STATISTICS, NATIONAL PLANNING AND NATIONAL DEVELOPMENT

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INTRODUCTION:

The most principal objective that has been emphasized, re-emphasized and perhaps over-emphasized by our leaders in this country is that of conspicuous and evident development in almost all aspects of life. Since 1999 that we returned into democratic rule, there has never been a government without words and promises of comfort to the citizen. There arise some simple questions: Are we truly on the path to Development? How long will it take us to get developed? Are the changes in some aspects of the nation evident enough to be credited development? These are some of the quiet and worrisome questions disturbing patriotic citizen.

Consequently, this presentation appraises three components of National Statistical System (Data Suppliers, Producers and Users) with a view of emphasizing their linkage and interrelationship to Statistics, Planning and national development.

A. THREE COMPONENTS OF NATIONAL STATISTICAL SYSTEM (NSS) AND THEIR ROLES

1. **Data Suppliers:** These are the agents or mediums that provide data about themselves. They are often refers to as respondents. They include individuals, households, organizations, group of individuals within specified organizations, establishments etc.

Roles: They are to cooperate with the data collecting agencies in giving out accurate data requested for and in the form they are required.

2. **Data Producers:** These are bodies or agencies responsible for data collection from data suppliers. They are also responsible for the compilation of the collected data. These bodies are at the federal, state and local government levels.

2.1 At Federal Level: National Bureau of Statistics (NBS), Central Bank of Nigeria (CBN), National Population Commission (NPC), Department of Planning, Research and Statistics (DPRS) of Ministries and Parastatals, Research and Training Institutions, etc.

2.2 At State Level: The State Statistical Agencies (SSAs).

2.3 At Local Government Level: Local Government Statistical Units, Budget and Planning Unit of Local Governments.

Roles: They are to ensure continuous flow of high quality and accessible data for a wide range of economic and social subject matter of interest to users for many purposes.

3. **Data Users:** These are the clientele of data production system. They demand and use of data to achieve some specific objectives. Data users are diverse and their number is ever increasing. The main users include Planners, Policy and decision makers in government ministries and institutions, Politicians, Authorities in States and Local Governments, Researches and Academicians, NGOs, Private

Sector Organizations, The donor community, International Organizations, The media, The general public, etc.

Roles: They are to ensure that findings in the usage of the data are appropriately utilized to achieve national development.

B. THE FIVE FUNDAMENTAL STEPS LEADING TO DEVELOPMENT

The following five (5) fundamental steps are inevitably needful to achieve development:

1. **Problem Identification:** Problem(s) to be solved have to be clearly stated and well defined. The goal(s) of the exercise also must be well spelt out. This may require identification and involvement of various expertises.

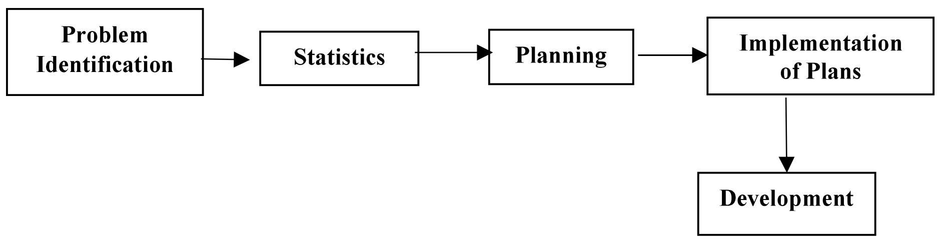
2. **Statistics:** At this stage, relevant information/data necessary to achieve the set goal(s) are gathered for effective planning. This has to be done thoroughly as it is bone which holds the flesh of planning together.

3. **Planning:** Planning has been seen as an examination of past trends to extrapolate into the future. It is inseparable from statistics as it utilizes information statistics provides. It is a necessary tool used by many governments and organizations to set their visions, missions, goals, and effective means of realizing development through effective direction and control. Well developed plans outline various steps and make adequate provision for all identified necessary resources to achieve the developmental objective.

4. **Implementation of Plans:** This is the most critical stage that leads directly to development. If our statistics and planning are thoughtfully done but lack implementation strategies, there cannot be development. With good planning, implementation cannot go wrong. It requires a holistic monitoring and objective examination of the various steps outlined under planning as they are being executed.

5. **Development:** Development is said to have occurred or been achieved if the provided workable solution(s) are bringing forth desired results. That is, a fair measure of success in the goal is achieved evidently.

FIGURE 1: The Five Fundamental Steps Leading to Development



Source: Self Motivated

C. PLANNING WITH AND WITHOUT STATISTICS

C.1 Let consider this historical incidence:

The Universal Free Primary Education Programme originated, planned and executed by late Chief Obafemi Awolowo, the then Premier Minister of Western Nigeria, recorded success because the five (5) fundamental steps leading to development were followed.

1. **Problem Identification:** He identified the need of his people in 1951 and decided to pursue it very rigorously by providing free education for the Primary School Students.
2. **Statistics:** With the assistance of United Nation, data were collected (census) on number of babies less than one (1) in 1951 and beyond. This was used to estimate the total number of children who would be ready for school in 1955.
3. **Planning:** The total number of children was used to plan for number of Primary Schools, teachers, desks and chairs, etc. Every taxable adult male

was asked to pay a development levy of five (5) shillings for the running of the programme.

