A MONTE CARLO STUDY OF HOERL AND KENNARD CLASSIFICATION-BASED

RIDGE PARAMETER ESTIMATION TECHNIQUES

[1]Bright Osagie Asemota [2]Kayode Ayinde, [3]Taiwo Stephen Fayose and [2]Olatayo Olusegun Alabi

[1]Rubber Research Institute of Nigeria,
Benin City, Edo State, Nigeria

[2]Department of Statistics,
Federal University of Technology, Akure, Ondo State, Nigeria
[3]Department of Mathematics and Statistics,
Federal Polytechnic, Ado Ekiti, Ekiti State, Nigeria

Email Address: *ase4me14@gmail.com; kayinde@futa.edu.ng;*
fayosets@fedpolyado.edu.ng; alabioo@futa.edu.ng

Abstract:

The presence of multicollinearity challenge influences the Ordinary Least Squares technique in producing inefficient estimates of the model parameters in linear regression model. The ridge regression becomes relevant and a panacea in producing efficient estimates in the presence of multicollinearity. This study examined the ridge parameter estimation techniques of Hoerl and Kennard and introduced the concept of various kinds into the classification of Hoerl and Kennard ridge parameters, and consequently proposed some new ridge parameters. Existing and proposed ridge parameter were compared by conducting Monte Carlo experiment 5000 times on a linear regression model with three (3) and six (6) explanatory variables exhibiting different degrees of multicollinearity (ρ = 0:8; 0:9; 095; 0:99; 0:999; 0.9999), error variance (σ^2 = 1; 25; 100) and seven levels of sample size (n = 10; 20; 30; 40; 50; 100; 150; 250), so as to identify those that would give most efficient estimates. Result

shows that newly proposed estimators are among those that provide efficient estimates. The study observed the implications of the presence of multicollinearity in estimating the parameters in a linear regression model and identified the ridge regression as a panacea to this problem
Keywords: Multicollinearity, Estimator, Ridge Parameter, Classification-Based

Introduction:

As useful as regression analysis, there are certain conditions that can lead to a misleading estimation or prediction. This could occur when the estimator used to estimate the regression parameters fail to meet certain criteria. The Ordinary Least Squares (OLS) estimator is the most popularly used estimator to estimate the parameters in a regression model, the estimator under certain assumptions has some very attractive statistical properties which have made it one of the most powerful estimators. One of the assumptions is that the explanatory variables are independent. However, in practice, there may be strong or perfect linear relationships among the explanatory variables. In this case, the OLS estimator could become unstable due to their large variance, which leads to poor prediction and wrong inference about model parameters. This problem is often referred to as multicollinearity problem (Gujarati, 1995).To remedy the problem of multicollinearity, there are several methods in existence used in tackling the challenge. This paper focuses on ridge regression technique. Ridge regression is a technique for analyzing multiple regression models that may be exposed to the multicollinearity problem. This technique operates by adding a degree of bias to the regression estimates, thus ridge regression reduces the standard errors; the net effect can be highly reliable estimates of the target parameters (Duzan and Shariff, 2016).
In this literature, ridge estimator has been proven to have a smaller mean square error (MSE) than the Ordinary Least Square (OLS) estimator.

Considering the standard regression model:

$$Y = X\beta + \varepsilon \qquad (1)$$

where Y is (*n* x *1*) vector of the response variable values, X is (*n* x *p*) matrix contains the values of *P* predictor variables and this matrix is full Rank (matrix of rank p), β is a (*p* x *1*) vector of unknown coefficients, and ε is a (*n* x *1*) vector of normally distributed random errors with zero mean and common variance, σ^2.

The ordinary least squares (OLS) estimate $\hat{\beta}$ of β is obtained as:

$$\hat{\beta} = (X^{/}X)^{-1}X^{/}Y, \tag{2}$$

$$\text{VAR}(\hat{\beta}) = \sigma^2(X^{/}X)^{-1}, \tag{3}$$

$$MSE(\hat{\beta}) = \hat{\sigma}^2 \sum_{i=1}^{P} \frac{1}{\lambda_i} \tag{4}$$

Duzan and Shariff (2016), by adding a degree of bias (k) to the diagonal elements of the $X^{'}X$ matrix, the ridge solution is given as:

$$\hat{\beta}_{Ridge}(K) = (X^{/}X + KI)^{-1}X^{/}Y, K \geq 0 \tag{5}$$

Note that, if K=0, the ridge estimator reduce to the OLS. If all K's are the same, the resulting estimators are called the ordinary ridge estimators (Judge, 1988).

