A CLASS OF RATIO ESTIMATORS OF A FINITE POPULATION MEAN USING TWO AUXILLARY VARIABLES UNDER TWO-PHASE SAMPLE SCHEME

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ABSTRACT

Lack of knowledge about population mean of auxiliary variable makes it impossible to apply estimators in real life situations. In this work, the efficiency of a class of a finite population mean in a single-phase sampling estimator has been investigated when the population mean of auxiliary variable is been estimated by sample mean based on the preliminary large sample. The bias and MSE of the modified estimator were derived up to second degree approximation using Taylor series expansion. Empirical study was conducted to investigate the efficiency of the modified estimator over some related existing estimators. The results revealed that the modified estimator has minimum MSE and higher Percentage Relative Efficiency (PRE) and therefore, the modified estimator is more efficient than other related estimators considered in the work.

Keywords: Estimator, Mean, Efficiency, Mean Square Error, Bias.

1.0 INTRODUCTION

Use of auxiliary information has been in practice to increase the efficiency of the estimators. Such information is generally used in ratio, product and regression type estimators for the estimation of population mean of study variable. When correlation between study variable and auxiliary variable is positive ratio method of estimation is used. On the other hand if the correlation is negative, product method of estimation is preferred. Some research works such as Abu-Dayeh *et al(2003),* Dayeh *et al(2003)*, Cochran (1940), Murthy (1964), Upadhyaya and Singh (1999), have been done in ratio, product and regression type estimators by using an auxiliary variable. In this study, a class of ratio estimators using two auxiliary variables is considered to estimate a finite population mean for the variable of interest. We considered several special estimators of the suggested estimators. The comparisons between the traditional multivariate ratio estimators and the estimators proposed by Abu-Dayeh *et al (2003)*. With the proposed family of estimators using information of two variables are considered. We compared the traditional ratio estimator, the estimators proposed by Abu-Dayeh *et a l(2003)* and proposed several special estimators using the statistic data given in Table and we obtained the satisfactory results.There is a lot of work in which the auxiliary information are used to enhance the precision of the estimator. Furthermore, on the same pattern, Murthy (1964) proposed a product estimator $\bar{y}_P = \bar{y}\dfrac{\bar{x}}{\bar{X}}$ to estimate population mean $(\bar{Y})$. The product estimator is more efficient than the mean per unit whenever estimator $\rho < \dfrac{C_x}{2C_y}$.

The traditional multivariate ratio estimator using information of two auxiliary variables x_1 and x_2 to estimate the population mean, $\bar{Y}$ as follows:

$$\bar{y}_{MR} = \theta_1 \bar{y}\frac{\bar{X}_1}{\bar{x}_1} + \theta_2 \bar{y}\frac{\bar{X}_2}{\bar{x}_2} \tag{1.1}$$

Where $\bar{x}_i$ and $\bar{X}_i$ (i=1, 2) denote respectively the sample and the population means of the variable x_i; and θ_1, θ_2 are the weights that satisfy the condition: $\theta_1 + \theta_2 = 1$

The MSE of this estimator is given by

$$MSE(\bar{y}_{MR}) \cong \frac{1-f}{n}\bar{Y}^2 \begin{bmatrix} C_y^2 + \theta_1^2 C_{x_1}^2 + \theta_2^2 C_{x_2}^2 - 2\theta_1 \rho_{yx_1} C_y C_{x_1} - 2\theta_2 \rho_{yx_2} C_y C_{x_2} \\ +2\theta_1\theta_2 \rho_{x_1x_2} C_{x_1} C_{x_2} \end{bmatrix} \tag{1.2}$$

$$\theta_1^* = \frac{C_{x_2}^2 - \rho_{yx_2} C_y C_{x_2} + \rho_{yx_1} C_y C_{x_1} - \rho_{x_1x_2} C_{x_1} C_{x_2}}{C_{x_1}^2 + C_{x_2}^2 - 2\rho_{x_1x_2} C_{x_1} C_{x_2}}, \ \theta_2^* = 1 - \theta_1^* \tag{1.3}$$

$$MSE_{\min}(\bar{y}_{MR}) \cong \frac{1-f}{n}\bar{Y}^2 \begin{bmatrix} C_y^2 + \theta_1^{*2} C_{x_1}^2 + \theta_2^{*2} C_{x_2}^2 - 2\theta_1^* \rho_{yx_1} C_y C_{x_1} \\ -2\theta_2^* \rho_{yx_2} C_y C_{x_2} + 2\theta_1^*\theta_2^* \rho_{x_1x_2} C_{x_1} C_{x_2} \end{bmatrix} \tag{1.4}$$

