

## COMPARATIVE PERFORMANCE OF ESTAR AND AFRIMA MODELS IN FORECASTING THE NIGERIAN EXCHANGE RATE.

<sup>1</sup>Akintunde, Mutairu Oyewale, <sup>2</sup>Dayo, Kayode Vincent, <sup>3</sup>Eriobu, Nkiru Obioma, <sup>4</sup>Ogunleke, Akinyemi Samuel, and <sup>1</sup>Adetona, Basirat Omotola.

<sup>1</sup>Department of Statistics, Federal University of Agriculture, Abeokuta, Ogun State.

<sup>2</sup> Department of Statistics, University of Abuja

<sup>3</sup> Department of Statistics, Nnamdi Azikwe University, Anambra State.

<sup>4</sup>IT- Business and Advanced Analytics Department. New Brunswick Community College, Saint John, NB, Canada.

Corresponding author's e-mail: [waleakintunde2004@gmail.com](mailto:waleakintunde2004@gmail.com)

### Abstract

This study examines and compares the predictive performance of the Exponential Smooth Transition Autoregressive (ESTAR) and Autoregressive Fractionally Integrated Moving Average (ARFIMA) frameworks for modelling and forecasting Nigeria's exchange rate dynamics. Using monthly observations spanning January 2000 to December 2025, the study investigates whether exchange rate behaviour is better captured by nonlinear adjustment mechanisms or long-memory dependence structures. Preliminary time-series diagnostics indicate evidence of persistence, gradual adjustment toward equilibrium, and nonlinear characteristics in the underlying data-generating process, suggesting that exclusive reliance on conventional linear specifications may be inadequate. The competing models were estimated and assessed using both in-sample adequacy and out-of-sample forecasting criteria. Model selection and forecast evaluation employed the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), Root Mean Square Error (RMSE), and Mean Absolute Percentage Error (MAPE). The empirical results indicate that the ESTAR specification achieves superior short-term forecasting performance, reflecting its capacity to capture regime-dependent movements and nonlinear correction dynamics in exchange rate fluctuations. Conversely, the ARFIMA model exhibits greater forecast stability over longer horizons, consistent with its ability to accommodate fractional persistence and long-range dependence. The findings underscore the importance of incorporating nonlinear and long-memory econometric structures into exchange rate forecasting, particularly in emerging market contexts characterised by structural adjustments and market frictions. By combining nonlinear transition modelling with long-memory representation, the study provides empirical evidence that may strengthen forecasting practice and inform macroeconomic surveillance, monetary policy formulation, and exchange-rate risk assessment in Nigeria.

Keywords: Exchange Rate; Forecasting; ESTAR Model; ARFIMA Model; Nonlinear Modeling; Model Evaluation.

### 1.0 Introduction

Contemporary scholarship on exchange rate behaviour in developing economies has increasingly rejected the adequacy of linear, memoryless representations of time-series dynamics. In economies such as Nigeria, currency fluctuations are now widely interpreted as the outcome of interacting structural distortions, policy-induced shifts, and information inefficiencies that jointly produce nonlinear adjustment patterns alongside persistent dependence in shocks. These empirical realities

undermine the assumptions embedded in traditional autoregressive integrated moving average structures, particularly their reliance on constant parameters, symmetric responses, and exponentially decaying innovation effects. As a result, modern econometric inquiry has progressively gravitated toward modelling strategies that accommodate both conditional nonlinearity and fractional persistence, with the ESTAR and ARFIMA frameworks emerging as leading alternatives.

Empirical studies further suggest that such dynamics cannot be meaningfully captured through a single linear specification. Salisu (2011) maintains that the Nigerian exchange rate process is inherently regime-dependent, with adjustment behaviour varying in response to changing macroeconomic and policy conditions. In a related contribution, Ayoade (2024) identifies sustained volatility clustering and nonlinear adjustment patterns in the naira market, reinforcing the argument that conventional linear time-series models are structurally inadequate for reliable forecasting. Taken together, these findings point to the necessity of employing more adaptable econometric frameworks capable of accommodating structural shifts, nonlinear dynamics, and time-variant behaviour in exchange rate modelling.

Ani, et.al. (2024) explored the implications of foreign exchange rate disturbances for the market performance of selected manufacturing firms listed on the Nigerian Exchange Group (NGX). Adopting an ex-post facto research framework and employing Ordinary Least Squares (OLS) estimation, the study assessed the extent to which exchange rate volatility influences stock valuation and trading activity. The empirical evidence revealed an inverse relationship between exchange rate disruptions and both share prices and transaction volumes, indicating that heightened currency instability tends to weaken market performance. The authors argued that firm-level adjustment mechanisms and market-specific factors may have moderated the transmission of exchange rate fluctuations to equity performance. Consequently, the study emphasized the importance of exchange rate stabilization policies and robust foreign exchange risk management practices as essential measures for enhancing investor confidence and sustaining corporate competitiveness within Nigeria's manufacturing sector.