The objectives of this study are to:

- propose some new ridge parameters resulting from the classification of Hoerl and Kennard.
- examine and compare the proposed ridge parameter with the existing ones.
- identify the best ridge parameter(s) that will produce most efficient estimates of the parameters of linear regression model with multicollinearity problem.

The justification for this study is that there are several Ridge Parameters in existence while many are still being proposed. However, only one is expected to be used in practice. Hence, the need to identify the one that would produce most efficient estimated regression model parameters in the presence of multicollinearity.

Review of methods of estimating the Ridge Parameter.

Since, ridge regression estimators are considered by several researchers at different times and under different simulation conditions, there are many suggestions for computing and estimating the value of K, which will be summarized briefly. In order to estimate the value of k, the following methods are available in the literature.

Hoerl and Kennard (1970) suggested *k* to be (denoted here by $\hat{K}_{HK}$)

$$\hat{K}_{HK} = \frac{\sigma^2}{\alpha_{Max}} \tag{6}$$

where $\hat{\alpha}_{Max}$ is the maximum element of $\hat{\alpha}$. Hoerl and Kennard claimed that (equation 6) gives smaller MSE than the OLS method.

Lawless and Wang (1976) suggested *k* to be (denoted here by K_{LW}) resulting from taking the Harmonic Mean of the ridge parameter $K_{LW} = \frac{\sigma^2}{\lambda_i \alpha_i^2}$. The estimator is defined as $\hat{K}_{LW}^{HM} = \frac{\sigma^2}{\sum_{i=1}^{p} \lambda_i \alpha_i^2}$, (7)

Where λ_i is the i^th^ eigenvalue of the *X'X.*

Hocking, *et al* (1979) suggested k to be (denoted here by $\hat{K}_{HSL}$)

$$\hat{K}_{HSL} = \sigma^2 \frac{\sum_{i=1}^{p} \lambda_i \alpha_i^2}{\left(\sum_{i=1}^{p} \lambda_i \alpha_i^2 \right)^2} \tag{8}$$

Kibria (2003) proposed the following estimators for *k* based on arithmetic mean (AM), geometric mean (GM), and median of $\frac{\sigma^2}{\alpha_i^2}$,. These are defined as follows:

The estimator based on Arithmetic mean AM (denoted by $\hat{K}_{AM}$)

$$\hat{K}_{AM} = \frac{1}{p} \sum_{i=1}^{p} \frac{\hat{\sigma}^2}{\alpha_i^2} \tag{9}$$

The estimator based on geometric mean GM (denoted by $\hat{K}_{GM}$)

$$\hat{K}_{GM} = \frac{\hat{\sigma}^2}{\left(\prod_{i=1}^{p} \hat{\alpha}_i^2\right)^2} \tag{10}$$

The estimator based on median (denoted by $\hat{K}_{MED}$)

$$\hat{K}_{MED} = Median\left\{\frac{\hat{\sigma}^2}{\alpha_i^2}\right\}, i = 1,2,..., p \tag{11}$$

Lukman and Ayinde (2015) proposed the estimator of the ridge parameter K as the median of the square root of $\hat{K}_{MAO}$,. The estimator is given as:

$$\hat{K}_{MAO}^{MSR} = Median\left(\sqrt{\hat{K}_{MAO}}\right) \tag{12}$$

Methodology

In simulating our data for the study, we first generated the error term follow by the explanatory variables then the dependent variable. This is illustrated below:

Consider a linear regression model for the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + ... + \beta_m X_{pt} + U_t \tag{13}$$

t = 1,2,…, n; p = 3,6 ; $U_t \sim N(0, \sigma^2)$

The model was studied with fixed regressors, $X_{it}, i = 1,2,..., p; t = 1,2,..., n$ such that there exist different levels of multicollinearity among the regressors.

