance estimation methods struggle with high-dimensional time series due to issues such as limited sample sizes, noise accumulation, and non-stationarity (Aubry et al., 2024). Effective covariance monitoring allows for early detection of anomalies, structural shifts, or emerging patterns that could indicate system failures, shifts, or other critical events.

The analysis of high-dimensional health time series often relies on classical statistical models that assume conditions such as stationarity, absence of outliers, and balanced sample-to-variable ratios (n>p). In practice, however, these assumptions are frequently violated, which undermines the validity of traditional estimation and monitoring methods (Cai, et al., 2013; Zhang, et al. 2019). This study explores state-of-the-art approaches for covariance estimation, change detection, and anomaly identification in high-dimensional time series, with an emphasis on scalable and computationally efficient methodologies. To address the challenges with high-dimensional time series, modern approaches leverage techniques such as regularized covariance estimation using Machine Learning Methods and Robust estimators. However, this study wishes to propose a robust estimator like robust covariance estimator and sparse covariance estimation for time series i.e. Minimum Covariance Determinant, Ridge and LASSO. It is expected that this method will enable scalable and adaptive monitoring of covariance structures, facilitate early detection of anomalies and improve decision-making in dynamic environments.

2. MATERIALS AND METHODS

2.1 Research Design

The methods used in collecting the data were simulations based on different distributions. A simulation study was conducted to assess the performance of the three robust estimators outlined in the previous sections, particularly their ability to correctly estimate the changing levels of the estimators under certain conditions like non-stationarity, presence of outliers and high dimensionality by the proposed robust and sparse covariance estimation for high dimensional time series i.e. Minimum Covariance Determinant, Ridge and LASSO. The simulated is firstly considered because the required number of data to assess the models or data that follows different distributions and levels may not be available. The population sizes were varied to assess the models' performance in simulation. Also, high-dimensional time series health data collected from healthcare institutions in Nigeria will also be used for the assessments. This data may include various health indicators such as vital signs, lab results, and disease incidence rates. Monthly reported cases over a specified period will be considered to ensure sufficient observations for analysis.

2.2 Method of Data Collection

Data were simulated from a high-dimensional multivariate autoregressive time series model with a specified covariance structure to provide a more robust way of simulation. Data were generated from linear autoregressive functions given as follows:

$$Y_{t_i} = \emptyset Y_{ti-1} + e_t, \quad (1)$$

$t = 1,2, \ldots ,10,\ 50, and\ 100.\ i = 1,2, \ldots , 1000$

Y_{ti} are present responses simulated from random samples of normal distribution and Y_{ti-1} are past responses e_{ti} is a random error which is known as white noise which is also distributed from normal distributions as follows:

$X_{ti} \sim N(2,4) and e_{ti} \sim N(1,2)$ for non-stationary data structures. The generation of the data used for this simulation study is in two cases. These are stationary and non-stationary cases. Random values were simulated for the response variables from Normal distribution. This was followed by injecting different level of outliers and dimensionality. The error term was generated differently from Normal, Exponential, Cauchy, and Uniform distributions. For the simulation study, the sample sizes simulated from each of autoregressive cases were n = 10, 50 and 100 to represent small, moderate and large sample sizes. At a particular choice of sample size, the number of variables were varied (p = 10, 50, 100 and 150) such that n < p, n = p and n > p and different proportions of outliers (0%, 10% and 20%) were injected in a fraction of observations by adding heavy-tailed noise, this indicates no outlier, low and high proportions of the outliers in the data respectively. Also, in the simulations, different shift in diagonal of the covariance matrix were introduced. The simulation study was performed 1000 times for different cases and the average values were be recorded.

2.3 Robust Estimators

This study simulates high-dimensional time series data and applie**s** Proposed Minimum Covariance Determinant (MCD), Ridge, and LASSO (Graphical LASSO) estimators while considering non-stationarity, outliers, and high-dimensionality.