## 2.0 Literature review

The Exponential Smooth Transition Autoregressive framework, formally introduced by Granger and Teräsvirta (1993), represents a class of nonlinear models in which system dynamics evolve continuously as a function of deviations from equilibrium rather than switching abruptly between discrete regimes. This gradual adjustment mechanism is particularly relevant in foreign exchange markets, where behavioural responses differ depending on the intensity of disequilibrium pressures. In such settings, small deviations may trigger weak corrective forces, whereas larger misalignments may induce stronger mean-reverting behaviour. Empirical work by Michael, Nobay, and Peel (1997) reinforces this interpretation by showing that real exchange rates tend to revert toward equilibrium in a nonlinear fashion, largely influenced by transaction costs and market frictions. Despite its theoretical appeal, applications of ESTAR within Nigeria's exchange rate literature remain relatively limited, suggesting that the nonlinear adjustment structure of the naira market is not yet fully captured in existing empirical models.

In contrast, the ARFIMA modelling class addresses persistence in economic time series through a fundamentally different mechanism. Originating from the seminal contributions of Granger and Joyeux (1980) and Hosking (1981), this framework extends standard integration concepts by permitting fractional orders of differencing. This modification enables the representation of long-range dependence, where shocks do not vanish rapidly but instead decay at a slow hyperbolic rate. Such behaviour implies that historical disturbances retain explanatory power over extended horizons, a feature frequently observed in financial and macroeconomic series. Baillie (1996) emphasises that this type of persistence is a defining attribute of exchange rate dynamics, especially in environments characterised by incomplete adjustment and delayed information transmission.

Empirical literature provides substantial evidence in support of fractional integration models as superior alternatives to conventional ARIMA specifications. Studies by Shittu and Yaya (2008; 2009) reveal that ARFIMA-based forecasts consistently outperform linear models in both fitting accuracy and predictive stability. Similarly, Alireza and Ahmad (2009) demonstrate that allowing for fractional differencing enhances model performance in financial datasets exhibiting prolonged dependence structures. These findings are echoed in later contributions such as Omekara et al. (2016) and Hamzaoui and Regaieg (2017), which identify persistent dependence structures in exchange rate-related variables and forward market expectations. More recent empirical extensions (Liu et al., 2018; Elmezouar et al., 2021; Azza et al., 2021; Jibrin et al., 2021) further confirm that fractional models tend to yield improved forecasting precision under conditions of sustained macroeconomic volatility and structural uncertainty.

Within Nigeria's macro-financial environment, exchange rate behaviour exhibits a combination of instability, regime sensitivity, and volatility clustering, suggesting that both nonlinear adjustment and persistence mechanisms are simultaneously active. Salisu (2011) highlights significant structural fragility and time-varying volatility in the naira market, indicating that linear specifications are insufficient for capturing its evolving nature. More recent analyses (Ayoade, 2024) further suggest that policy reforms and external macroeconomic shocks continue to generate prolonged deviations from equilibrium, reinforcing the relevance of persistence-aware modelling strategies.

Despite this recognition, the existing body of research largely treats nonlinear adjustment models and long-memory models as independent analytical alternatives rather than competing explanations of the same underlying process. ESTAR models emphasize conditional, state-contingent adjustment speeds, while ARFIMA frameworks focus on unconditional dependence that spans long temporal horizons. This conceptual separation has led to a fragmented empirical literature in which the relative explanatory strength of each approach remains unresolved, particularly in the context of emerging market currencies such as the Nigerian naira.

Consequently, the literature reveals a fundamental methodological gap: the absence of a unified comparative assessment that directly evaluates whether nonlinear mean reversion mechanisms or fractional integration processes more accurately describe exchange rate evolution. Addressing this gap is essential, not only for improving forecasting performance but also for advancing theoretical

understanding of how adjustment and persistence coexist in currency markets characterized by structural instability and policy-driven distortions. A comparative evaluation of ESTAR and ARFIMA therefore provides a more comprehensive framework for interpreting exchange rate behaviour, as it integrates both regime-dependent adjustment and long-memory dependence within a single empirical inquiry.

