USE OF HYBRIDIZED LINEAR-RATIO ESTIMATOR FOR SUCCESSIVE SAMPLING ON TWO OCCASIONS OVER NIGERIA POPULATION CENSUS

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ABSTRACT

Successive sampling is a known technique that can be used in longitudinal surveys to estimate population parameter. Therefore, this study aimed at proposing efficient estimator technique in successive sampling schemes. The proposed estimator was obtained through mathematical expectation and statistical assumptions to derived an unbiased estimate of mean (μ), minimum variance (σ^2) and Relative efficiency comparison (REC). A Real life data were used to validate the expression in the study. The data were National Population Commission (NPC) conducted in Nigeria for 1991 and 2016 population census in Nigeria. The findings of the study include a proposed estimator obtained by hybridization of linear and ratio estimator, mathematical estimations of the mean in first and second occasions, minimum variance with maximum precision and relative gain in efficiency of the four estimators; SEst, REst, LEst and PHLREst derived; estimates of change and sum of the population parameter for the two occasions were derived and the estimates of change achieved minimum variance and maximum efficiency when correlation coefficient $\rho = 1.0$ and proportion matched $\lambda = 0.2$. The study concluded that the proposed hybridized linear-ratio estimator is more efficient than the conventional *ones in term of precision. Also, the results were validated with the real life data with the same conclusion. Therefore, the proposed estimator is recommended for use in successive sampling scheme.*

Keywords: Successive Sampling, Estimators, Hybridization, Population census

1. INTRODUCTION

Successive sampling is used extensively in applied sciences, sociology and economic researches. Many survey these days are repetitive in character. Government agencies like the National Bureau of Statistics and other research based institutes collect information regularly on the same population to estimate some population parameters for current occasion. When same population is sampled repeatedly, it is said that a first sample has been taken (on one occasion) from a population of N units and a second sample is to be taken (on another occasion) on the same population, there is thus an opportunity of making use of the information contained in the first sample. Therefore, problem is how best to learn from past experience and use it for improving precision of future estimates. Estimates can be made not only for the existing time period (current estimates) but also of the change that has taken place since the previous occasion and of the average over a given period. One characteristics growth of theoretical statistics is the emergence of a large body of the theory which discusses how to make good estimate from data. In the development of theory specifically for sample surveys, little use has been made of this knowledge. There are two principal reasons. Firstly, a survey that contains a large number of items has a greater advantage in estimation procedure that requires little more than simple addition, whereas the superior methods of estimate is statistical theory, such as maximum likelihood, may necessitate a series of successive approximations before the estimate can be found, Secondly, there has been a difference in attitude in the two lines of research. Most of the estimation methods in theoretical statistics take for granted that we know the functional form of the frequency distribution followed by the data in the sample, and the method of estimation is carefully geared to this type of distribution.

AIM AND OBJECTIVES OF THE STUDY

The aim of the study is to Propose Ratio Estimator Techniques in the context of devising efficient estimator strategies in Successive Sampling Schemes. The objectives aimed to derive, determine, compare and investigate:

i. the performance of the Simple Estimator (SE_{st}), Linear Estimator (LE_{st}) over Proposed Ratio Estimator ($PRATE_{st}$) with respect to their precisions.
ii. the effect(s) of the Correlation coefficient (ρ), Proportion matched (λ) and Proportion unmatched (θ) samples on the estimates and the estimators respect toChange and Sum in two occasions using Simple Estimator (SE_{st}), Linear Estimator (LE_{st}) and Proposed Ratio Estimator ($PRATE_{st}$)

The problem of sampling on Two Successive Occasions with a partial replacement of sampling units was first considered by Jessen (1942) in a survey of farm data. Sen (1961) also applied this sampling plan with success in designing a mail survey in Ontario of waterfowl hunter who hunted successively during (1967 - 1968) and (1968 - 1969). In the study, an estimate was developed for the current season (1968 – 1969), based on the relationship between the value of characteristics during the current season and its value during the previous season, that yielded more precise estimates of the kill of waterfowl than the usual estimates based on simple random sampling and using the hunter's current season's performance only. In a similar study, Singh and Srivastara (1974) presented on different level of repeated sampling and established that for one level of repeated sampling, only sample values that have been drawn from the population of current time can be added to the sample pattern. And for higher levels, both the earlier sample values and current values can be added. Singh and Srivastara (1974) also developed a repeated sampling method to obtain a minimum variance estimate of population values (mean and total) by suitably constructing a linear function of sample at different times.