4. **Implementation of Plans:** The set plans were carefully followed.
5. **Development:** The programme recorded great success.

On the contrast, the Premier of Eastern Part of the Nigeria, Dr. Azikwe, saw what was going on and decided to start the same programme in 1956. The programme was started as determined but could not last more than one (1) year. Why? **No Statistics, bad planning and implementation, and no development (evidence of achievement) at all.**

C.2 Let consider Lagos state in the recent past.

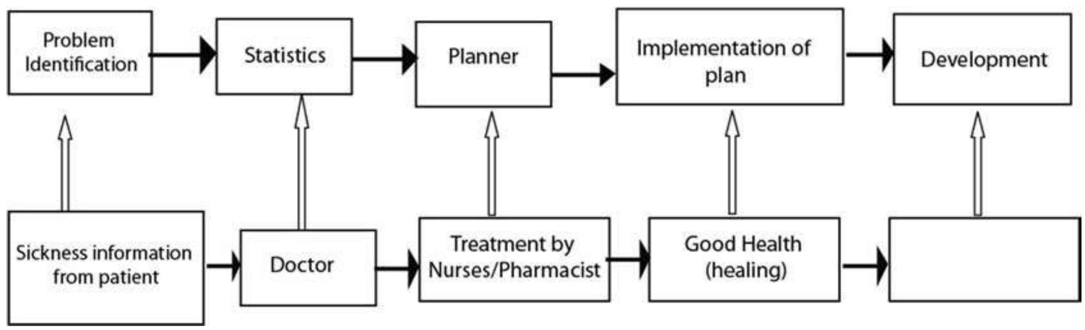
The past Governors of Lagos State played key and active roles in the development of the state. The conspicuous changes in State is consequential on statistics obtained past leaders or experience gathered by them while serving at one position or the other in the state before being elected in as governors. For example, Governor Babatunde Raji Fashola served under Governor Bola Ahmed Tinubu and this enabled him to be well informed of the strategy to implement to achieve development.

Comments: Planning without Statistics is like a ship **without Rudder and Compass**. Without a Rudder, a ship is unable to exert control over its course. Without a Compass, a ship suffers and is unable to determine its course.

D. NATIONAL PLANNING WITH INCORRECT STATISTICS

Let examine a medical doctor and a patient: A doctor cannot diagnose a patient effectively without collecting adequate information about the nature and the severity of the sickness of his/her patient. He has to be very thoughtful in his/her assessment to give correct prescriptions of medicine.

FIGURE 2: The Five Fundamental Steps Leading to Development applied a medical doctor and a patient



Source: *Self Motivated.*

Now, suppose instead of using the correct information, he/she uses incorrect information. Perhaps, the doctor fails to examine the patient at all and just diagnose the patient. The following problems may arise:

1. Continuous visits to hospital
2. Long period of sickness
3. Long suffering / Deformity
4. Death

1. **Continuous visits to hospital:** In an attempt to make wrong things right, many often revisit collected wrong data, alter and torture them thinking that when they confess and repent they will behave well to have desired results. Unfortunately, the data are still wrong if not worse.
2. **Long period of sickness:** When wrong data are used in development planning, there will be no development. The problem(s) to be solved continue to raise its ugly head and become perpetual.
3. **Long suffering / Deformity:** When prescribed wrong drugs are used to cure a disease continuously, it can lead to long suffering and deformity. The use of

incorrect data in planning will cause other problems rather than providing solution. Thus, problems are on the increase.

4. **Death:** This is the last bus stop. Incorrect information used in planning can lead to collapse of a system, society, state etc. at long run.

Sources of Incorrect Statistics: Assumption, guessed value, altered data etc.

Comments: All these are avoidable provided we all resolve to give out facts, correct information, and reliable data needed for the development of our constituencies, state and the nation.

E. NATIONL PLANNING WITH INADEQUATE STATISTICS

Planners are not always patients enough to get all necessary data collected as this may take long time. Most planners are not prepared to delay all planning activities until inadequacies in data are overcome. They often feel that development planning cannot await the building of a comprehensive system of statistics. To them, it does not make good sense to delay action on the ground of absence of good statistics. If good statistics are regarded as an absolute prerequisite for planning, the day when serious economic planning will commence is not yet in sight.

Comments:

1. The use of experts and personnel who were sufficiently well acquainted with the problem has been suggested to make reasonably contribution to an analysis of the situation of inadequate statistics. One fundamental danger associated with this is that, planning is then precipitated on the opinion of the experts and their guesses.
2. The consequence of planning with inadequate data often results into errors, creation of numerous bottlenecks which prevent completion of plans and a serious waste of resources.
3. When data are inadequate, it has been strongly suggested to start planning on the limited scale permitted by the existing data because of the menaces inadequate data can cause.

F. ESSENTIALS OF A GOOD DEVELOPMENT PLANNING:

1. Existence of facts (reliable data) about the past and current activities especially on natural and human resources. The usefulness of these data depends on its accuracy, coverage, form and timeliness. Reliable data are primary requirement for development planning and without these there cannot be effective planning.
2. Reliable data are not to be used as guidelines in preparations of plans but are to be embraced absolutely in their provision of truth and direction.
3. Existence of a body containing a group of professionals who knows the significance of facts (reliable data) and ready to make use of the same for planning.