2.3.1 Minimum Covariance Determinant (MCD)

Given a dataset X={x_1,x_2,…,x_n} the aim is to find a subset of size h (with h ≤ n) such that the determinant of the empirical covariance matrix of this subset is minimized, Let $H \subset \{1,2, \ldots , n\}$ be a subset of indices with $\mid H \mid = h$. Define the subset mean and covariance are respectively given as

$$\bar{x}_H = \frac{1}{h}\sum_{i=1}^{H} x_i$$

$$\bar{S}_H = \frac{1}{h}\sum_{i=1}^{H}(x_i - \bar{x}_H)(x_i - \bar{x}_H)' \quad (2)$$

Then, the MCD estimator finds the subset H^* that minimizes the determinant of S_H

$$\underset{H\subset\{1,2,\ldots,n\}}{\text{argmin}}\ |S_H|$$

The corresponding MCD location and scatter estimates are:

$$\hat{\mu}_{MCD} = \bar{x}_H$$

$$\hat{\sum}_{MCD} = \bar{x}_H = \mathrm{c(h,n)}.\, S_H$$

Where c(h,n) is a consistency factor used to correct for bias and ensure consistency at the multivariate normal model (often close to 1, depends on h, n and p). A common choice is

$$h = \frac{n+p+1}{2}$$

which gives maximum breakdown point (about 50%). Larger h includes more observations, giving more efficient estimates but lower robustness. Robust estimator that minimizes the determinant over subsets of data.

2.3.2 Ridge Estimator:

Derive the parameter of the following ridge estimator equation

$$\Sigma_{ridge} = S + \lambda I$$

where S is the sample covariance and λ is a tuning parameter.

Model Setup

We start with the standard linear regression model:

$$y = X\beta + \varepsilon$$

y: $n \times 1$ vector of observations
X: $n \times p$ matrix of predictors
β: $p \times 1$ vector of unknown coefficients
The OLS estimator minimizes the residual sum of squares:

$$\beta_{OLS} = \underset{\beta}{\operatorname{argmin}} |y - X\beta|^2$$

$$\beta_{OLS} = (X'X)(X'Y) \qquad (3)$$

If $X'X$ is ill-conditioned or singular (due to multicollinearity or p>n), OLS becomes unstable or undefined.
Add an L2 penalty to the OLS criterion, we have the ridge estimator as follows

$$\beta_{ridge} = \underset{\beta}{\operatorname{argmin}} (|y - X\beta|^2 + \boldsymbol{\lambda}(|\beta|^2)) \qquad (4)$$

Where,
$\lambda \geq 0$, is the ridge penalty (shrinkage) parameter. $|\beta|^2 = \beta'\beta$

$$L(\beta) = (y - X\beta)'(y - X\beta) + \boldsymbol{\lambda}\beta'\beta$$
$$= y'y - 2y'X\beta + \beta'X'X\beta + \boldsymbol{\lambda}\beta'\beta$$

Taking the derivative with respect to β and set it to zero:

$$\frac{\partial L}{\partial \beta} = -2X'y + 2X'X\beta + 2\lambda\beta = 0$$
$$(X'X + \lambda I)\beta = X'y$$
$$\beta_{ridge} = (X'X + \lambda I)^{-1}X'y \qquad (5)$$

The ridge estimator shrinks the OLS coefficients towards zero. The term λI stabilizes the inversion and reduces variance. As $\lambda \to 0$, $\hat{\beta}_{ridge} \to \hat{\beta}_{OLS}$. As $\lambda \to \infty$, $\hat{\beta}_{ridge} \to 0$

Mean Square Error (MSE) and Bias of Ridge Estimator

For any estimator $\hat{\beta}$, the MSE (risk) is

$$MSE(\hat{\beta}) = E\left|\hat{\beta} - \beta\right|^2 = \left|E(\hat{\beta}) - \beta\right|^2 + E|(\beta - E(\beta)|^2$$
$$= bias^2 + tr(Var(\hat{\beta})$$
$$E(\beta_{ridge}) = (X'X + \lambda I)^{-1}X'X\beta$$
$$Bias = E(\beta_{ridge}) - \beta = [(X'X + \lambda I)^{-1}X'X - I]\beta \qquad (6)$$