However, the use of ratio method of estimation in successive sampling was first suggested by Avdhani (1968) and later Sen et al. (1975). Gupta (1970) suggested the use of product method and later it was embraced by Artes et al. (1988), Artes and Garcia (2001). In is contributions, Okador (1992) gave some estimators of the population ratio when sampling is done with partial replacement of units. In, this case, the estimate of the population total of the character y_1 on the recent occasion is first obtained by a suitable combination of two independent estimates of the population totals from the matched and unmatched sample. The estimate of the population total of y_2 is similarly obtained. These two estimates of the population totals of y_1 and y_2 are then used to derive the estimate of the population ratio.

The approach that was used for estimation of current population mean, further extended to develop general theory of estimation in repeated surveys for the current population variance. For the first time in successive sampling, Sud et al. (2001) considered the problem of estimating the population variance on current occasion. Azam et al. (2001) proposed an estimator by using a linear combination of available sample variances for estimating the current population variance based on matched and unmatched portions of the samples at both occasions and most recently, Singh et al. (2011) suggested a class of estimators for estimation of finite population variance on current occasion. Sodipo et al. (2013) use unistage sampling over two occasion using SRSWOR and regression estimator was applied in obtaining current estimates with one auxiliary variable.

2, METHODOLOGY

The data used was collected from National Population Commission (NPC) of the census conducted in Nigeria for year 1991 and 2006.

Notation and meaning with derivation of methods under study.

N - is the population size

n - is the sample size taken on the first occasion

m - is the number of matched or retained units from the first occasion and used as part of second occasion

u - is the number of unmatched or a fresh unit on the second occasion from the remaining unit of the population

ρ - is the correlation coefficient between the matched units of x and y

λ - is the proportion of matched or retained units

θ - is the proportion of unmatched or new units

σ^2- is the pooled variance of S_x^2 and S_z^2

S_x^2 - is the sample variance of units (x) on first occasion

$\bar{x}_{1u}$- *is the sample mean of unmatched units of x from first occasion.*
$\bar{x}_{1m}$ - *is the sample mean of matched units of x from first occasion.*
$\bar{y}_{2u}$ - *is the sample mean of unmatched units of y from second occasion.*
$\bar{y}_{2m}$- *is the sample mean of matched units of y from second occasion.*
$\bar{z}_{1u}$ *is the sample mean of unmatched units of z from first occasion.*
$\bar{z}_{2m}$- *is the sample mean of matched units of z from second occasion.*
$\bar{z}_{2u}$ - *is the sample mean of unmatched units of z from second occasion.*
$\bar{z}_{1m}$ - *is the sample mean of matched units of z from first occasion.*
$\bar{X}$ - *is the population mean of x units.*$\bar{Y}$ - *is the population mean of y units.*
$\bar{Z}$ - *is the population mean of z units.*

Derivation of Estimate, Variance and Relative Efficiency of the Sampling Scheme

This section discusses the derivation of the formula for estimate, variance and relative efficiency of the sampling scheme. Basically, we would be employing the method of Yates (1949), Patterson (1950), Das(1982), Chaturvedi (1983) and Okafor (1985) as template

1. Simple Estimator (SE_{st})
   a) The unbiased Simple Estimator of mean for first and second occasion respectively
      i) $\mu_{1SE_{st}} = \bar{X}$

$$= \left(\frac{u\bar{x}_u + m\bar{x}_m}{n}\right) \quad (1)$$

      ii) $\mu_{2S\ \ st} = \bar{Y}$

$$= \left(\frac{u\bar{y}_u + m\bar{y}_m}{n}\right) \quad (2)$$

   b) i) The estimate of this change, $\widehat{\Delta}_{(SE_{st})}$, is

$$\widehat{\Delta}_{(SE_{st})} = \bar{Y} - \bar{X}$$

$$\widehat{\Delta}_{(SE_{st})} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u) \quad (3)$$

ii) Variance of Estimate of change

$$\hat{\Delta}_{SE_{st}} = \lambda(\bar{y}_m - \bar{x}_m) - \theta(\bar{y}_u - \bar{x}_u)$$