CONCLUSION

The problems associated with the use of any other statistics apart those provided by reliable data in planning are often more complex than the original problems themselves. To achieve progressive and meaningful development in the nation, it is fundamental to know that planning and statistics are bedrocks. Planning without Statistics is like a ship without Rudder and Compass while planning with incorrect statistics leads to repetition of issues and revolving round the same stage of development.

EXCELENT DEVELOPMENT = f (RELIABLE STATISTICS, EFFECTIVE PLANNING) + E

ON THE DEVELOPMENT OF SYSTEM UPDATES FROM SYMMETRIC NONLINEAR STATE-SPACE MODEL

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***ABSTRACT**:*

A new class of nonlinear Time Series model called Symmetric Nonlinear State-Space Model (SNSSM) was developed using Kalman filter technique. Some important components for estimating the SNSSM were successfully derived. These components are estimate of the state, prediction and updating equations which in turn served as system updates of the developed model.

KEY WORDS: State-Space model, Kalman filter, Predicted State, Kalman Gain, Filter State Covariance

1.0 INTRODUCTION

A State-Space model consists of a transition/state equation and a measurement equation. The transition equation formulates the dynamics of the state variables and the measurement equation relates the observed variables to the unobserved transition vector. The state vector can contain trend, seasonal, cyclical and regression components together with an error term also known as innovation. However, the stochastic behavior of the state variable, its association to the data and the covariance structure of the errors depend on parameters that are almost always unknown,(Sascha, 2009).

The goal of the State-Space model is to infer information about the states, given the observations/measurements, as new information arrives. A well-known algorithm for carrying out this procedure is known as Kalman filter (Anons, 2020). A Kalman filter is an optimal recursive estimator which infers parameters of interest from indirect, inaccurate and uncertain measurements. It is recursive so that new measurements can be processed as they arrive (Lindsay, 2017). Thus, Kalman filter is a set of recursion equations for determining the optimal estimates of the state vector given information available at time t. The filter consist of two sets of equations: prediction equations and updating equations (Shyam*et al.*, 2015). Kalman filter and State-Space model formulation together, provide a very powerful tool for the recursive treatment of dynamic systems, (Amold*et al.*, 2008).

The purpose of filtering is to update our knowledge of the state vector as soon as a new observation becomes available (Raphael, 2016) and (Robert*et al.,* 2016). *Note: in this research, we refer to the original linear State-Space models as Classical State-Space Models (CSSM)*. However, the CSSMs are related to hidden Markov Models; the distinction between the two is that the underlying Markov process (the State) is continuous in the former and discrete in the later.

Researchers from different fields across the world are contributing immensely to the development of the State-Space/Kalman filter models both theoretically and emphatically. Theoretically for example, Hamilton (1994) gave a State-Space representation of a linear dynamic system. The wisdom behind this representation is to capture all the dynamics of the unobserved measurement vector $\mathbf{Y}_t$ in terms of unobserved state vector, *say* $\mathbf{X}_t$. He imposed some restrictions on the parameters of the measurement vector that would ensure the stability of the process. He further proposed a general form of linear State-Space model with a constant parameter; and he derived an optimal forecast of the system via a well-established result for normal variables; all the needed components of the linear Kalman filter algorithm have been derived. The major limitation of the Hamilton's work on the State-Space modeling and Kalman filtering is the assumption of linearity, the frame work was designed to handle a linear or approximately linear system, and therefore, it cannot handle any nonlinear system.

It was observed that the CSSMs had a strong limitation of linearity in its state equation: the model frame work was designed to handle linear or approximately linear systems alone; and majority of real life situations followed nonlinear system! As an improvement, Raphael (2016), proposed a Modified State-Space Model (MSSM). The MSSM allows for the introduction of nonlinear function: Logistic Smooth Transition Autoregressive (LSTAR) model in the state equation of the CSSM and this transformed the CSSM from linear to nonlinear model. The basic limitation of the Raphael's work is the asymmetric behaviour of its state equation as claimed/stated in the Liew (2002) and Olukayode's (2010) arguments.
We seek to address the above limitation by proposing a new class of nonlinear Time Series Model in State-Space form with symmetric nonlinear state equation which is expected to model any symmetric nonlinear series.

2.0 METHODOLOGY

2.1 CLASSICAL STATE-SPACE MODELS (CSSM)

The State-Space model is a system of two equations as given in (1) and (2):

$$Y_{t+1} = HX_{t+1} + V_{t+1} \tag{1}$$

$$X_{t+1} = \psi X_t + \omega_{t+1} \tag{2}$$

The first equation called measurement (observation) equation, describes the relation between the observed Time Series, Y_{t+1} and the (possibly unobserved) state X_{t+1}. The second equation called the (State) transition equation, describes the evolution of the state variables as being driven by the stochastic process of innovations ω_t.

The terms V_{t+1} and ω_{t+1} are the measurement and the process noise respectively. Usually one assumes normal innovations, such that $V_{t+1} \sim N\left(0, \sigma_V^2\right)$ and

$\omega_{t+1} \sim N\left(0, \sigma_{\omega}^{2}\right)$. Similarly, these error terms V_{t+1} and ω_{t+1} are assumed to be serially independent and independent of each other at all time periods as well as uncorrelated with the initial state. The role of V_{t+1} in the output equation (1) is to account for any uncertainty in the measurement of the output (i. e. it tells us how much or little we can trust the equation).

The parameter H is an unknown that links the unobservable variables and regression effects of the state equation with the observation equation, ψ is an unknown parameter that determines how the observation and state equations evolve (change) in time.