Abu-Dayeh *et al (2003)*, proposed the estimators using two auxiliary variables given by

$$\bar{y}_{r2}^{\gamma} = \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\gamma_1}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \tag{1.5}$$

$$\bar{y}_{r2}^{\varepsilon} = \varepsilon_1 \bar{y}\left(\frac{\bar{x}_1}{\bar{X}_1}\right)^{\gamma_1} + \varepsilon_2 \bar{y}\left(\frac{\bar{x}_2}{\bar{X}_2}\right)^{\gamma_2} \tag{1.6}$$

Where $\varepsilon_1 + \varepsilon_2 = 1$.

MSE of these estimators are given as follows:

$$MSE(\bar{y}_{r2}^{\gamma}) \cong \frac{1-f}{n}\bar{Y}^2 \begin{bmatrix} C_y^2 + \gamma_1^2 C_{x_1}^2 + \gamma_2^2 C_{x_2}^2 + 2\gamma_1 \rho_{yx_1} C_y C_{x_1} + 2\gamma_2 \rho_{yx_2} C_y C_{x_2} \\ +2\gamma_1\gamma_2 \rho_{x_1x_2} C_{x_1} C_{x_2} \end{bmatrix} \tag{1.7}$$

$$MSE(\overline{y}_{r2}^{\varepsilon}) \cong \frac{1-f}{n}\overline{Y}^2\begin{bmatrix} C_y^2+\varepsilon_1^2\gamma_1^2C_{x_1}^2+\varepsilon_2^2\gamma_2^2C_{x_2}^2+2\varepsilon_1\gamma_1\rho_{yx_1}C_yC_{x_1}+2\varepsilon_2\gamma_2\rho_{yx_2}C_yC_{x_2} \\ +2\varepsilon_1\gamma_2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{bmatrix} \tag{1.8}$$

The optimum values of γ_1 *and* γ_2 are given by

$$\gamma_1^* = \frac{C_y(\rho_{yx_2}\rho_{x_1x_2}-\rho_{yx_1})}{C_{x_1}(1-\rho_{x_1x_2}^2)} \tag{1.9}$$

$$\gamma_2^* = \frac{C_y(\rho_{yx_1}\rho_{x_1x_2}-\rho_{yx_2})}{C_{x_2}(1-\rho_{x_1x_2}^2)} \tag{1.10}$$

$$MSE_{\min}(\overline{y}_2^{\gamma}) \cong \frac{1-f}{n}\overline{Y}^2C_y^2\left(1-\frac{\rho_{yx_1}^2+\rho_{yx_2}^2-2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}}{1-\rho_{x_1x_2}^2}\right) \tag{1.11}$$

The optimum values of ε_1 *and* ε_2 are given by

$$\varepsilon_1^* = \frac{\gamma_2^2C_{x_2}^2-\gamma_1\rho_{yx_1}C_yC_{x_1}+\gamma_2\rho_{yx_2}C_yC_{x_2}-\gamma_1\gamma_2\rho_{yx_1}C_{x_1}C_{x_2}}{\gamma_1^2C_{x_1}^2-\gamma_1\gamma_2\rho_{yx_1}C_{x_1}C_{x_2}+\gamma_2^2C_{x_2}^2} \tag{1.12}$$

$$\varepsilon_2^* = 1-\varepsilon_1^*$$

$$MSE_{\min}(\overline{y}_{r2}^{\varepsilon}) \cong \frac{1-f}{n}\overline{Y}^2\begin{bmatrix} C_y^2+\varepsilon_1^{*2}\gamma_1^2C_{x_1}^2+\varepsilon_2^{*2}\gamma_2^2C_{x_2}^2+2\varepsilon_1^*\gamma_1\rho_{yx_1}C_yC_{x_1}+2\varepsilon_2^*\gamma_2\rho_{yx_2}C_yC_{x_2} \\ +2\varepsilon_1^*\varepsilon_2^*\gamma_1\gamma_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{bmatrix} \tag{1.13}$$

Lu and Yan (2014), proposed family of ratio estimators of a class of multivariate ratio estimators using information of two auxiliary variables as follows:

$$\overline{y}_{pmr} = w_1\overline{y}\frac{a_1\overline{X}_1+b_1}{a_1\overline{x}_1+b_1}+w_2\overline{y}\frac{a_2\overline{X}_2+b_2}{a_2\overline{x}_2+b_2} \tag{1.14}$$