The error term U_t was generated to be normally distributed with mean zero and variance $U_t \sim N(0, \sigma^2)$. In this study, σ^2 values were 1, 25 and 100.

In generating the explanatory variable, the study followed the procedure used by McDonald and Galarneau (1975), Wichern and Churchill (1978), Gibbons (1981) and Kibria (2003) Alkhamisi *et al* (2006), Muniz and Kibria (2009) Muniz *et al*. (2012), Lukman and Ayinde (2015). This is given as:

$$X_{ti},= \left(1-p^2\right)^{1/2} Z_{ti} + \rho Z_{tp} \tag{14}$$

$$t = 1,2,\dots,n; i = 1,2,\dots,p$$

where Z_{ti} is independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two explanatory variables and p is the number of explanatory variables. The different degrees of multicollinearity (ρ) considered were taken as 0.8, 0.9, 0.95, 0.99, 0.999 and 0.9999. Also, the number of explanatory variables (p) was taken to be three (3) and six (6).

In generating the dependent variable, the regression model is

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \dots + \beta_p X_{pt} + U_t \qquad t = 1,2,\dots, n; \qquad p = 3,6$$

β was taken to be identically zero. When p = 3, β_0 = 10, β_1 = 4, β_2 = 1, β_3 = 8, When p = 6*,* the values of β were chosen to be: β_0 = 10, β_1 = 4, β_2 = 1, β_3 = 8, β_4 = 1.8, β_5= 2.5, β_6= 3.2. We varied the sample sizes between 10, 20, 30, 50, 100, 150, and 250. Three different values of σ^2: 1, 25 and 100 were also used. At a specified value of *n; p* and σ^2, the fixed *Xs* are first generated; followed by the *U*, and the values of *Y* are then obtained using the regression model. The experiment is repeated 5000 times.

This literature introduced the concept of various kinds based on the Hoerl and Kernard (1970) ridge parameter estimation technique.

$$k_i(\text{HK}) = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} \tag{15}$$

Lukman, (2015) introduced the concept of various type and forms. The different forms are Fixed Maximum (FM), Varying Maximum (VM), Arithmetic Mean (AM), Geometric Mean (GM), Harmonic Mean (HM) and Median (M) and the various types are Original (O), Reciprocal (R), Square Root (SR) and Reciprocal of Square Root (RSR).

This study introduced a new type called Mid-Range (MR) and six (6) new concepts of various kinds into the existing classification of various types and forms of Hoerl and Kernard. The various kinds are as follows:

Table I: Newly introduced Various Kinds

S/N	Various Kinds	Equation(s)
1	Original kind (O),	$\frac{\hat{\sigma}^2}{\alpha_i^2}$
2	Square Root of the original kind (S),	$\sqrt{\frac{\hat{\sigma}^2}{\alpha_i^2}}$
3	P^{th} Root of the original kind (P)	$\sqrt[p]{\frac{\hat{\sigma}^2}{\alpha_i^2}}$
4	Reciprocal of the original kind (OR),	$\frac{\alpha_i^2}{\hat{\sigma}^2}$
5	Square Root of the Reciprocal of original kind (SR)	$\sqrt{\frac{\alpha_i^2}{\hat{\sigma}^2}}$
6	P^{th} Root Reciprocal of the original kind (PR).	$\sqrt[p]{\frac{\alpha_i^2}{\hat{\sigma}^2}}$

The existing and newly proposed estimators are summarized in table 2 below.