Moreover, one can decides to look at (1) and (2) as vectors/matrices which can subsequently be written as

$$\underset{(m\times 1)}{\mathbf{Y}_{t+1}} = \underset{(m\times n)(n\times 1)}{\mathbf{H}\,\mathbf{X}_{t+1}} + \underset{(m\times 1)}{\mathbf{V}_{t+1}} \tag{3}$$

$$\underset{(n\times 1)}{\mathbf{X}_{t+1}} = \underset{(n\times n)(n\times 1)}{\boldsymbol{\psi}\,\mathbf{X}_{t}} + \underset{(n\times 1)}{\boldsymbol{\omega}_{t+1}} \tag{4}$$

In this case, $E\left(\mathbf{V}_{t+1}\mathbf{V}'_{t+1}\right) = \underset{m\times m}{\mathbf{L}}$ and $E\left(\boldsymbol{\omega}_{t+1}\boldsymbol{\omega}'_{t+1}\right) = \underset{n\times n}{\mathbf{Z}}$

However, the matrices $\mathbf{V}_{t+1}$ and $\boldsymbol{\omega}_{t+1}$ are not really implemented/included in evaluations of (3) and (4) because they are assumed to be random innovations with zero mean, but instead are always used in determination of any information about the observation and state error covariance matrices $\mathbf{L}$ and $\mathbf{Z}$.

The system matrices $\mathbf{H}$ and $\boldsymbol{\psi}$ are in general vary with time, but would not change with respect to states/transitions. In most cases, they are regarded as constants. $\boldsymbol{\psi}$ Contains the coefficients of the transition terms in the state equation (4) and $\mathbf{H}$ performs similar task in the measurement equation (3).

The above system of equations has a basic limitation in Kalman filter theory as it is linear in parameter. Hence this linearity problem makes it inadequate to handle any nonlinear system; and majority of real life problems in our modern world followed nonlinear system. This necessitates the need for a nonlinear model to overcome this great challenge.

The Extended Kalman Filter (EKF) proposed by Stanley F. Schmidt (1926-2015) and Uncented Kalman Filter (UKF) developed by Jeffrey K. Uhlmann (1965-) were part of the efforts initiated to address the linearity problem of the CSSM. Additionally, Raphael (2016) proposed a modification of the CSSM with the aid of Smooth Transition Autoregressive (STAR) model. This modification is also regarded as a good development in the areas of Time Series as well as Kalman filter literature.

The STAR model is a nonlinear Time Series model that allows for state-dependent or regime-switching behavior. For example, changes in government policy may instigate a change in regime. With a view to modeling this type of Time Series data, a family of Smooth Transition Autoregressive (STAR) models has been proposed by Terasvirta (1994).

The data-generating process to be modeled is viewed as a linear process that switches between numbers of regimes according to some rules. It has been assumed that there is a continuum of switches, that is, there is a smooth transition from one extreme regime to the other. It consists of three stages: specification, estimation and evaluation; Iquebal (2016). The STAR model of order *p* is given as,

$$x_{t+1} = G\left(x_{t+1};\theta\right) = \varphi_1 x_t \left[1 - G\left(X_t;\gamma,c\right)\right] + \varphi_2 x_t G\left(X_t;\gamma,c\right) + \omega_{t+1} \qquad (5)$$

where

$G\left(X_t;\gamma,c\right)$ is bounded between 0 and 1, which realizes the "smooth transition" between regimes dynamically rather than an abrupt/sudden jump from one regime to the other. C is the threshold value and the parameter γ determines the speed and smoothness of the transition.

Note that when $\gamma \to \infty$, $G\left(X_t;\gamma,c\right) = 1$ then the propose model becomes linear which also happens when $\gamma \to 0$ .Transition Function, $G\left(X_{t-d};\gamma,c\right)$ causes the nonlinear dynamics in the model, and can have different functional choices. For each choice of transition function, we get different regime switching behavior. The most common choices are logistic and exponential forms as given in equation (6) and (7) respectively.

$$G\left(X_{t-d};\gamma,c\right)=\frac{1}{1+\exp[-\gamma\left(X_{t-d}-c\right)]} \qquad (6)$$

$$G\left(X_{t-d};\gamma,c\right)=1-\exp[-\gamma\left(X_{t-d}-c\right)^{2}] \qquad (7)$$

Note: If (6) is considered as $G\left(X_{t-d};\gamma,c\right)$ in (5), then (5) is called Logistic Smooth Transition Autoregressive (LSTAR) Model. Similarly, if (7) is considered as $G\left(X_{t-d};\gamma,c\right)$ in (5), then (5) is called Exponential Smooth Transition Autoregressive (ESTAR) Model.

Comparing between the two transition functions: (6) and (7), the logistic is changing monotonically with X_t, while the exponential is changing symmetrically at C with X_t. To visualize the asymmetric and symmetric features of the two transition functions: logistic and exponential, see figure 1.