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$$Var(\beta_{ridge}) = \sigma^2(X'X+\lambda I)^{-1}X'X(X'X+\lambda I)^{-1}\beta$$

Putting together

$$MSE(\beta_{ridge}) == [(X'X+\lambda I)^{-1}X'X\beta-\beta]^2+\sigma^2(X'X+\lambda I)^{-1}X'X(X'X+\lambda I)^{-1}\beta \quad (7)$$

2.3.3 LASSO-based Estimator:

Sparse inverse covariance estimation using graphical LASSO:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\big(tr(S\theta)\big) - log|\theta| + \lambda|\theta| \quad (8)$$

Below is a step-by-step treatment of the Lasso (ℓ_1-penalized) estimator. We start with the same linear model

$$y = X\beta+\varepsilon, \varepsilon \sim (0,\sigma^2 I),$$

and add an ℓ_1 penalty

$$\beta_{lasso} = \underset{\beta}{\operatorname{argmin}}(|y-X\beta|^2+\lambda|\beta|_1), \lambda|\beta|_1 = \sum_{j=1}^{p}|\beta_j|, \lambda \geq 0 \quad (9)$$

Because $\lambda|\beta|_1$ is not differentiable at zero, we use sub gradients. Define the residual vector $r = y - X\beta$. The sub-gradient condition for optimality is

$0 = -2X'r + \lambda\partial|\beta|_1$

$$2X_j(y-X\beta) = \lambda s_j, s_j = \begin{cases} \sin(\beta_j), \beta_j \neq 0 \\ -1,1, \quad \beta_j = 0 \end{cases}$$

Mean Square Error (MSE) and Bias of Lasso Estimator

Suppose

$$X'X = Ip, y = X\beta+\varepsilon, \varepsilon \sim N(0,\sigma^2 I_n)$$

Define the "raw" OLS coordinate

$$z_j = y = X\beta+\varepsilon, \varepsilon \sim N(0,\sigma^2 I_n)$$

The Lasso solution is

$$\hat{\beta}_j = S\left(z_j,\frac{\lambda}{2}\right) = \sin\,(z_j)max\left[|z_j|-\frac{\lambda}{2},0\right]$$

$$Bias(\hat{\beta}_j) = E(\hat{\beta}_j)-\beta_j$$

Since

$$\hat{\beta}_j = S\left(z_j,\frac{\lambda}{2}\right), \text{one shows (using truncated- normal integrals), then}$$

$$E(\hat{\beta}_j) = \int_{-\infty}^{\infty} S\left(z_j,\frac{\lambda}{2}\right)\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(z-\beta_j)^2}{2\sigma^2}}$$

which simplifies to the sum of two terms,

$$E(\hat{\beta}_j) = \left(\beta_j-\frac{\lambda}{2}\right)\emptyset\left(\frac{\beta_j-\frac{\lambda}{2}}{\sigma}\right)+\left(\beta_j+\frac{\lambda}{2}\right)\left[1-\emptyset\left(\frac{\beta_j+\frac{\lambda}{2}}{\sigma}\right)\right]$$
$$+\sigma\left[\varphi\left(\frac{\beta_j+\frac{\lambda}{2}}{\sigma}\right)-\varphi\left(\frac{\beta_j-\frac{\lambda}{2}}{\sigma}\right)\right]$$

Here Φ and φ are the standard normal CDF and PDF. Therefore

$$Bias(\hat{\beta}_j) = E(\hat{\beta}_j)-\beta_j$$

given by the above expression minus β_j

$$Var(\hat{\beta}_j) = E(\hat{\beta}_j^2) - [E(\hat{\beta}_j)]^2$$

$$E(\hat{\beta}_j^2) = \int_{-\infty}^{\infty} S\left(z_j, \frac{\lambda}{2}\right)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\beta_j)^2}{2\sigma^2}}$$

Closed-form expressions again involve φ and φ, but are lengthier (omitted here for brevity).

$$MSE(\hat{\beta}) = bias^2 + tr(Var(\hat{\beta})$$

$$= (E|\hat{\beta}_j| - \beta)^2 + [E|\hat{\beta}_j^2| - E(|\hat{\beta}_j|)^2] \quad (10)$$

sum these over $j = 1, \ldots, p$to get the overall parameter-vector MSE.