Different Forms	VARIOUS TYPES					
	O	S	P	R	SR	PR
FM1	$\hat{K}_1^{FM1}=\frac{\hat{\sigma}^2}{Max(\hat{\alpha}_i^2)}$ Hoerl and Kennard (1970)	$\hat{K}_1^{FM1SR}=\sqrt{\hat{K}_1^{FM1}}$ Lukman and Ayinde (2015	$\hat{K}_1^{FM1PR}=\sqrt[p]{\hat{K}_1^{FM1}}$ Proposed	$\hat{K}_1^{FM1R}=\frac{1}{\hat{K}_1^{FM1}}$ Lukman and Ayinde (2015	$\hat{K}_1^{FM1RSR}=\frac{1}{\sqrt{\hat{K}_1^{FM1}}}$ Lukman and Ayinde (2015	$\hat{K}_1^{FM1RP}=\frac{1}{\sqrt[p]{\hat{K}_1^{FM}}}$ Proposed
FM2	$\hat{K}_2^{FM2}=\frac{\hat{\sigma}^2}{[Max(\hat{\alpha}_i)]^2}$ Proposed	$\hat{K}_2^{FM2SR}=\sqrt{\hat{K}_2^{FM2}}$ Proposed	$\hat{K}_2^{FM2PR}=\sqrt[p]{\hat{K}_2^{FM2}}$ Proposed	$\hat{K}_2^{FM2R}=\frac{1}{\hat{K}_2^{FM2}}$ Proposed	$\hat{K}_2^{FM2RSR}=\frac{1}{\sqrt{\hat{K}_2^{FM2}}}$ Proposed	$\hat{K}_2^{FM2RP}=\frac{1}{\sqrt[p]{\hat{K}_2^{FM}}}$ Proposed
VM	$\hat{K}_3^{VM}=Max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i}\right)$ Lukman and Ayinde (2015)	$\hat{K}_3^{VMSR}=\sqrt{\hat{K}_3^{VM}}$ Lukman and Ayinde (2015)	$\hat{K}_2^{VMPR}=\sqrt[p]{\hat{K}_3^{VM}}$ Proposed	$\hat{K}_3^{VMR}=\frac{1}{\hat{K}_3^{VM}}$ Lukman and Ayinde (2015)	$\hat{K}_3^{VMRSR}=\frac{1}{\sqrt{\hat{K}_3^{VM}}}$ Lukman and Ayinde (2015)	$\hat{K}_3^{VMRPR}=\frac{1}{\sqrt[p]{\hat{K}_3^{VM}}}$ Proposed
GM	$\hat{K}_4^{GM}=\sqrt[p]{\prod_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i}}$ Kibria (2003)	$\hat{K}_4^{GMSR}=\sqrt{\hat{K}_4^{GM}}$ Lukman and Ayinde (2015)	$\hat{K}_4^{GMPR}=\sqrt[p]{\hat{K}_4^{GM}}$ Proposed	$\hat{K}_4^{GMR}=\frac{1}{\hat{K}_4^{GM}}$ Lukman and Ayinde (2015)	$\hat{K}_4^{GMRSR}=\frac{1}{\sqrt{\hat{K}_4^{GM}}}$ Lukman and Ayinde (2015)	$\hat{K}_4^{GMRPR}=\frac{1}{\sqrt[p]{\hat{K}_4^{GM}}}$ Proposed
HM	$\hat{K}_5^{HM}=\frac{p}{\sum_{i=1}^{p}\frac{\hat{\alpha}_i^2}{\hat{\sigma}}}$ Hoerl *et al.* (1975)	$\hat{K}_5^{HMSR}=\sqrt{\hat{K}_5^{AM1}}$ Lukman and Ayinde (2015)	$\hat{K}_5^{HMPR}=\sqrt[p]{\hat{K}_5^{AM1}}$ Proposed	$\hat{K}_5^{HMR}=\frac{1}{\hat{K}_5^{AM1}}$ Lukman and Ayinde (2015)	$\hat{K}_5^{HMRSR}=\frac{1}{\sqrt{\hat{K}_5^{AM1}}}$ Lukman and Ayinde (2015)	$\hat{K}_5^{HMRPR}=\frac{1}{\sqrt[p]{\hat{K}_5^{AM}}}$ Proposed
AM	$\hat{K}_6^{AM}=\frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$ Kibria (2003)	$\hat{K}_6^{AMSR}=\sqrt{\hat{K}_6^{AM2}}$ Lukman and Ayinde (2015)	$\hat{K}_6^{AMPR}=\sqrt[p]{\hat{K}_6^{AM2}}$ Proposed	$\hat{K}_6^{AMR}=\frac{1}{\hat{K}_6^{AM2}}$ Lukman and Ayinde (2015)	$\hat{K}_6^{AMRSR}=\frac{1}{\sqrt{\hat{K}_6^{AM2}}}$ Lukman and Ayinde (2015)	$\hat{K}_6^{AMRPR}=\frac{1}{\sqrt[p]{\hat{K}_6^{AM}}}$ Proposed
MR	$\hat{K}_7^{MR}=\frac{Max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)+Min\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)}{2}$ Proposed	$\hat{K}_7^{MRSR}=\sqrt{\hat{K}_7^{MR}}$ Proposed	$\hat{K}_7^{MRPR}=\sqrt[p]{\hat{K}_7^{MR}}$ Proposed	$\hat{K}_7^{MRR}=\frac{1}{\hat{K}_7^{MR}}$ Proposed	$\hat{K}_7^{MRRSR}=\frac{1}{\sqrt{\hat{K}_7^{MR}}}$ Proposed	$\hat{K}_7^{MRRPR}=\frac{1}{\sqrt[p]{\hat{K}_7^{MR}}}$ Proposed