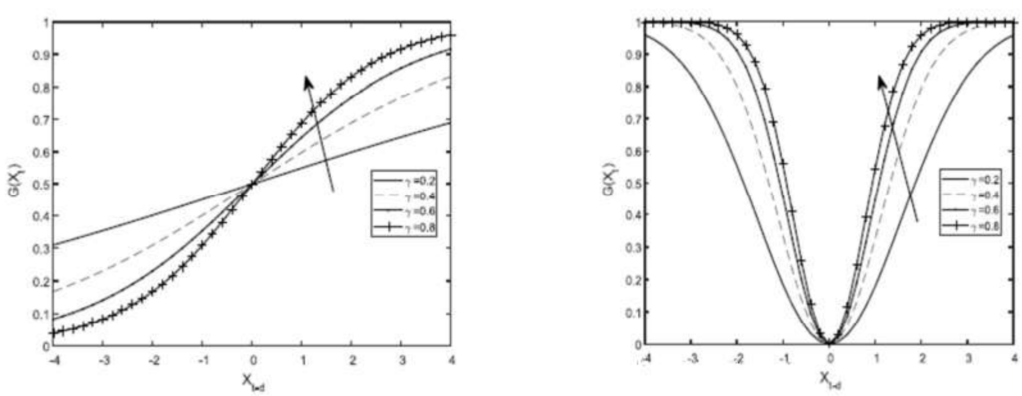

Figure 1: Logistic and Exponential transition functions with varying values of gamma (γ).

3.3 DERIVING THE ESSENTIAL COMPONENTS OF THE PROPOSE SNSSM MODEL

The essential components of the proposed model are Predicted State (PS), and optimal forecast (system update). These would be derive sequentially.

3.3.1 DERIVING THE PREDICTED STATE

Note: equation (7) is known as exponential transition function; if appeared in the STAR model, the result is called an Exponential-STAR (ESTAR) model as in (5) above which is also symmetric in nature. The symmetrical property of equation (5) makes it capable of handling/modeling any symmetric nonlinear series such as exchange rate.

Substituting (7) in (5) we have

$$\begin{aligned} X_{t+1} &= \psi_1 X_t \left(1-\left(1-e^{-\gamma\,(X_t - C)^2}\right)\right) + \psi_2 X_t \left(1-e^{-\gamma\,(X_t - C)^2}\right) + \omega_{t+1} \\ &= \psi_1 X_t e^{-\gamma\,(X_t - C)^2} + \psi_2 X_t \left(1-e^{-\gamma\,(X_t - C)^2}\right) + \omega_{t+1} \end{aligned} \tag{8}$$

Now, the focus is to get the predicted state estimate which is an important component in the development of the Kalman filter algorithm. To achieve that we differentiate (8) with respect to the current state,

$$\begin{aligned} \frac{\partial X_{t+1}}{\partial X_t} = {} & \psi_1 X_t \left(-2\left(\gamma\left(X_t - C\right)\right)\right) e^{-\gamma\,(X_t - C)^2} + \psi_1 e^{-\gamma\,(X_t - C)^2} \\ & + \psi_2 X_t \left(2\gamma\left(X_t - C\right) e^{-\gamma\,(X_t - C)^2}\right) + \psi_2 \left(1-e^{-\gamma\,(X_t - C)^2}\right) \end{aligned} \tag{9}$$

Which gives

$$\begin{aligned} = {} & -2\psi_1 \gamma X_t \left(X_t - C\right) e^{-\gamma(X_t - C)^2} + \psi_1 e^{-\gamma(X_t - C)^2} + 2\psi_2 \gamma X_t \left(X_t - C\right) e^{-\gamma(X_t - C)^2} \\ & + \psi_2 \left(1-e^{-\gamma(X_t - C)^2}\right) \end{aligned} \tag{10}$$

expanding and equation to zero, we have

$$\left(\psi_1-\psi_2\right)\left(1-G\left(X_t\right)\right)\left(-2\gamma X_t^2+2\gamma C X_t+1\right)+\psi_2=0 \tag{11}$$

Simplifying further we have

$$-2\gamma\left(X_t-\frac{C}{2}\right)^2=\frac{-\psi_2}{\left(\psi_1-\psi_2\right)\left(1-G\left(X_t\right)\right)}-\frac{\left(2+\gamma C^2\right)}{2} \tag{12}$$

Taking the L. C. M. of the R. H. S. of (12) and simplifying further gives

$$\hat{X}_t=\frac{-C\pm\sqrt{\dfrac{2\psi_2+\left(\gamma C^2+2\right)\left(\psi_1-\psi_2\right)\left(1-G\left(X_t\right)\right)}{\gamma\left(\psi_1-\psi_2\right)\left(1-G\left(X_t\right)\right)}}}{2} \tag{13}$$

Note that (13) is the predicted state estimate with the following regularity conditions:

$C\geq 0,\ 0<\gamma<\infty,\ \psi_1>0,\ \psi_2>0,\ \psi_1>\psi_2,$ and $0<G\left(X_t\right)<1.$

Recall that $G\left(X_t\right)=G\left(X_{t-d};\gamma,c\right)=1-\ell^{-\gamma\left(X_{t-d}-c\right)^2}$

One would ask whether the predicted state: equation (13) inherited the symmetrical feature of the exponential transition function given in (7) or not? To answer this, we need to visualize (13) to see if it is really symmetric; even though it will give us two graphs because of the presence of $[\pm]$ signs, the two graphs are given in figures2a and 2b.