We will run simulations across multiple iterations and compare estimators in different settings (stationary, non-stationary, with/without outliers and multicollinearity). The MSE and Bias of the estimators will also be used to determine the relative performances of the three estimators.

2.4 Covariance Estimation

The covariance estimation were carried out using Sample Covariance Matrix (SCM).The Sample Covariance Matrix (SCM) is a fundamental statistical tool used to estimate the covariance structure of multivariate data. It is widely applied in healthcare analytics, finance, and machine learning to understand relationships between variables. Given a dataset X with n observations and p variables:

$$\begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1p} \\ \vdots & & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{np} \end{bmatrix}$$

The Sample Covariance Matrix (SCM) is computed as:

$$S = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})(X_i - \bar{X})'$$

where:

S is the p×p covariance matrix.

X_i is the ith row vector of observations.

$\bar{X}$is the mean vector of each variable

$(X_i - \bar{X})'$ represents the deviation from the mean.

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

Each entry in S represents the covariance between two variables X_i and X_j

$$S_{ij} = \frac{1}{n-1} \sum_{i,j=1}^{n} (X_i - \bar{X})(X_j - \bar{X})'$$

If $i = j$ then S_{ij} is the variance of variable X_j

If $i \neq j$ then S_{ij} is the variance of variable X_j

2.5 Evaluation Techniques

Performance evaluation of covariance monitoring methods involves assessing accuracy, robustness, computational efficiency, and adaptability to real-world health data. The key metrics and techniques used for evaluation include:

(i) Mean Squared Error (MSE) of the Covariance Estimator

To measure how well an estimated covariance matrix $\hat{\Sigma}$ approximates the true covariance matrix Σ, we use:

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$$MSE(\hat{\Sigma}) = \frac{1}{p^2}\sum_{i=1}^{p}\sum_{j=1}^{p}(\hat{\Sigma}_{ij} - \Sigma_{ij})^2$$

The Lower MSE indicates a better estimation of the covariance structure.

(ii) Frobenius Norm Error (FNE)

Measures the deviation between the estimated and true covariance matrix:

$$FNE(\hat{\Sigma}) = |\hat{\Sigma}_{ij} - \Sigma_{ij}| = \sqrt{\sum_{i=1}^{p}\sum_{j=1}^{p}(\hat{\Sigma}_{ij} - \Sigma_{ij})^2}$$

(iii) Condition Number (κ)

$$k(\Sigma) = \frac{\lambda_{max}(\Sigma)}{\lambda_{min}(\Sigma)}$$

where λ_{max} and λ_{min} are the largest and smallest eigenvalues of Σ, respectively. High κ values indicate numerical instability, common in high-dimensional settings. The flow chart framework illustrates the relationship between high-dimensional healthcare data, the challenges associated with analyzing such data, traditional and robust/sparse covariance estimation methods, change-point detection strategies, evaluation metrics, and healthcare applications. The flow shows how robust covariance monitoring will address the limitations of traditional methods to improve disease surveillance, patient monitoring, and treatment effectiveness.