### 3.0 Mathematical frame work of ESTAR and AFRIMA MODELS

#### 3.1 Exponential Smooth transition Autoregressive model

The Exponential Smooth Transition (ESTAR) model is of the form

$$y_t = \phi' X_{t-1} + \theta y_{t-d} \cdot G(y_{t-d}; \gamma, c) + \varepsilon_t \quad (1)$$

Where transition function is given as

$$G(y_{t-d}; \gamma, c) = 1 - \exp\left(-\gamma(y_{t-d} - c)^2\right)$$

Equation 1 could be further expressed as a two regimes models as shown below

$$y_t = \phi_1' y_{t-d} \left(-\exp-\gamma(y_{t-d} - c)^2\right) + \phi_2' y_{t-d} \left(1 - \exp-\gamma(y_{t-d} - c)^2\right) + \varepsilon_t \quad (2)$$

#### 3.2 Autoregressive Fractional Integrated Moving Average (AFRIMA) Models

General AFRIMA model is expressed as

$$\phi(L)(1-L)^d y_t = \theta(L)\varepsilon_t \quad (3)$$

Where

$$\phi(L) = 1 - \phi_1 L - \dots - \phi_p L^p \quad (\text{AR polynomial})$$

$$\theta(L) = 1 + \theta_1 L + \dots + \theta_q L^q \quad (\text{MA polynomial})$$

The main feature of AFRIMA model is the fractional differencing which is of the form

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k \quad (4)$$

Which could be expanded as

$$(1-L)^d y_t = y_t - d y_{t-1} + \frac{d(d-1)}{2!} y_{t-2} + \frac{d(d-1)(d-2)}{3!} y_{t-3} + \dots \quad (5)$$

This infinite-order representation permits a gradual and hyperbolic attenuation of shocks over time, thereby effectively capturing the presence of long-range dependence and persistent temporal correlation within the stochastic process.

#### 3.3 Performance measure indices

Error metrics were employed to evaluate the performance of the series analyzed in this study. Specifically, the assessment of forecast accuracy focused on error measures based on squared deviations. The selected metrics include Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Scaled Error (MASE).

**3.3.1 Mean Squared Error (MSE):** Mathematically, it is expressed as:

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_t)^2$$

**3.3.2 Root Mean Squared Error (RMSE):** it is expressed as

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_t)^2}$$

**3.3.3 Mean Absolute Error (MAE):** The formula for MAE is:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_t|$$

**3.3.4 Mean Absolute Percentage Error (MAPE):** It is expressed as:

$$MAPE = \frac{100}{n} \sum_{i=1}^n \left| \frac{y_i - \hat{y}_t}{y_t} \right|$$

#### 4.0 DATA ANALYSIS

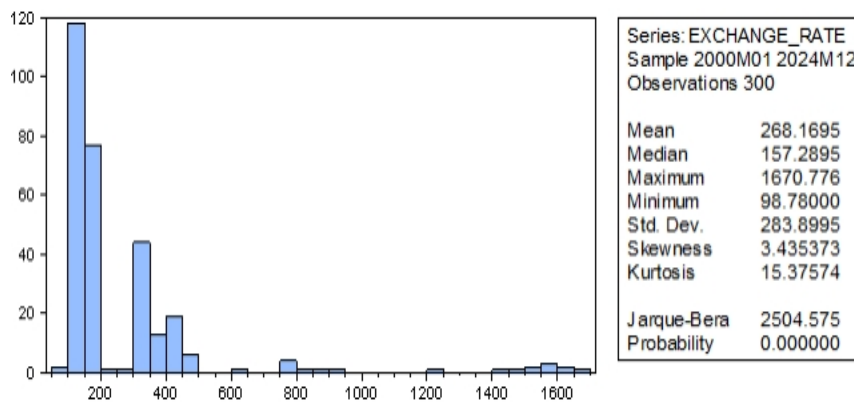


Figure 1: Descriptive Analysis

The histogram together with the summary statistics for the exchange rate series (2000M01–2024M12;  $N=300$ ) points to a distribution that departs markedly from normality, exhibiting strong asymmetry, fat tails, and pronounced instability over time.

A clear divergence between the mean (268.17) and the median (157.29) suggests substantial positive skewness. In practical terms, most observations are concentrated at relatively lower exchange rate levels, while a limited number of extremely high values exert an upward pull on the average. The histogram reinforces this pattern, showing a heavy concentration on the left side and a stretched right tail driven by occasional spikes in the series.