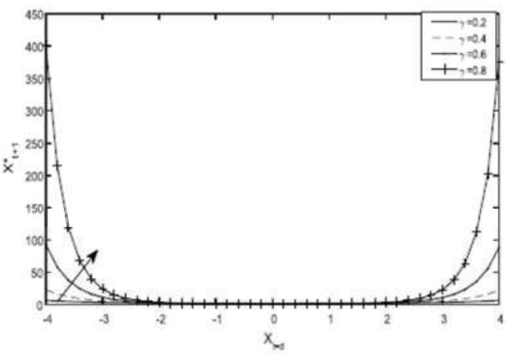

Figure 2a: Estimate of predicted state with varying values of Gamma (γ) for positive sign.

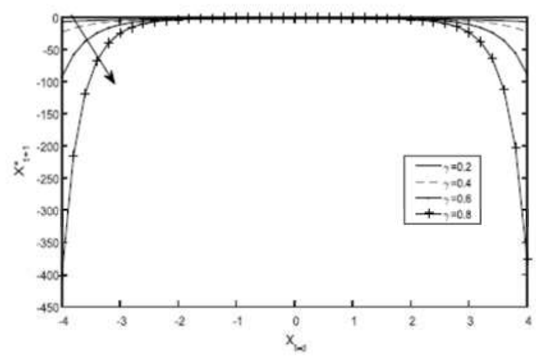

Figure 2b: Estimate of predicted state with varying values of Gamma (γ) for negative sign.

Figure 2a and 2b clearly show that the symmetrical properties of equation (7) were inherited by (13); hence the system of equation: equations: (1) and (2) is now nonlinear as well as symmetric. The system is therefore capable of handling/modeling any symmetric nonlinear series such as exchange rate.

3.4 DERIVATION OF THE OPTIMAL FORECAST

We can get the optimal forecasts from an established marginal and conditional property of the multivariate normal distribution; [see, Rencher (2002), Timm (2002) and Hamilton (1994) for more details]. It should be noted that the easiest way to derive the recursive equations (Kalman recursion) is by using normality assumption. The wisdom behind using the normality assumption is indeed for robustness; that is the solution that may be obtain is optimal in the class of all possible solutions (be it linear and/or nonlinear). We can recall our system of equations (our State-Space equations given in (3) and (4) above) and let $\mathbf{Y}_{t+1}$ and $\mathbf{X}_{t+1}$ denote $(m \times 1)$ and $(n \times 1)$ subvectors respectively whose joint normal distribution is given as

$$\mathbf{J} = \begin{pmatrix} \mathbf{Y}_{t+1} \\ \mathbf{X}_{t+1} \end{pmatrix} \sim \mathbf{N}_{n+r}\left(\begin{pmatrix} \boldsymbol{\mu}_y \\ \boldsymbol{\mu}_x \end{pmatrix}, \begin{pmatrix} \boldsymbol{\tau}_{yy} & \boldsymbol{\tau}_{yx} \\ \boldsymbol{\tau}_{xy} & \boldsymbol{\tau}_{xx} \end{pmatrix} \right) \qquad (14)$$

Similarly, whenever $\mathbf{X}_{t+1}$ becomes available, we use the above normality property to update the distribution of $\mathbf{Y}_{t+1}$.

Hence, the conditional distribution of $\mathbf{Y}_{t+1}$ given $\mathbf{X}_{t+1}$ is also multivariate normal with mean $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\tau}$; i. e. $\mathbf{Y}_{t+1} \mid \mathbf{X}_{t+1} \sim \mathbf{N}(\boldsymbol{\mu}, \boldsymbol{\tau})$; where

$$\boldsymbol{\mu} = \boldsymbol{\mu}_{y} + \boldsymbol{\tau}_{yx}\boldsymbol{\tau}_{xx}^{-1}\left(\mathbf{X}_{t+1} - \boldsymbol{\mu}_{x}\right) \tag{15}$$

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{yy} - \boldsymbol{\tau}_{yx}\boldsymbol{\tau}_{xx}^{-1}\boldsymbol{\tau}_{xy} \tag{16}$$

Now, the optimal forecast value of $\mathbf{Y}_{t+1}$ conditional on having known $\mathbf{X}_{t+1}$ is given as

$$E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t+1}\right) = \boldsymbol{\mu}_{y} + \boldsymbol{\tau}_{yx}\boldsymbol{\tau}_{xx}^{-1}\left(\mathbf{X}_{t+1} - \boldsymbol{\mu}_{x}\right) \tag{17}$$

It is important to point out that the quantity $\boldsymbol{\tau}_{yx}\boldsymbol{\tau}_{xx}^{-1}$ is called a matrix of regression coefficient as it facilitate the role of relating the $E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t+1}\right)$ to $\mathbf{X}_{t+1}$; Rencher, (2002). and the **MSE** of the forecast is given as

$$E\left\{\left(\mathbf{Y}_{t+1} - \boldsymbol{\mu}\right)\left(\mathbf{Y}_{t+1} - \boldsymbol{\mu}\right)' \mid \mathbf{X}_{t+1}\right\} = \boldsymbol{\tau}_{yy} - \boldsymbol{\tau}_{yx}\boldsymbol{\tau}_{xx}^{-1}\boldsymbol{\tau}_{xy} \tag{18}$$

Now, for us to apply equations (15) through (18) in our developed methodology, we need to find $\boldsymbol{\mu}_{y}$, $\boldsymbol{\mu}_{x}$, $\boldsymbol{\tau}_{yy}$, $\boldsymbol{\tau}_{yx}$, $\boldsymbol{\tau}_{xy}$ and $\boldsymbol{\tau}_{xx}$