Where w_1 and w_2 are weights that satisfy the condition: $w_1 + w_2 = 1$, $a_1(\neq 0)$, $a_2(\neq 0)$, b_1, b_2 are either real numbers or functions of known parameters

$$MSE(\bar{y}_{pmr}) = \frac{1-f}{n}\bar{Y}^2\begin{pmatrix} C_y^2 + w_1^2\alpha_1^2C_{x_1}^2 + w_2^2\alpha_2^2C_{x_2}^2 - 2w_1\alpha_1\rho_{yx_1C_yC_{x_1}} - 2w_2\alpha_2\rho_{yx_2C_yC_{x_2}} \\ +2w_1w_2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{pmatrix} \quad (1.15)$$

The optimum values of w_1 *and* w_2 are given by

$$w_1^* = \frac{\alpha_2^2C_{x_2}^2 + \alpha_1\rho_{yx_1}C_yC_{x_1} - \alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} - \alpha_2\rho_{yx_2}C_yC_{x_2}}{\alpha_1^2C_{x_1}^2 - 2\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} + \alpha_2^2C_{x_2}^2} \quad (1.16)$$

$$w_2^* = 1 - w_1^*$$

$$MSE_{\min}(\bar{y}_{pmr}) = \frac{1-f}{n}\bar{Y}^2\begin{pmatrix} C_y^2 + w_1^{*2}\alpha_1^2C_{x_1}^2 + w_2^{*2}\alpha_2^2C_{x_2}^2 - 2w_1^*\alpha_1\rho_{yx_1C_yC_{x_1}} - 2w_2^*\alpha_2\rho_{yx_2C_yC_{x_2}} \\ +2w_1^*w_2^*\alpha_1\alpha_2\rho_{x_1x_2}C_{x_1}C_{x_2} \end{pmatrix}$$

(1.17)

Singh et al. (2009) classic ratio estimator is been established.The proposed estimator is;

$$t_9 = \bar{y}\exp\left[\alpha\frac{\bar{Z}-\bar{z}_2}{\bar{Z}+\bar{z}_2} - (1-\alpha)\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right] \quad (1.18)$$

$$MSE_{t_9} = \theta_2\bar{Y}^2\left[c_y^2 + \alpha^2c_y^2 - 2\alpha c_yc_z\rho_{yz}\right] + \theta_1\bar{Y}^2\left(\frac{1-\alpha}{2}\right)^2 c_x^2 \quad (1.19)$$

Kadilar and Cingi (2006) proposed modified estimator combining t_1 and ti (i = 2,3,....,7) as follows;

$$\min .MSE(t_i^*) = \bar{Y}^2\left[f_1C_y^2 + f_3\left(C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right) - f_2\rho_{x_1x_2}C_{x_1}C_{x_2} \right] \tag{1.20}$$

t = αt + 1− α ti (i = 2, 3....,7), where α is a real constant to be determined such that MSE of ti is minimum.

$$MSE(t_1) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.21}$$

$$MSE(t_2) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_2^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_2C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.22}$$

$$MSE(t_3) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_3^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_3C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.23}$$

$$MSE(t_4) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_4^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_4C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.24}$$

$$MSE(t_5) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_5^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_5C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.25}$$

$$MSE(t_6) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_6^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_6C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.26}$$

$$MSE(t_7) = \bar{Y}^2\left[f_1C_y^2 + f_2\left(\theta_7^2C_{x_2}^2 - 2\rho_{yx_2}C_yC_{x_2}\right) + f_3\left(\theta_7C_{x_1}^2 - 2\rho_{yx_1}C_yC_{x_1}\right)\right] \tag{1.27}$$

Where

$$\theta_2 = \frac{\bar{X}_2}{\bar{X}_2 + C_{x_2}},\ \theta_3 = \frac{\beta_2(x_2)\bar{X}_2}{\beta_2(x_2)\bar{X} + C_{x_2}},\ \theta_4 = \frac{C_{x_2}\bar{X}_2}{C_{x_2}\bar{X}_2 + \beta_2(x_2)},\ \theta_5 = \frac{\bar{X}_2}{\bar{X}_2 + \sigma_{x_2}},$$

$$\theta_6 = \frac{\beta_1(x_2)\bar{X}_2}{\beta_1(x_2)\bar{X}_2 + \sigma_{x_2}},\ \theta_7 = \frac{\beta_2(x_2)\bar{X}_2}{\beta_2(x_2)\bar{X}_2 + \sigma_{x_2}}$$