3. Results and Discussion

Table 1: Performance Measures for n=100, p=10

Estimator	Outlier_Prop	Shift_Level	Frobenius_Error	MSE	ASN	Probability_Signal
MCD	0	0	1.0094	0.0102	0.0001	0
Ridge	0	0	1.0643	0.0113	0.0001	0
GLASSO	0	0	1.4219	0.0202	0.0001	0
MCD	0.05	0	2.5792	0.0665	0.0003	1
Ridge	0.05	0	197.0045	388.1076	1.9405	1
GLASSO	0.05	0	196.2617	385.1864	1.9259	1
MCD	0.1	0	0.9003	0.0081	0.0000	0
Ridge	0.1	0	364.2128	1326.5094	6.6325	1
GLASSO	0.1	0	363.0720	1318.2125	6.5911	1
MCD	0	1	1.6801	0.0282	0.0001	1
Ridge	0	1	2.0786	0.0432	0.0002	1
GLASSO	0	1	1.4470	0.0209	0.0001	0
MCD	0.05	1	1.5316	0.0235	0.0001	1
Ridge	0.05	1	212.8484	453.0446	2.2652	1
GLASSO	0.05	1	211.8880	448.9653	2.2448	1
MCD	0.1	1	1.0632	0.0113	0.0001	0
Ridge	0.1	1	406.2089	1650.0564	8.2503	1
GLASSO	0.1	1	405.9685	1648.1046	8.2405	1
MCD	0	5	1.0939	0.0120	0.0001	0
Ridge	0	5	1.8137	0.0329	0.0002	1

GLASSO	0	5	1.5098	0.0228	0.0001	1
MCD	0.05	5	1.8787	0.0353	0.0002	1
Ridge	0.05	5	302.8174	916.9838	4.5849	1
GLASSO	0.05	5	301.8366	911.0535	4.5553	1
MCD	0.1	5	1.2785	0.0163	0.0001	0
Ridge	0.1	5	553.6010	3064.7402	15.3237	1
GLASSO	0.1	5	554.4485	3074.1312	15.3707	1

Table 1 compares the performance of the MCD, Ridge, and LASSO estimators under varying outlier proportions and shift levels for a small sample size and low dimensional setting. In the absence of outliers and mean shifts, all estimators perform comparably, though MCD attains the lowest Frobenius error and MSE, indicating superior estimation accuracy. As the outlier proportion increases to 5% and 10%, the performance of Ridge and LASSO deteriorates drastically, with extremely large Frobenius errors, MSE, and ASN values, reflecting severe sensitivity to contamination. In contrast, MCD remains stable across all contamination levels, maintaining low estimation errors and near-zero ASN. Under increasing shift levels, MCD continues to outperform Ridge and LASSO by a wide margin, demonstrating strong robustness. Overall, Table 1 shows that MCD is markedly superior in contaminated and shifted environments, while Ridge and LASSO are only competitive in clean data settings.

Table 2: Performance Measures for n=100, p=15

Estimator	Outlier_Prop	Shift_Level	Frobenius_Error	MSE	ASN	Probability_Signal
MCD	0	0	4.1531	0.0767	0.0004	1
Ridge	0	0	3.6188	0.0582	0.0003	1
GLASSO	0	0	2.5485	0.0289	0.0001	1
MCD	0.05	0	1.3654	0.0083	0.0000	0
Ridge	0.05	0	291.8605	378.5892	1.8929	1
GLASSO	0.05	0	290.6203	375.3786	1.8769	1
MCD	0.1	0	4.0027	0.0712	0.0004	1
Ridge	0.1	0	531.9773	1257.7769	6.2889	1
GLASSO	0.1	0	530.6353	1251.4392	6.2572	1
MCD	0	1	1.8120	0.0146	0.0001	1
Ridge	0	1	2.0058	0.0179	0.0001	1
GLASSO	0	1	1.5156	0.0102	0.0001	1
MCD	0.05	1	2.2769	0.0230	0.0001	1
Ridge	0.05	1	303.3261	408.9187	2.0446	1
GLASSO	0.05	1	302.3234	406.2197	2.0311	1
MCD	0.1	1	1.4711	0.0096	0.0000	0
Ridge	0.1	1	597.0500	1584.3054	7.9215	1
GLASSO	0.1	1	598.6706	1592.9175	7.9646	1
MCD	0	5	3.1827	0.0450	0.0002	1
Ridge	0	5	3.3342	0.0494	0.0002	1
GLASSO	0	5	2.3899	0.0254	0.0001	1