Now, we can let

$$\boldsymbol{\mu}_{y} = E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right) = \mathbf{H}\hat{\mathbf{X}}_{t+1|t} \tag{19}$$

Since we already know that $E\left(\mathbf{V}_{t+1}\right) = 0$

Considering (3) and (19), we can write the **forecast error** as

$$\left\{\mathbf{Y}_{t+1} - E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right)\right\} = \left(\mathbf{H}\mathbf{X}_{t+1} + \mathbf{V}_{t+1}\right) - \left(\mathbf{H}\hat{\mathbf{X}}_{t+1|t}\right)$$

which resulted to

$$\left\{\mathbf{Y}_{t+1} - E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right)\right\} = \mathbf{H}\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right) + \mathbf{V}_{t+1} \tag{20}$$

where $\mathbf{V}_{t+1}$ is as previously defined and it is independent of both $\mathbf{X}_{t+1}$ and $\hat{\mathbf{X}}_{t+1|t}$.

We can now write the **conditional variance** of the forecast error given in (20) as

$$\boldsymbol{\tau}_{yy} = E\left\{\left(\mathbf{Y}_{t+1} - E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right)\right)\left(\mathbf{Y}_{t+1} - E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right)\right)' \mid \mathbf{X}_{t}\right\} \tag{21}$$

by substituting (20) into (21), we have

$$\boldsymbol{\tau}_{yy} = E\left\{\left(\mathbf{H}\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right) + \mathbf{V}_{t+1}\right)\left(\mathbf{H}'\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)' + \mathbf{V}'_{t+1}\right)\right\}$$

by expanding and setting all cross products to zero, we have

$$\boldsymbol{\tau}_{yy} = \mathbf{H}E\left(\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)'\right)\mathbf{H}' + E\left(\mathbf{V}_{t+1}\mathbf{V}'_{t+1}\right)$$

which finally gives

$$\boldsymbol{\tau}_{yy} = \mathbf{Q}_{t+1|t} = \mathbf{H}'\mathbf{F}_{t+1|t}\mathbf{H} + \mathbf{L} \tag{22}$$

Furthermore, we can write the **conditional covariance** between the errors in forecasting the observation vector (20) and the state vector as

$$\boldsymbol{\tau}_{yx} = E\left\{\left(\mathbf{Y}_{t+1} - E\left(\mathbf{Y}_{t+1} \mid \mathbf{X}_{t}\right)\right)\left(\mathbf{X}_{t+1} - E\left(\mathbf{X}_{t+1} \mid \mathbf{X}_{t}\right)\right)' \mid \mathbf{X}_{t}\right\} \tag{23}$$

by substituting (20) into (23), we have

$$\boldsymbol{\tau}_{yx} = E\left\{\left(\mathbf{H}\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right) + \mathbf{V}_{t+1}\right)\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)' \mid \mathbf{X}_{t}\right\}$$

by expanding and setting all cross products to zero, we have

$$\boldsymbol{\tau}_{yx} = \mathbf{H}E\left(\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)\left(\mathbf{X}_{t+1} - \hat{\mathbf{X}}_{t+1|t}\right)'\right)$$

therefore

$$\boldsymbol{\tau}_{yx} = \mathbf{H}\mathbf{F}_{t+1|t} \tag{24}$$

Now, we can easily write an updated form of $\mathbf{J}$ by substituting the functional forms of $\boldsymbol{\mu}_y$, $\boldsymbol{\mu}_x$, $\boldsymbol{\tau}_{yy}$, $\boldsymbol{\tau}_{yx}$, $\boldsymbol{\tau}_{xy}$ and $\boldsymbol{\tau}_{xx}$ into (14) as

$$\mathbf{J} = \begin{pmatrix} \mathbf{Y}_{t+1} \mid \mathbf{X}_t \\ \mathbf{X}_{t+1} \mid \mathbf{X}_t \end{pmatrix} \square\ \mathbf{N}_{n+r}\left(\begin{pmatrix} \mathbf{H}\hat{\mathbf{X}}_{t+1|t} \\ \hat{\mathbf{X}}_{t+1|t} \end{pmatrix}, \begin{pmatrix} \mathbf{H}'\mathbf{F}_{t+1|t}\mathbf{H} + \mathbf{L} & \mathbf{H}\mathbf{F}_{t+1|t} \\ \mathbf{H}'\mathbf{F}_{t+1|t} & \mathbf{F}_{t+1|t} \end{pmatrix}\right) \tag{25}$$

Note that everything about the previous/pass that is needed for the determination of the future values of the observation vector $\mathbf{Y}_{t+1}$ have been summarized and captured by $\mathbf{X}_t$.