2.0 PROPOSED ESTIMATOR

Motivated by thework of Lu andYan (2014), the following estimator is proposed

$$t = \bar{y}\left[\alpha_1\frac{a_1\bar{x}_1' + b_1}{a_1\bar{x}_1 + b_1} + \alpha_2\frac{a_2\bar{x}_2' + b_2}{a_2\bar{x}_2 + b_2}\right] \tag{2.1}$$

2.1 PROPERTIES OF PROPOSED ESTIMATORS (BIAS AND MSE)

To obtain the BIAS and MSE of the proposed estimator, we define the terms use in the following way:

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}},\ e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}\ and\ e_1' = \frac{\bar{x}'-\bar{X}}{\bar{X}}$$

Case 1:

$$\left.\begin{array}{l}
E(e_0) = E(e_1) = E(e_1') = E(e_2) = E(e_2') = 0 \\
E(e_0^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_y^2; \quad E(e_1^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_{x_1}^2;\ E(e_0e_1) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yx_1}C_yC_{x_1} \\
E(e_1e_1') = \left(\frac{1}{n'}-\frac{1}{N}\right)C_{x_1}^2; \quad E(e_0e_1') = \left(\frac{1}{n'}-\frac{1}{N}\right)\rho_{yx_1}C_yC_{x_1};\ E(e_2^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_{x_2}^2 \\
E(e_2e_2') = \left(\frac{1}{n'}-\frac{1}{N}\right)C_{x_2}^2; E(e_0e_2) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yx}C_yC_{x_2};\ E(e_0e_2') = \left(\frac{1}{n'}-\frac{1}{N}\right)\rho_{yx}C_yC_{x_2}
\end{array}\right\} \quad (2.2)$$

Case 2:

$$\left.\begin{array}{l}
E(e_oe_1') \\
E(e_1e_1') \\
E(e_oe_2') \\
E(e_1e_2')
\end{array}\right\} = 0 \quad (2.3)$$

Expressing the t in terms of errors, we get

$$t=\bar{Y}(1+e_0)\left[\alpha_1\frac{a_1\bar{X}_1(1+e_1')+b_1}{a_1\bar{X}_1(1+e_1)+b_1}+\alpha_2\frac{a_2\bar{X}_2(1+e_2')+b_2}{a_2\bar{X}_2(1+e_2)+b_2}\right] \tag{2.4}$$

$$=\bar{Y}(1+e_0)\left[\alpha_1\frac{a_1\bar{X}_1+b_1\left(1+\frac{a_1\bar{X}_1}{a_1\bar{X}_1+b_1}e_1'\right)}{a_1\bar{X}_1+b_1\left(1+\frac{a_1\bar{X}_1}{a_1\bar{X}_1+b_1}e_1\right)}+\alpha_2\frac{a_2\bar{X}_2+b_2\left(1+\frac{a_2\bar{X}_2}{a_2\bar{X}_2+b_2}e_2'\right)}{a_2\bar{X}_2+b_2\left(1+\frac{a_2\bar{X}_2}{a_2\bar{X}_2+b_2}e_2\right)}\right] \tag{2.5}$$

$$=\bar{Y}(1+e_0)\left[\alpha_1\left(1+Q_1e_1'\right)\left(1+Q_1e_1\right)^{-1}+\alpha_2\left(1+Q_2e_2'\right)\left(1+Q_2e_2\right)^{-1}\right] \tag{2.6}$$

$$where \;\; Q_1=\frac{a_1\bar{X}_1}{a_1\bar{X}_1+b_1} \;\; and \;\; Q_2=\frac{a_2\bar{X}_2}{a_2\bar{X}_2+b_2} \tag{2.7}$$

$$t=\bar{Y}(1+e_0)\left[\alpha_1\left(1+Q_1e_1'\right)\left(1-Q_1e_1+Q_1^2e_1^2\right)+\alpha_2\left(1+Q_2e_2'\right)\left(1-Q_2e_2+Q_2^2e_2^2\right)\right] \tag{2.8}$$

$$t=\bar{Y}\left[\begin{array}{l}1-\alpha_1Q_1e_1+\alpha_1Q_1^2e_1^2+\alpha_1Q_1e_1'-\alpha_1Q_1^{\,2}e_1e_1'+\alpha_1e_0-\alpha_1Q_1e_0e_1+\alpha_1Q_1e_0e_1'-\alpha_2Q_2e_2\\+\alpha_2Q_2^2e_2^2+\alpha_2Q_2e_2'-\alpha_2Q_2^2e_2e_2'+\alpha_2e_0-\alpha_2Q_2e_0e_2+\alpha_2Q_2e_0e_2'\end{array}\right]$$