$$V(\hat{\Delta}_{SE_{st}}) = 2(1 - \lambda\rho)\, \sigma^2/_n \quad (4)$$

c) i) The Estimate of Sum

$$\hat{\Sigma}_{SE_{st}} = \bar{X} + \bar{Y}$$

$$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u) \quad (5)$$

ii) Variance of the Sum

$$\hat{\Sigma}_{SE_{st}} = \lambda(\bar{y}_m + \bar{x}_m) + \theta(\bar{y}_u + \bar{x}_u)$$

$$v(\hat{\Sigma}_{SE_{st}}) = 2(1 + \lambda\rho)\, \sigma^2/_n \quad (6)$$

2) Linear Estimator (LE_{st})

a) The linear estimator for $\hat{\mu}_{1L}$ and $\hat{\mu}_{2L}$ can also be sought from the form.

i) $\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1 - d)\bar{x}_{1u}$

Where *b* and *d* are constant

$$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{y}_{2u} - \bar{y}_{2m}) + \lambda\bar{x}_{1m} + \theta(1 - \rho^2\theta)\bar{x}_{1u}\} \quad (7)$$

ii) $\hat{\mu}_{2L\ st} = a\,(\overline{x_{1u}} - \overline{x_{1m}}) + C\,\overline{y_{2m}} + (1 - C)\,\overline{y_{2u}}.$

Where *a* and *c* are constant

$$= \frac{1}{1-\rho^2\theta^2}\{\lambda\rho\theta(\bar{x}_{1u} - \bar{x}_{1m}) + \lambda\bar{y}_{2m} + \theta(1 - \rho^2\theta)\bar{y}_{2u}\} \quad (8)$$

b) i) The estimate of change

$$\hat{\Delta}_{LE_{st}} = \hat{\mu}_{2L} - \hat{\mu}_{1L}$$

$$= \frac{1}{(1-\rho\theta)} \lfloor \theta(1-\rho)(\bar{y}_{2u} - \bar{x}_{1u}) + \lambda(\bar{y}_{2m} - \bar{x}_{1m}) \rfloor$$

(9)

ii) Variance of Estimate of change

$$V(\hat{\Delta}_{LE_{st}}) = \frac{2(1-\rho)}{(1-\rho\theta)} \sigma^2/n \qquad (10)$$

c) i) The estimate of Sum

$$\hat{\Sigma}_{LE_{st}} = \hat{\mu}_{2L} + \hat{\mu}_{1L}$$

$$= \frac{1}{(1+\rho\theta)} \lfloor \lambda(\bar{x}_{1m} + \bar{y}_{2m}) + \theta(1+\rho)(\bar{x}_{1u} + \bar{y}_{2u}) \rfloor$$

(11)

iii) Variance of Sum

$$V(\hat{\Sigma}_{TSO_L}) = \frac{2(1+\rho)}{(1+\rho\theta)} \sigma^2/n \qquad (12)$$

Properties of the Proposed Ratio Estimator

1. Biased
2. Mean square Error (MSE)
3. Minimum Mean Square Errors (MMSE)
4. Efficiency comparison

Considera population containing of N unit. Let a character under study on first (second) occasion be denoted by x *and* y. It is assumed that the information on auxiliary variable z is available on the first as well as on the second occasion. We consider the population to be large enough and the sample size is constant on each occasion. Using sample random without replacement (SRSWOR), we select a sample of size n on the first occasion of these n units, a sub-sample of size $m = n\lambda$ is retained on the second occasion. This sub sample is supplemented by selecting of $u = (n-m) = n\mu$ units afresh from the units that were not selected on the first occasion. Following Yates (1949), Petterson (1950) and Singh (2005) methods, we use

$$\hat{\mu}_{1LE_{st}} = b(\bar{y}_{2u} - \bar{y}_{2m}) + d\bar{x}_{1m} + (1-d)\bar{x}_{1u} \qquad (13)$$

$$\mu_{2_{LE_{st}}} = a(\bar{x}_{1u} - \bar{x}_{1m}) + c\bar{y}_{2m} + (1-c)\bar{y}_{2u} \quad (14)$$