With this; we can generalize using the facts from (15) and (16) that $\mathbf{X}_{t+1} \mid \mathbf{X}_{t+1} = \mathbf{X}_{t+1} \mid \mathbf{Y}_{t+1}$ is distributed $\mathbf{N}_r\left(\hat{\mathbf{X}}_{t+1}, \mathbf{F}_{t+1}\right)$, where $\hat{\mathbf{X}}_{t+1}$ is given below

$$\hat{\mathbf{X}}_{t+1|t+1} = \hat{\mathbf{X}}_{t+1|t} + \mathbf{H}\mathbf{F}_{t+1|t}\left(\mathbf{H}'\mathbf{F}_{t+1|t}\mathbf{H} + \mathbf{L}\right)^{-1}\left(\mathbf{Y}_{t+1} - \mathbf{H}\hat{\mathbf{X}}_{t+1|t}\right) \tag{26}$$

Note: we can write (26) as

$$\hat{\mathbf{X}}_{t+1|t+1} = \hat{\mathbf{X}}_{t+1|t} + \mathbf{K}_{t+1}\left(\mathbf{Y}_{t+1} - \mathbf{H}\hat{\mathbf{X}}_{t+1|t}\right) \tag{27}$$

Note: Equation (27) is called blending equation; which is obtained by expressing the estimate of the current state $\hat{\mathbf{X}}_{t+1}$ as a linear combination of predicted state $\hat{X}_t$ plus the difference between the actual measurements Y_{t+1} and the predicted state $\hat{X}_t$

multiply by some gain factor called the Kalman gain K_{t+1}. The quantity $\left(\mathbf{Y}_{t+1} - \mathbf{H}\hat{\mathbf{X}}_{t+1|t}\right)$ is called a correction term.

So, the whole idea is if a predicted state is really good, it will be equal to the actual measurement, so the correction term will be zero; and a predicted state will be exactly the estimated state (that is a perfect prediction). On the other hand, if a predicted state is not so good, then the correction term will return a value greater than zero, and the role of the Kalman gain to tellhow much information is needed from the actual measurement to correct a predicted state estimate to get a final more accurate state estimate.

Note: The **MSE** of the forecast given in (18) can now be updated as

$$\mathbf{F}_{t+1|t+1} = \mathbf{F}_{t+1|t} - \mathbf{H}\mathbf{F}_{t+1|t}\left(\mathbf{H}'\mathbf{F}_{t+1|t}\mathbf{H} + \mathbf{L}\right)^{-1}\mathbf{H}'\mathbf{F}_{t+1|t} \qquad (28)$$

which can be written (28) as

$$\mathbf{F}_{t+1|t+1} = \mathbf{F}_{t+1|t} - \mathbf{K}_{t+1}\mathbf{H}'\mathbf{F}_{t+1|t} \qquad (29)$$

Hence, (29) finally becomes

$$\mathbf{F}_{t+1|t+1} = \left(\mathbf{I} - \mathbf{K}_{t+1}\mathbf{H}'\right)\mathbf{F}_{t+1|t} \qquad (30)$$

4.0 FINDINGS AND CONCLUSION

4.1 FINDINGS: KEY EQUATIONS OF THE PROPOSED SNSSM

As stated earlier, the Kalman filter consists of two sets of equations: prediction equations and updating equations. These were derived in the previous section and presented here for crystal clear.

Prediction equations:

$$\underset{(n\times 1)}{\mathbf{X}_{t+1|t}} = \underset{(n\times n)}{\boldsymbol{\psi}} \underset{(n\times 1)}{\mathbf{X}_{t|t}}$$
$$\underset{(n\times n)}{\mathbf{F}_{t+1|t}} = \underset{(n\times n)}{\boldsymbol{\psi}} \underset{(n\times n)}{\mathbf{F}_{t|t}} \underset{(n\times n)}{\boldsymbol{\psi}'} + \underset{(n\times n)}{\mathbf{Z}} \tag{31}$$

where $\mathbf{X}_{t|t}$ is as given in (13)

Updating equations:

$$\underset{(n\times 1)}{\mathbf{X}_{t+1|t+1}} = \underset{(n\times 1)}{\mathbf{X}_{t+1|t}} + \underset{(n\times m)}{\mathbf{K}_{t+1}} \left(\underset{(m\times 1)}{\mathbf{Y}_{t+1}} - \underset{(m\times 1)}{\mathbf{Y}_{t+1|t}} \right)$$
$$\underset{(n\times n)}{\mathbf{F}_{t+1|t+1}} = \left(\underset{(n\times n)}{\mathbf{I}} - \underset{(n\times m)}{\mathbf{K}_{t+1}} \underset{(m\times n)}{\mathbf{H}'} \right) \underset{(n\times n)}{\mathbf{F}_{t+1|t}} \tag{32}$$

where $\mathbf{Y}_{t+1|t}$ and $\mathbf{K}_{t+1}$ were given in (19) and (26) respectively.

4.2 CONCLUSION

It is very important to note that (31) and (32) are the system updates developed SNSSM. Running the algorithm at $t=0$ gives one complete Kalman filter's iteration also known as the Kalman recursion. Repeating the same process at $t=1,2,...,T$ (where T is the number of observations) yields the Kalman recursions in Kalman filter literature.

It is customary to initializes/starts the filter with some arbitrary values (a prior information) say $\boldsymbol{\mu}_{0|0}$ and $\mathbf{F}_{0|0}$, use it to predicts $\mathbf{Y}_{1|0}$ and $\mathbf{Q}_{1|0}$; whenever the observation $\mathbf{Y}_1$ becomes available, it will be use in the updating equations and compute $\boldsymbol{\mu}_{1|1}$ and $\mathbf{F}_{1|1}$ which at the same time considered as prior for the subsequent observation. This process completes one Kalman recursion. It is very important to note that the effect of initial prior $\mathbf{\mu}_{0|0}$ and $\mathbf{F}_{0|0}$ is decreasing with the increase of time t. This is also consistent with (Yu, 2015), (Tsay, 2010) and (Shyam e*t al.*, 2015).

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