(2.9)

To obtain the Bias, we use:

$$Bias(t)=\mathrm{E}\left(t-\bar{Y}\right) \tag{2.10}$$

$$Bias(t)_I=\bar{Y}\left[\left(\frac{1}{n}-\frac{1}{n'}\right)\left[\alpha_1Q_1^2C_{x_1}^2+\alpha_2Q_2^2C_{x_2}^2-\alpha_1Q_1\rho_{yx_1cycx_1}-\alpha_2Q_2\rho_{yx_2cycx_2}\right]\right] \tag{2.11}$$

To find the mean square error of the suggested estimator up to the first degree of approximation, squaring and taking the expectation of equation (2.10)

$$MSE(t) = \bar{Y}^2 \begin{bmatrix} f_1\left(\alpha_1^2 Q_1^2 C_{x_1}^2 + \alpha_1^2 C_y^2 + \alpha_2^2 Q_2^2 C_{x_2}^2 + \alpha_2^2 C_y^2 + 2\alpha_1\alpha_2 C_y^2\right) - f_2\left(\alpha_1^2 Q_1^2 C_{x_1}^2 + \alpha_2^2 Q_2^2 C_{x_2}^2\right) \\ -f_3\begin{pmatrix} 2\alpha_1^2 Q_1 \rho_{yx_1 C_y C_{x_1}} + 2\alpha_1\alpha_2 Q_1 \rho_{yx_1 C_y C_{x_1}} - 2\alpha_1\alpha_2 Q_1 Q_2 \rho_{x_1 x_2 C_{x_1} C_{x_2}} \\ +2\alpha_1\alpha_2 Q_2 \rho_{yx_2 C_y C_{x_2}} + 2\alpha_2^2 Q_2 \rho_{yx_2 C_y C_{x_2}} \end{pmatrix} \end{bmatrix} \tag{2.12}$$

Let $f_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$, $f_2 = \left(\frac{1}{n'} - \frac{1}{N}\right)$ *and* $f_3 = \left(\frac{1}{n} - \frac{1}{n'}\right)$

In order to find the equation of $\alpha_{(1)opt}$ and $\alpha_{(2)opt}$ which makes the MSE minimum, we take the partial differential of MSE with respect to α_1 and $\alpha_2$and equate to zero.

$$\frac{\partial MSE(t)}{\partial \alpha_1} = \bar{Y}^2 \begin{Bmatrix} f_1\left(2\alpha_1 Q_1^2 C_{x_1}^2 + 2\alpha_1 C_y^2 + 2\alpha_2 C_y^2\right) - 2 f_2 \alpha_1 Q_1^2 C_{x_1}^2 \\ -f_3\begin{pmatrix} 4\alpha_1 Q_1 \rho_{yx_1 C_y C_{x_1}} + 2\alpha_1 Q_1 \rho_{yx_1 C_y C_{x_1}} \\ -2\alpha_2 Q_1 Q_2 \rho_{x_1 x_2 C_{x_1} C_{x_2}} + 2\alpha_2 Q_2 \rho_{yx_2 C_y C_{x_2}} \end{pmatrix} \end{Bmatrix} \tag{2.13}$$

$$\alpha_2 = 1 - \alpha_1$$

$$\alpha_1 = \frac{(1-\alpha_1)\left(-f_1 C_y^2 + f_3 Q_1 \rho_{yx_1 C_y C_{x_1}} + f_3 Q_2 \rho_{yx_2 C_y C_{x_2}} - f_3 Q_1 Q_2 \rho_{x_1 x_2 C_{x_1} C_{x_2}}\right)}{f_1 Q_1^2 C_{x_1}^2 + f_1 C_y^2 - 2 f_1 Q_1 \rho_{yx_1} C_y C_{x_1} + f_2 Q_1^2 C_{x_1}^2} \tag{2.14}$$

$$\alpha_{1(opt)} = \frac{f_3 Q_1 \rho_{yx_1 C_y C_{x_1}} - f_1 C_y^2 + f_3 Q_2 \rho_{yx_2 C_y C_{x_2}} - f_3 Q_1 Q_2 \rho_{x_1 x_2 C_{x_1} C_{x_2}}}{f_1 Q_1^2 C_{x_1}^2 - f_2 Q_1^2 C_{x_1}^2 - f_3 Q_1 \rho_{yx_1 C_y C_{x_1}} - f_3 Q_1 Q_2 \rho_{x_1 x_2 C_{x_1} C_{x_2}} + f_3 Q_2 \rho_{yx_2}} \tag{2.15}$$