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MCD	0.05	5	1.5501	0.0107	0.0001	1
Ridge	0.05	5	457.5557	930.4766	4.6524	1
GLASSO	0.05	5	458.1215	932.7790	4.6639	1
MCD	0.1	5	1.5261	0.0104	0.0001	1
Ridge	0.1	5	846.8212	3187.1383	15.9357	1
GLASSO	0.1	5	849.7997	3209.5983	16.0480	1

Table 2 presents results for a moderate dimensional setting with the same sample size. Under no contamination, LASSO slightly outperforms MCD and Ridge in Frobenius error and MSE, suggesting an advantage in higher dimensional clean data. However, once outliers are introduced, Ridge and LASSO experience explosive growth in estimation errors and ASN, whereas MCD maintains low Frobenius error and MSE across all outlier proportions. Increasing shift levels further amplify the instability of Ridge and LASSO, while MCD remains relatively unaffected. These results indicate that although LASSO may be competitive in clean high-dimensional settings, MCD remains the most reliable estimator under data contamination.

Table 3: Performance Measures for n=100, p=150

Estimator	Outlier_Prop	Shift_Level	Frobenius_Error	MSE	ASN	Probability_Signal
MCD	0	0	3.1928	0.0255	0.0001	1
Ridge	0	0	2.6796	0.0180	0.0001	1
GLASSO	0	0	1.8686	0.0087	0.0000	1
MCD	0.05	0	2.4860	0.0155	0.0001	1
Ridge	0.05	0	392.1401	384.4347	1.9222	1
GLASSO	0.05	0	390.4555	381.1387	1.9057	1
MCD	0.1	0	3.1472	0.0248	0.0001	1
Ridge	0.1	0	731.2538	1336.8304	6.6842	1
GLASSO	0.1	0	729.8624	1331.7478	6.6587	1
MCD	0	1	2.3810	0.0142	0.0001	1
Ridge	0	1	2.0774	0.0108	0.0001	1
GLASSO	0	1	1.8478	0.0085	0.0000	1
MCD	0.05	1	2.3297	0.0136	0.0001	1
Ridge	0.05	1	411.8551	424.0615	2.1203	1
GLASSO	0.05	1	409.8556	419.9541	2.0998	1
MCD	0.1	1	2.1203	0.0112	0.0001	1
Ridge	0.1	1	799.7529	1599.0119	7.9951	1
GLASSO	0.1	1	797.7363	1590.9578	7.9548	1
MCD	0	5	2.6953	0.0182	0.0001	1
Ridge	0	5	3.0551	0.0233	0.0001	1
GLASSO	0	5	2.2432	0.0126	0.0001	1
MCD	0.05	5	2.5189	0.0159	0.0001	1
Ridge	0.05	5	587.9361	864.1721	4.3209	1
GLASSO	0.05	5	586.2564	859.2415	4.2962	1
MCD	0.1	5	6.2145	0.0966	0.0005	1

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Ridge	0.1	5	1132.3950	3205.7962	16.0290	1
GLASSO	0.1	5	1137.7462	3236.1662	16.1808	1

Table 3 evaluates estimator performance under higher dimensionality. In the absence of outliers, LASSO achieves the lowest Frobenius error and MSE, followed closely by Ridge and MCD. However, the introduction of outliers results in dramatic performance degradation for Ridge and LASSO, with very large Frobenius errors and MSE values. MCD remains stable and consistently achieves the smallest errors across all contamination and shift scenarios. These findings highlight the vulnerability of penalized estimators in small-sample, high-dimensional contaminated settings and reaffirm the robustness advantage of MCD.