We Proposed a ratio estimator $\bar{z}$ on the both occasion which is based on a sample of size m common to both the occasion and is given by

$$\mu_{1_{PRATE_{st}}} = b\left(\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + d\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + (1-d)\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} \quad (15)$$

$$\mu_{2_{PRATE_{st}}} = a\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + c\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + (1-c)\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} \quad (16)$$

Where a, b, c and d are constants

- ✓ To determine the value of constants a, b, c and d.
- ✓ Find the variance of $\mu_{1_{PRATE_{st}}}$ and $\mu_{2_{PRATE_{st}}}$ then take the derivatives with respect to constants a, b, c and d, therefore equate the resulting equations to zero to obtain a, b, c and d.

Hence,

a) The Proposed Ratio Estimator of the means are:

i) $$\mu_{1_{PRATE_{st}}} = \frac{cH-M}{G}\left(\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z} - \frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z}\right) + \frac{GF-M}{GD-HE}\frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z} + \left(\frac{GD+ME-HE-G}{GD-HE}\right)\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}$$

ii) $$\mu_{2_{PRATE_{st}}} = \frac{CH-M}{G}\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right) + \frac{GF-M}{GD-HE}\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \left(\frac{GD+ME-HE-GF}{GD-HE}\right)\frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z}$$

Where

$$C = \frac{GF-M}{GD-H} \qquad D = \frac{A_3A_4}{\theta\lambda} - \frac{A_0k^2B_3}{n} \; E = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda} \; F = \frac{A_5A_6}{\theta\lambda}$$

$$G = \frac{A_1A_2}{\theta\lambda} - \frac{A_0k^2B_1}{n} \; H = \frac{A_0k^2B_2}{n} - \frac{A_7A_8}{\theta\lambda} \; M = \frac{A_9A_{10}}{\theta\lambda}$$

Also, where

$$A_0 = \frac{1}{\lambda^2} - \frac{1}{\theta\lambda} + \frac{1}{\theta^2} \; A_1 = 4 - R_1^{-2} - 2R_1^{-1}\rho$$

$A_2 = 1 - 2{R_1}^{-2} - 2{R_1}^{-1} A_3 = 1 + 2{R_2}^{-2} - 3{R_2}^{-1}\rho$

$A_4 = 1 - 2{R_2}^{-2} A_5 = 1 + 3{R_2}^{-2} - 3{R_2}^{-1}\rho$

$A_6 = {R_2}^{-2} + {R_2}^{-1}\rho A_7 = {R_1}^{-1}{R_2}^{-1} - {R_2}^{-1}\rho - \rho$

$A_8 = 2{R_1}^{-1}{R_2}^{-1} - 2{R_2}^{-1}\rho - {R_1}^{-1}\rho A_9 = 2{R_2}^{-2} - 2{R_2}^{-1}\rho - {R_1}^{-1}\rho + \rho$

$A_{10} = {R_2}^{-2} - {R_2}^{-1}\rho$

Where

$B_1 = {R_1}^{-2} - {R_1}^{-1}\rho + \rho^2 B_2 = {R_1}^{-1}\rho - {R_1}^{-1}{R_2}^{-1} + {R_2}^{-1}\rho - \rho^2$

$B_3 = {R_2}^{-2} - 2{R_2}^{-1}\rho + \rho^2$

Where

$$R_1 = \frac{\bar{z}}{\bar{x}} R_2 = \frac{\bar{z}}{\bar{y}} k^2 = \frac{S^2 x}{\bar{z}^2} \rho = \rho xy = \rho xz = \rho yz$$

b) Estimate of change:

i) The estimate required $\hat{\Delta}_{PRATE_{st}} = \mu_{2_{PRATE_{st}}} - \mu_{1_{PRATE_{st}}}$