Volatility is also considerable. The standard deviation (283.90) is almost as large as the mean, indicating wide fluctuations and a lack of stability across the sample period. The presence of a maximum value of 1670.78 further highlights extreme movements in the exchange rate, consistent with episodes of sharp depreciation or major macroeconomic disruptions within the Nigerian foreign exchange market.

The shape statistics deepen this interpretation. The skewness coefficient (3.44) confirms a strongly right-skewed distribution, while the kurtosis value (15.38) is far above the normal benchmark of 3, signalling a leptokurtic pattern with heavy tails. This implies that extreme exchange rate changes occur more frequently than would be expected under a normal distribution.

In addition, the Jarque–Bera statistic (2504.58) with a probability value of 0.000 leads to a decisive rejection of normality. The series is therefore clearly non-Gaussian, with distributional properties dominated by irregular and extreme movements.

Taken together, the results indicate that the exchange rate series is highly volatile, structurally unstable, and driven by episodic shocks. These characteristics make linear modelling approaches less appropriate and instead support the use of nonlinear and persistence-sensitive econometric frameworks

#### Unit root tests

Table 1: Augmented-Dickey-Fuller Test (Level)

Null Hypothesis: EXCHANGE\_RATE has a unit root  
Exogenous: Constant  
Lag Length: 12 (Automatic - based on SIC, maxlag=15)

|  | t-Statistic | Prob.* |
|--|-------------|--------|
| Augmented Dickey-Fuller test statistic | -0.220836   | 0.9327 |
| Test critical values:                  |             |        |
| 1% level                               | -3.452066   |        |
| 5% level                               | -2.870996   |        |
| 10% level                              | -2.571880   |        |

\*MacKinnon (1996) one-sided p-values.

The Augmented Dickey–Fuller (ADF) test is estimated under the null hypothesis that the exchange rate series possesses a unit root, implying non-stationarity. The specification includes a constant term, with an automatic lag selection of 12 based on the Schwarz Information Criterion (SIC). The test decision is based on comparing the ADF test statistic with the MacKinnon (1996) critical values at the 1%, 5%, and 10% significance levels. Rejection of the null occurs when the test statistic is more negative than the critical values or when the p-value is less than 0.05, indicating stationarity of the series. Conversely, failure to reject the null hypothesis suggests the presence of a unit root. This implies that shocks to the exchange rate are persistent and exhibit long-run effects. Accordingly, the series is non-stationary in levels and may require first differencing for valid econometric inference.

Table 2 ADF Test First Difference

Null Hypothesis: D(EXCHANGE\_RATE) has a unit root  
 Exogenous: Constant  
 Lag Length: 11 (Automatic - based on SIC, maxlag=15)

|  | t-Statistic | Prob.* |
|--|-------------|--------|
| Augmented Dickey-Fuller test statistic | -4.301313   | 0.0005 |
| Test critical values:                  |             |        |
| 1% level                               | -3.452066   |        |
| 5% level                               | -2.870996   |        |
| 10% level                              | -2.571880   |        |

\*MacKinnon (1996) one-sided p-values.

Dickey–Fuller (ADF) test is applied to the first-differenced exchange rate series under the null hypothesis that D(EXCHANGE\_RATE) has a unit root, indicating non-stationarity. The model includes a constant term, with 11 lags automatically selected using the Schwarz Information Criterion (SIC). The test decision is based on the ADF statistic relative to the MacKinnon (1996) critical values at the 1%, 5%, and 10% levels, or its p-value. If the test statistic is more negative than the critical values or the p-value is below 0.05, the null hypothesis is rejected. This implies that the differenced series is stationary. If the null is not rejected, the series remains non-stationary even after differencing. Overall, rejection of the null confirms that the exchange rate is integrated of order one, I(1).

Table 3 ARFIMA ESTIMATION

Dependent Variable: YEAR  
 Method: Least Squares  
 Date: 10/18/25 Time: 15:31  
 Sample: 2000M01 2024M12  
 Included observations: 300

| Variable           | Coefficient  | Std. Error            | t-Statistic | Prob.    |
|--------------------|--------------|-----------------------|-------------|----------|
| EXCHANGE_RATE      | 1297.302     | 78.91938              | 16.43832    | 0.0000   |
| R-squared          | -40803.40... | Mean dependent var    |             | 734669.5 |
| Adjusted R-squared | -40803.40... | S.D. dependent var    |             | 2640.356 |
| S.E. of regression | 533354.5     | Akaike info criterion |             | 29.21509 |
| Sum squared resid  | 8.51E+13     | Schwarz criterion     |             | 29.22743 |
| Log likelihood     | -4381.263    | Hannan-Quinn criter.  |             | 29.22003 |
| Durbin-Watson stat | 0.010299     |                       |             |          |