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Table 4: Performance Measures for n=300, p=10

Estimator	Outlier_Prop	Shift_Level	Frobenius_Error	MSE	ASN	Probability_Signal
MCD	0	0	1.1129	0.0124	0.0000	0
Ridge	0	0	1.3711	0.0188	0.0001	0
GLASSO	0	0	1.2203	0.0149	0.0000	0
MCD	0.05	0	0.7492	0.0056	0.0000	0
Ridge	0.05	0	192.5104	370.6027	1.2353	1
GLASSO	0.05	0	191.4560	366.5541	1.2218	1
MCD	0.1	0	1.0363	0.0107	0.0000	0
Ridge	0.1	0	357.1370	1275.4685	4.2516	1
GLASSO	0.1	0	356.5841	1271.5220	4.2384	1
MCD	0	1	2.0014	0.0401	0.0001	1
Ridge	0	1	2.3294	0.0543	0.0002	1
GLASSO	0	1	1.6992	0.0289	0.0001	1
MCD	0.05	1	1.2662	0.0160	0.0001	0
Ridge	0.05	1	206.4403	426.1761	1.4206	1
GLASSO	0.05	1	205.3102	421.5229	1.4051	1
MCD	0.1	1	1.0991	0.0121	0.0000	0
Ridge	0.1	1	403.5471	1628.5030	5.4283	1
GLASSO	0.1	1	402.6035	1620.8957	5.4030	1
MCD	0	5	0.7924	0.0063	0.0000	0
Ridge	0	5	1.1149	0.0124	0.0000	0
GLASSO	0	5	1.1173	0.0125	0.0000	0
MCD	0.05	5	1.2339	0.0152	0.0001	0
Ridge	0.05	5	297.8361	887.0633	2.9569	1
GLASSO	0.05	5	296.6605	880.0742	2.9336	1
MCD	0.1	5	1.9388	0.0376	0.0001	1
Ridge	0.1	5	561.8102	3156.3075	10.5210	1
GLASSO	0.1	5	561.6769	3154.8095	10.5160	1

Table 4 shows that increasing the sample size improves the performance of all estimators under clean data. MCD retains the lowest Frobenius error and MSE across most scenarios. When outliers

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are introduced, Ridge and LASSO still exhibit substantial increases in error and ASN, although less severe than in the smaller sample case. MCD continues to demonstrate strong robustness, with minimal sensitivity to both outliers and mean shifts. Thus, even with moderate sample sizes, MCD remains the most reliable estimator in the presence of contamination.

4. Discussion of Results

The results obtained in this study are largely consistent with existing literature on high-dimensional covariance estimation and robust statistical methods. The simulation outcomes demonstrate that the Minimum Covariance Determinant (MCD) estimator consistently outperforms the Ridge and Graphical LASSO estimators when data contain outliers, structural shifts, or heavy-tailed distributions. These findings support earlier theoretical and empirical studies that highlight the robustness properties of MCD estimators.

First, the strong performance of the MCD estimator under contaminated data conditions aligns with the foundational work of Rousseeuw *et al* (1999), who introduced high-breakdown robust estimators for multivariate location and scatter. His work demonstrated that MCD can maintain reliable covariance estimates even when a substantial portion of observations are contaminated. Later work by Mia Hubert and collaborators further confirmed that MCD-based estimators remain stable in high-dimensional datasets with outliers. The stability observed in this study, even when contamination levels increased to 10%, is therefore consistent with these theoretical robustness guarantees.

The findings also support previous research showing that penalized estimators such as Ridge and Graphical LASSO perform well under ideal conditions but deteriorate under data contamination. The sparse inverse covariance estimation method proposed by Jiang *et al* (2023), has been widely applied for high-dimensional covariance estimation due to its ability to impose sparsity in precision matrices. However, several studies have reported that such estimators are sensitive to outliers because the ℓ_1-penalty does not inherently provide robustness against extreme observations. The dramatic increase in Frobenius norm error and mean squared error observed in the present study when outliers were introduced confirms this limitation.

Similarly, the instability of Ridge estimation observed in the simulation experiments agrees with earlier findings that shrinkage-based covariance estimators are primarily designed to address multicollinearity and high dimensionality rather than contamination. Studies by Olivier and Wolf (2004) demonstrated that shrinkage estimators improve conditioning of covariance matrices in high-dimensional settings but do not necessarily provide protection against heavy-tailed distributions or outliers. The explosive increase in estimation errors for Ridge under contamination in this study therefore corroborates these earlier observations.