Hence

$$\Delta_{PRATE_{st}} = \frac{1}{G(GD - HE)}\left\{\left[(G^2F + GFH - GME - MGD)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} - \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[\left((G^2D - G^2F + GME + MGD - GHE - GFH)\right)\left(\frac{\bar{y}_{2u}}{\bar{Z}_{2u}}\bar{Z} - \frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z}\right)\right]\right\} \quad (17)$$

ii) Variance of Estimate of change

$$V\left(\Delta_{PRATE_{st}}\right) = \frac{2}{G(GD-HE)}\left\{(G^2D - GHE) - (G^2F + GFH - GME - MGD)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \quad (18)$$

c) Estimate of Sum:

i) The estimate required $\Sigma_{PRATE_{st}} = \hat{\mu}_{2PRATE_{st}} + \hat{\mu}_{1PRA\quad st}$

$$\Sigma_{PRATE_{st}} = \frac{1}{G(GD-HE)}\left\{\left[(G^2F + MGD - GME - GFH)\left(\frac{\bar{y}_{2m}}{\bar{z}_{2m}}\bar{Z} + \frac{\bar{x}_{1m}}{\bar{z}_{1m}}\bar{Z}\right)\right] + \left[(G^2D - G^2F + GFH + GME - MED - GHE)\left(\frac{\bar{x}_{1u}}{\bar{z}_{1u}}\bar{Z} + \frac{\bar{y}_{2u}}{\bar{z}_{2u}}\bar{Z}\right)\right]\right\}$$

(19)

ii) Variance of Estimate of Sum

$$V\left(\Sigma_{PRATE_{st}}\right) = \frac{2}{G(GD-HE)}\left\{(G^2D - MED) - (G^2F - GME)\frac{\rho}{\lambda}\right\}\frac{\sigma^2}{n} \quad (20)$$

4) Relative Gain in Precision

a) Relative gain in precision of change in Linear Estimator (LE_{st}) over Simple Estimator(SE_{st})

$$R\left(\frac{\Delta_{SE_{st}}}{\hat{\Delta}_{LE_{st}}}\right) = \frac{v\left(\hat{\Delta}_{SE_{st}}\right) - v\left(\hat{\Delta}_{LE_{st}}\right)}{v\left(\hat{\Delta}_{LE_{st}}\right)}$$

Hence,

$$R\left(\frac{\Delta_{SE_{st}}}{\hat{\Delta}_{LE_{st}}}\right) = \frac{\rho^2\lambda\theta}{1-\rho}$$

(21)

b) Relative gain in precision of sum in Linear Estimator (LE_{st}) over Simple Estimator(SE_{st})$R\left(\frac{\hat{\Sigma}_{SE_{st}}}{\hat{\Sigma}_{LE_{st}}}\right) = \frac{v(\hat{\Sigma}_{SE_{st}}) - v(\hat{\Sigma}_{SE_{st}})}{v(\hat{\Sigma}_{LE_{ST}})}$

Hence, $R\left(\frac{\hat{\Sigma}_{SE_{st}}}{\hat{\Sigma}_{LE_{st}}}\right) = \frac{\rho^2\lambda\theta}{1+\rho}$ (22)

c) Relative gain in precision of change in Proposed Ratio Estimator ($PRATE_{st}$) over Simple Estimator (SE_{st})

$$R\left(\frac{\Delta_{SE_{st}}}{\Delta_{PRATE_{st}}}\right)=\frac{V(\Delta_{SE_{st}})-V(\Delta_{PRATE_{st}})}{v(\Delta_{PRATE_{st}})}\times 100\%$$

Hence,

$$=\frac{G(1-\lambda\rho)(GD\qquad)}{(G^2D-GHE)-(G^2F+GFH-GME-M\qquad)}\left(\frac{\rho}{\lambda}\right) \qquad (23)$$

d) Relative gain in precision of sum in Proposed Ratio Estimator ($PRATE_{st}$) over Simple Estimator (SE_{st})

$$R\left(\frac{\Sigma_{SE_{st}}}{\Sigma_{PRATE_{st}}}\right)=\frac{v(\Sigma_{SE_{st}})-v(\Sigma_{PRATE_{st}})}{v(\Sigma_{PRATE_{st}})}\times 100\%$$

Hence, $=\frac{G(1+\lambda\rho)(GD-HE)}{(G^2D-ME\quad)-(G^2F-GMF)}\left(\frac{\rho}{\lambda}\right)$ (24)

3. RESULT OF THE ANALYSIS

Empirical Study I

The data from census conducted in Nigeria in 1991 (1st occasion) and 2006 (2nd occasion) was considered. We define the variables X and Y as the population of males and female in each state and Z is defined as the auxiliary variable which is the total number of households in each state and T is the total number of both sex. For simplicity