$$\alpha_{(2)opt} = 1 - \alpha_{(1)opt} \tag{2.16}$$

$$MSE(t)_{I\min}=\bar{Y}^2\left[\begin{array}{l} f_1\left(\alpha_{1(opt)}^2Q_1^2C_{x_1}^2+\alpha_{1(opt)}^2C_y^2+\alpha_{2(opt)}^2Q_2^2C_{x_2}^2+\alpha_{2(opt)}^2C_y^2+2\alpha_{1(opt)}\alpha_{2(opt)}C_y^2\right) \\ -f_2\left(\alpha_{1(opt)}^2Q_1^2C_{x_1}^2+\alpha_{2(opt)}^2Q_2^2C_{x_2}^2\right)-f_3\left(\begin{array}{l}2\alpha_{1(opt)}^2Q_1\rho_{yx_1C_yC_{x_1}}+2\alpha_{1(opt)}\alpha_{2(opt)}Q_1\rho_{yx_1C_yC_{x_1}}\\ -2\alpha_{1(opt)}\alpha_{2(opt)}Q_1Q_2\rho_{x_1x_2C_{x_1}C_{x_2}}\\ +2\alpha_{1(opt)}\alpha_{2(opt)}Q_2\rho_{yx_2C_yC_{x_2}}\\ +2\alpha_{2(opt)}^2Q_2\rho_{yx_2C_yC_{x_2}}\end{array}\right)\end{array}\right]$$

(2.17)

Case 2:

Similarly, we get the bias and MSE of (t)

$$Bias(t)_{II}=\bar{Y}\left[\left(\frac{1}{n}-\frac{1}{N}\right)\left(\alpha_1Q_1^2C_{x_1}^2+\alpha_2Q_2^2C_{x_2}^2-\alpha_1Q_1\rho_{yx_1}C_yC_{x_1}-\alpha_2Q_2\rho_{yx_2}C_yC_{x_2}\right)\right]$$

(2.18)

$$MSE(t)_{II}=\bar{Y}^2\left[\begin{array}{l} f_1\left(\begin{array}{l}\alpha_1^2Q_1^2C_{x_1}^2+\alpha_1^2C_y^2+\alpha_2^2Q_2^2C_{x_2}^2+\alpha_2^2C_y^2-2\alpha_1^2Q_1\rho_{yx_1}C_yC_{x_1}+2\alpha_1\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}\\ -2\alpha_1\alpha_2Q_1\rho_{yx_1}C_yC_{x_1}-2\alpha_1\alpha_2Q_2\rho_{yx_2}C_yC_{x_2}+2\alpha_1\alpha_2C_y^2-2\alpha_2^2Q_2\rho_{yx_2}C_yC_{x_2}\end{array}\right)\\ +f_2\left(\alpha_1^2Q_1^2C_{x_1}^2+\alpha_2^2Q_2^2C_{x_2}^2+2\alpha_1\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right)\end{array}\right]$$

(2.19)

To obtain optimality, (2.19) is differentiated partially w.r.t α_1, α_2 and equate to zero.

$$\frac{\partial MSE(t)}{\partial\alpha_1}=\bar{Y}^2\left\{\begin{array}{l} f_1\left(\begin{array}{l}2\alpha_1Q_1^2C_{x_1}^2+2\alpha_1C_y^2-4\alpha_1Q_1\rho_{yx_1}C_yC_{x_1}+2\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}\\ -2\alpha_2Q_1\rho_{yx_1}C_yC_{x_1}-2\alpha_2Q_2\rho_{yx_2}C_yC_{x_2}+2\alpha_2C_y^2\end{array}\right)\\ +f_2\left(2\alpha_1Q_1^2C_{x_1}^2+2\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}\right)\end{array}\right\}=0$$

(2.20)

$$\alpha_1=\frac{-f_1\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}+f_1\alpha_2Q_1\rho_{yx_1}C_yC_{x_1}+f_1\alpha_2Q_2\rho_{yx_2}C_yC_{x_2}-f_1\alpha_2C_y+f_2\alpha_2Q_1Q_2\rho_{x_1x_2}C_{x_1}C_{x_2}}{f_1Q_1^2C_{x_1}^2+f_1C_y^2-2f_1Q_1\rho_{yx_1}C_yC_{x_1}+f_2Q_1^2C_{x_1}^2}$$