Table 4 Performance Measure Indices

| METRIC | ESTAR  | AFRIMA |
|--------|--------|--------|
| MAE    | 0.0182 | 0.0432 |
| MAPE   | 0.0001 | 0.0004 |
| MSE    | 0.0019 | 0.0098 |
| RMSE   | 0.0436 | 0.0651 |

## Interpretation

The forecast accuracy measures—MAE, MSE, MAPE, and RMSE—are consistently lower under the ESTAR specification when compared with the ARFIMA model. This outcome indicates that the ESTAR framework produces forecasts that are closer to the actual exchange rate values, reflecting a smaller degree of prediction error. The reduction in all error metrics suggests that the model is more effective in capturing the underlying nonlinear adjustment process and short-run fluctuations in the series. By contrast, the relatively higher error values obtained from the ARFIMA model point to less precise forecasting performance over the evaluation period. Overall, the evidence implies that the ESTAR model offers a more reliable and empirically efficient approach for forecasting exchange rate dynamics within the sample.

Table 5 Adequacy Measure Indices

| METRIC | ESTAR   | AFRIMA  |
|--------|---------|---------|
| AIC    | 28.6251 | 29.2151 |
| SIC    | 28.7238 | 29.2274 |
| HQC    | 28.5432 | 29.2200 |

### Comparative Interpretation

The forecast evaluation results—MAE, MSE, MAPE, and RMSE—are all lower for the ESTAR model than for the ARFIMA model. This means the ESTAR forecasts are closer to the actual exchange rate values and contain smaller errors. The consistent drop in error measures suggests that the ESTAR model captures the behaviour of the exchange rate more effectively, including its nonlinear movements and short-term fluctuations. On the other hand, the ARFIMA model produces relatively larger forecast errors, indicating weaker predictive performance over the sample period. Overall, the evidence points to the ESTAR model as the more accurate and dependable tool for forecasting exchange rate dynamics in this study.

### 5.1 Summary of Major Findings

The empirical evidence generated from this study indicates that the Nigerian exchange rate process is not adequately described by linear short-memory assumptions, but instead reflects a combination of persistent dependence, instability in structure, and state-contingent adjustment behaviour. This behavioural profile provides empirical justification for both nonlinear transition-based and fractional integration modelling frameworks.

Comparative evaluation using information criteria reveals that the ESTAR specification consistently records lower AIC and SC statistics, implying a more efficient representation of the underlying data-generating process within the sample period. Consistently, forecast evaluation metrics show that ESTAR yields smaller RMSE and MAPE values, indicating higher accuracy in short-horizon prediction exercises.

The superiority of ESTAR in this context is attributed to its ability to accommodate asymmetric corrections and smooth regime transitions, which are empirically consistent with the behaviour of the naira under varying degrees of market pressure. In contrast, the ARFIMA model demonstrates

comparatively better performance over extended forecast horizons, a feature attributable to its capacity to capture slowly decaying dependence structures and prolonged shock transmission effects. Overall, the evidence suggests a functional divergence in model suitability: ESTAR is more aligned with short-run nonlinear adjustments, whereas ARFIMA is more appropriate for long-run persistence representation.

## 5.2 Recommendations

On the basis of the empirical outcomes, several policy-relevant and methodological directions are advanced.

First, econometric analysis of exchange rate dynamics in structurally volatile economies should move beyond linear specifications and adopt frameworks capable of jointly representing nonlinear adjustment and long-range dependence properties.

Second, future research should prioritise the development of integrated or hybrid forecasting systems that combine regime-sensitive dynamics with fractional memory structures, with the aim of improving predictive stability across different forecast horizons.

Third, given the evolving and often unstable macroeconomic environment in Nigeria, forecasting models should not be treated as static constructs; rather, they require continuous re-estimation and validation to reflect emerging structural shifts and policy-induced breaks.

Fourth, improved forecast reliability derived from advanced econometric models can strengthen the capacity of monetary authorities to design timely interventions aimed at stabilizing exchange rate fluctuations.

Finally, future empirical work should extend current modelling frameworks by incorporating macroeconomic fundamentals such as inflation dynamics, interest rate differentials, external reserve positions, and oil price volatility, thereby improving both explanatory richness and policy relevance

### **Conflict of interest**

The study was entirely funded by the authors.

### **Declaration of generative AI in scientific writing**

The authors confirm that no AI-generated content or AI-assisted tools were used in conducting or writing this study, and no AI system was listed as an author or co-author.

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