Table 1: Distribution of Variables

NOTATION	DESCRIPTION OF VARIABLES
1st Occasion	Population census report in 1991
2nd Occasion	Population census report in 2006
X_1	Population of male in 1st occasion (M_1)
Y_1	Population of female in 1st occasion (F_1)
Z_1	No of household in 1st occasion (H_1)
X_2	Population of male in 2nd occasion (M_2)
Y_2	Population of female in 2nd occasion (F_2)
Z_2	No of household in 2nd occasion (H_2)
T_1	Population of both sexes in 1st occasion (B_1)
T_2	Population of both sexes in 2nd occasion (B_2)

Table 2: Relative Gain in precision of Sum (∑) with varying λ's and ρ's

ρ	$\lambda = 0.2$			$\lambda = 0.5$			$\lambda = 0.8$		
	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	26.3	55.4	179.4	47.3	83.5	193.2	56.1	118.2	222.4
0.15	25.1	54.8	165.6	46.8	82.7	181.3	55.1	117.8	214.7
0.25	24.8	53.3	155.6	45.2	81.6	176.4	54.3	116.9	201.6
0.35	23.4	52.9	146.7	44.9	80.4	165.4	53.7	115.3	199.6
0.45	22.1	51.7	137.8	43.2	79.6	155.8	52.9	114.6	186.8
0.55	21.9	50.2	126.7	42.6	78.5	145.6	51.2	113.2	175.9
0.65	20.6	49.6	116.8	41.8	77.6	136.7	50.1	112.6	165.1
0.75	19.1	47.6	107.6	40.1	76.5	126.7	49.6	103.7	155.8
0.85	18.9	46.3	98.6	39.8	75.5	116.8	48.9	98.4	146.5
0.95	17.6	45.9	88.6	38.2	74.9	98.7	47.2	88.5	136.7
1.00	16.8	44.9	76.5	37.6	73.4	86.8	46.5	76.7	128.9

Table: 3 Relative Gain in precision of Change (Δ) with varying λ'sand ρ's

ρ	$\lambda = 0.2$			$\lambda = 0.5$			$\lambda = 0.8$		
	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$	RE_{st}	LE_{st}	$PHLRE_{st}$
0.05	11.9	24.8	83.9	8.2	10.2	52.1	5.5	8.6	39.8
0.15	21.7	34.2	95.3	11.8	24.9	63.7	6.7	9.7	42.5
0.25	30.9	43.8	102.1	12.0	36.4	75.5	7.9	10.6	44.9
0.35	40.0	53.5	133.2	14.6	41.0	89.1	8.6	11.0	58.9
0.45	50.6	62.9	135.4	16.6	50.9	94.5	9.5	12.4	61.3
0.55	60.5	71.8	140.6	19.6	62.3	108.0	10.8	13.5	75.4
0.65	70.6	80.7	143.0	20.9	73.7	119.0	11.2	14.2	96.5
0.75	80.4	90.2	147.0	24.4	84.9	127.0	12.9	15.7	103.2
0.85	91.2	101.8	150.9	30.5	92.1	130.9	13.8	16.8	117.3
0.95	93.4	108.5	158.1	40.8	98.5	149.5	16.9	18.3	125.7
1.00	100.5	128.2	169.6	50.1	102.6	156.0	18.8	20.1	130.4

$N = 37\ n = 25$

4. DISCUSSION OF RESULTS

i. From Table 2, it was observed that the correlation coefficient (ρ) increases in each estimators and proportion matched (λ) increases as the estimators decreases with different λ's and ρ's under the estimate of the sum(Σ). So, the relative or gain in precision was achieved when efficiency$\rho's \to 1$ and $\lambda's \to 0$ under estimate of change with subtended gain in $PHLRE_{st}$

ii. From Table 12, it was observed that the relative efficiency or gain in precision was achieved when $\rho's \to 0$ and $\lambda's \to 1$ under estimate of sum, with substantial gain in $PHLRE_{st}$.

5. CONCLUSION

In view of the above result, based on the available information, we concluded that, the comparative and empirical study from Table 2 and table 3 of the proposed hybridized linear ratio estimator (PHLREest.) establishes its superiority in the sense of relative efficiency over some of the existing estimators (REest. and LEest.).

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