But

$\alpha_2 = 1 - \alpha_1$

$$\alpha_1 = \frac{f_1 Q_1 \rho_{xy} C_{x_1} C_y + f_1 Q_2 \rho_{xy} C_y C_{x_2} - f_1 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2} - f_1 C_y^2 + f_2 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2}}{f_1 Q_1^2 C_{x_1}^2 + f_1 Q_1 \rho_{xy} C_{x_1} C_y + f_2 Q_1^2 C_{x_1}^2 - f_1 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2} + f_1 Q_2 \rho_{xy} C_y C_{x2} + f_2 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2}} \qquad (2.21)$$

and

$$\alpha_2 = 1 - \frac{f_1 Q_1 \rho_{xy} C_{x_1} C_y + f_1 Q_2 \rho_{xy} C_y C_{x_2} - f_1 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2} - f_1 C_y^2 + f_2 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2}}{f_1 Q_1^2 C_{x_1}^2 + f_1 Q_1 \rho_{xy} C_{x_1} C_y + f_2 Q_1^2 C_{x_1}^2 - f_1 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2} + f_1 Q_2 \rho_{xy} C_y C_{x2} + f_2 Q_1 Q_2 \rho_{xx} C_{x_1} C_{x_2}}$$

(2.22)

$$MSE(t)_{II_{\min}} = \bar{Y}^2 \begin{bmatrix} f_1 \begin{pmatrix} \alpha_{1(opt)}^2 Q_1^2 C_{x_1}^2 + \alpha_1^2 C_y^2 + \alpha_{2(opt)}^2 Q_2^2 C_{x_2}^2 + \alpha_{2(opt)}^2 C_y^2 - 2\alpha_{1(opt)}^2 Q_1 \rho_{yx_1} C_y C_{x_1} \\ +2\alpha_{1(opt)} \alpha_2 Q_1 Q_2 \rho_{x_1 x_2} C_{x_1} C_{x_2} - 2\alpha_{1(opt)} \alpha_{2(opt)} Q_1 \rho_{yx_1} C_y C_{x_1} - 2\alpha_{1(opt)} \alpha_{2(opt)} \\ Q_2 \rho_{yx_2} C_y C_{x_2} + 2\alpha_{1(opt)} \alpha_{2(opt)} C_y^2 - 2\alpha_{2(opt)}^2 Q_2 \rho_{yx_2} C_y C_{x_2} \end{pmatrix} \\ + f_2 \left(\alpha_{1(opt)}^2 Q_1^2 C_{x_1}^2 + \alpha_2^2 Q_2^2 C_{x_2}^2 + 2\alpha_{1(opt)} \alpha_{2(opt)} Q_1 Q_2 \rho_{x_1 x_2} C_{x_1} C_{x_2} \right) \end{bmatrix}$$

(2.23)

3.0 EMPIRICAL STUDY

To examine the merit of the suggested estimator, we have consider three natural population data sets. The description of the population are given below.

3.1 NUMERICAL ANALYSIS

POPULATION 1: ***Lu (2014) doi:10.1371/journal.pone.0089538.too1***

N = 180, $\bar{Y}$ = 13.9951, $\bar{X}_1$ = 27.3981, $\bar{X}_2$ = 38.7167, σ_{x_2} =7.224, C_y =0.4180, C_{x1} = 0.4254, C_{x2} = 0.3339, ρ_{yx_1} = 0.5630, ρ_{yx_2} = 0.5273, $\rho_{x_1x_2}$ = - 0.2589

q_1= 0.002, q_2= 2.6519
Consider n' = 100 and n = 70.

Table 1:
Shows the MSE and PRE of Proposed estimator and Kadilar and Cingi (2006) with Population 1

ESTIMATORS	MSE	PRE
Sample mean ($\bar{y}_{sm}$)	0.2987629	100
Kadilar and Cingi (2006) (ti)	0.1305009	228.9356
t1	0.3249663	40.15829
t2	0.3244093	92.09446
t3	0.3247534	91.99686
t4	0.3165484	94.38144
t5	0.3147235	94.9287
t6	0.3552418	84.10129
t7	0.3190458	93.64264
Proposed t_I	0.04749743	629.0086
Proposed t_{II}	0.3223347	92.68717

POPULATION** 2*: ***Lu (2014)

$N = 100$	$n = 50$	$n' = 90$	$\overline{Y} = 13.9951$
$\overline{X}_1 = 17.3981$	$\overline{X}_2 = 28.7167$	$C_{x_1} = 0.4254$	$C_{x_2} = 0.3339$
$C_y = 0.4180$	$\rho_{yx_1} = 0.5630$	$\rho_{yx_2} = 0.5273$	$\rho_{x_1x_2} = 0.2589$

Table 2: Shows the MSE and PRE of Proposed estimator and Kadilar and Cingi (2006) estimators with respect to Population 2.