The results further align with the work of Jiang *et al* (2023), who emphasized that high-dimensional covariance estimation becomes highly sensitive to deviations from classical assumptions, particularly when the number of variables exceeds the number of observations. Their studies showed that estimation errors increase rapidly in such settings unless robust or structured estimators are employed. In the present study, the deterioration of Ridge and LASSO performance when dimensionality increased to p=150 with contamination provides additional empirical support for these theoretical insights.

Finally, the findings contribute to recent literature on high-dimensional change-point and covariance monitoring, such as the work of Jiang *et al* (2023), who highlighted the need for reliable covariance estimation methods when monitoring evolving dependence structures in time series. The present study extends this literature by demonstrating that robust covariance estimators like

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MCD can provide stable monitoring performance even in the presence of structural shifts and contaminated observations.

5. Conclusion

This study systematically evaluated the performance of robust and sparse covariance estimators Minimum Covariance Determinant (MCD), Ridge, and Graphical LASSO under varying conditions of dimensionality, contamination, structural shifts, and sample size in high-dimensional time series settings. The results consistently demonstrate that estimator performance is highly sensitive to violations of classical assumptions such as normality, absence of outliers, and stationarity.

Across all simulation scenarios, a clear pattern emerged. In clean data environments (no outliers and no structural shifts), Ridge and Graphical LASSO occasionally achieved slightly lower Frobenius Norm Errors and Mean Squared Errors, particularly in moderate to high-dimensional settings (e.g., $p=15$ and $p = 150$). This confirms that penalized estimators are competitive and efficient when classical assumptions hold.

However, once contamination was introduced even at low levels (5%) the performance of Ridge and Graphical LASSO deteriorated dramatically. The Frobenius errors and MSE values increased explosively, often by several orders of magnitude. The Average Signal Number (ASN) also became excessively large, indicating instability and unreliable change detection. These effects were amplified in higher dimensions and under stronger structural shifts. This confirms that shrinkage- and sparsity-based estimators are highly sensitive to outliers and heavy-tailed disturbances, particularly in small-sample high-dimensional settings ($n < p$).

In contrast, the Minimum Covariance Determinant (MCD) estimator demonstrated remarkable stability across all experimental conditions. Its Frobenius errors and MSE values remained consistently low even under 10% contamination and substantial mean shifts. Importantly, increasing dimensionality from $p=10$ to $p=150$ did not lead to catastrophic failure of the MCD estimator. Furthermore, as the sample size increased from $n=100$, $n=300$, overall estimation accuracy improved for all methods, but MCD maintained its robustness advantage. This indicates that while larger sample sizes help mitigate instability, robustness to contamination is fundamentally more important than sample size alone in high-dimensional monitoring problems.

The findings therefore establish three major empirical conclusions:

1. While Ridge and LASSO may be efficient under ideal conditions, their vulnerability to outliers makes them unreliable for real-world health time series data where contamination and structural changes are common.
2. MCD provides stable covariance monitoring under assumption violations. The estimator maintains low estimation error, numerical stability, and reliable change detection even in high-dimensional and non-stationary environments.
3. As dimensionality increases, penalized estimators become increasingly unstable in the presence of outliers, whereas MCD retains strong performance.

Overall, the simulation evidence strongly supports the adoption of robust covariance estimation particularly MCD for high-dimensional healthcare time series monitoring. In practical applications such as disease surveillance, patient monitoring, and anomaly detection in Nigerian health systems, data contamination, structural shifts, and heavy-tailed errors are unavoidable. Under such realistic conditions, robust covariance-based monitoring provides a more reliable and trustworthy framework than conventional shrinkage or sparsity-based estimators.

Therefore, this study concludes that robust covariance monitoring using MCD offers a scalable, stable, and practically applicable solution for high-dimensional time series analysis under violation

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of selected assumptions. Future research may extend this framework to adaptive robust-sparse hybrid methods and real-time monitoring systems for large-scale healthcare data environments.

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