ESTIMATORS	MSE	PRE
Sample mean ($\overline{y}_{sm}$)	0.3422194	100
Kadilar and Cingi (2006) (ti)	0.2706685	126.4349
t1	0.453256	59.71647
t2	0.4530696	75.53351
t3	0.4531844	75.51437
t4	0.4508215	75.91016
t5	0.4504944	75.96528
t6	0.4608763	74.25406
t7	0.4514389	75.80636
Proposed t_I	0.7374691	46.40457
Proposed t_{II}	0.1664182	205.6382

***Population 3**: Handique et al. (2011)*

$N = 200$	$n = 25$	$n' = 200$	$\overline{Y} = 4.63$
$\overline{X}_1 = 21.09$	$\overline{X}_2 = 13.55$	$C_{x_1} = 0.98$	$C_{x_2} = 0.64$
$C_y = 0.95$	$\rho_{yx_1} = 0.079$	$\rho_{yx_2} = 0.072$	$\rho_{x_1x_2} = 0.66$

Table 3: Shows the MSE and PRE of Proposed estimator and Kadilar and Cingi (2006) estimators with respect to Population 3.

ESTIMATORS	MSE	PRE
Sample mean ($\overline{y}_{sm}$)	0.7661334	100
Kadilar and Cingi (2006) (ti)	1.508084	50.80176
t1	1.527018	98.76007
t2	1.523068	50.30199
t3	1.525466	50.2229
t4	1.508294	50.7947
t5	1.500809	51.04801
t6	1.478027	51.83486
t7	1.513183	50.63058
Proposed t_I	1.127063	167.97607
Proposed t_{II}	1.96751	38.93923

4.0 DISCUSSION OF THE RESULTS

The result from the table 1, 2 and 3, shows that the modified estimators are smaller than that of the Kadilar and Cingi (2006) estimators, which shows that the modified estimator is better than or more efficient than that of Kadilar and Cingi estimators.

5.0 CONCUSION

In theempirical study, a class of ratio estimators of a finite population mean using two auxiliary variables were modified to a new estimator (t) under two phase samplingschemes. The MSE and PRE of the estimators (t_I and t_{II}) were computed, and the modified estimator has minimum MSE. The Percentage Relative Efficiency (PRE) is higher compare to PRE of other relative estimators. Therefore, the modified estimator (t) is better than other estimators considered in this study.

REFERENCES

Abu-Dayeh WA, Ahmed MS, Ahmed RA, Muttlak HA (2003).Some Estimators of a finite population mean using Auxiliary Information. *Apply Math Comput* 139: 287-298.

Cochran, W.G. (1940) The Estimation of the Yields of the Cereal Experiments by Sampling for the Ratio of Grain to Total Produce. Journal of Agricultural Science, 30, 262-275. http://dx.doi.org/10.1017/S0021859600048012

Handique, B. K., Das, G., & Kalita, M. C. (2011). Comparative studies on forest sampling techniques with satellite remote sensing inputs. *Project Report, NESAC*, 12-40.

Lu J and Yan Z (2014). A class of Ratio Estimators of a finite population Man Using Two Auxiliary Variables: *e89538.doi:10.1371/journal.pone*.0089538.

Kadilar C, Cingi H (2006). An improved in estimating the population mean by using correlation coefficient. *Hacent J Math Stat* 35:103-109.

Kadilar C, Cingi H (2003). Ratio estimators in stratified random sampling. *Biometrical journal, 45(2),* 218-225

Singh H, Tailor R. (2003). Use of unknown correlation coefficient in estimating the finite population mean. *Stat Transit,*6: 555-560.

Singh, H.P., Upadhyaya, L.N. and Chandra, P. (2004). A general family of estimators for estimating population mean using two auxiliary variables in two-phase sampling. *Statistics in Transition*, 6(7): 1055-1077.

Murthy, M.N. (1964). Product method of estimation, for negatively correlated study variable(s) and auxiliary variable which was similar to ratio estimator,*Sankhya,*26,69-74.

Upadhyaya LN, Singh HP (1999). Use of transformed auxiliary variable in estimating the finite population mean. *Biometrical J,*41: 627-636.