TABLE II: CLASSIFICATION OF BOTH THE EXISTING AND PROPOSED ESTIMATORS IN DIFFERENT FORMS, VARIOUS

TYPES AND ORIGINAL KIND $K_i = \frac{\hat{\sigma}^2}{\alpha_i^2}$

The mean square error (MSE) has been applied by several authors to evaluate and compare the performance of ridge regression estimator with that of the ordinary least square estimator when multicollinearity is present. Some of these are Hoerl and Kennard (1970), Lawless and Wang (1976), Saleh and Kibria (1993), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi *et al*. (2006), Mansson *et al*. (2010), Lukman and Ayinde (2015) and so on. To investigate whether the ridge estimator is better than the OLS estimator, the MSE is calculated using equation defined already:

$$MSE(\beta) = \frac{1}{R}\sum_{j=1}^{R}\sum_{i=0}^{p}\left(\hat{\beta}_{ij} - \beta_i\right)^2 \quad (16)$$

Where Replication (R) = 5000; P = 3, 6; $\hat{\beta}_{ij}$ is the ith element of the estimator β in the jth replication which gives the estimate of β_i which are the true value of the parameter previously mentioned.

Results and Discussion

The results for each of the newly introduced kinds, in light of various types and different forms were presented by counting the number of time the MSE is minimum over the levels of multicollinearity and standard error. The most efficient ordinary ridge estimator under the various kinds, various types and their different form is identified.

The results from the simulation study carried out on the original kind, various types and different forms of the Hoerl and Kennard ridge parameter are presented. A sample of result at different level of multicollinearity, sample size and variance when p= 3 and p=6 is shown in Table III shows the number of times the various ridge estimators ranked $1, \leq 1.5 and \leq 2.5$ when counted over the levels of multicollinearity and variance when p =3 and p = 6 at different sample sizes. From Table II, the following can be observed:

i. When p=3, the estimator **KOTSFFM1** performs best while estimators **KOTORFFM1** and **KOTSRFFM1** also perform well. This order of performance is still maintained when the ranks of the estimators with values less than or equal 1.5 and 2.5 were used.

ii. When p=6, the estimators **KOTSFHM, KOTORFFM1** and **KOTORFFM1** performs best. However, the estimator **KOTSFFM1** which performs best when p=3, is not doing well when p=6. This order of performance is still maintained when the ranks of the estimators with values less than or equal 1.5 and 2.5 were used.

iii. The estimator **KOTSFFM1** which performs best when p=3, does not perform well when p=6, while the estimator **KOTSFHM** which performs the best when p=6, does not perform well when p=3.

The illustration above is further presented pictorially in Figure I and II

Table III. Number of times the different estimators produced the minimum MSE with original kind

ORIGINAL KIND																		OT
R	ESTMATOR	P=3								P=6								
		10	20	30	50	100	150	250	T	10	20	30	50	100	150	250	T	
	OLS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	KOTOFGR	1	0	0	0	1	0	0	2	0	0	1	0	1	0	0	2	4
	KOTOFFM1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTSFFM1	0	0	0	5	0	5	5	**15**	2	0	0	0	0	1	0	**3**	18
	KOTPFFM1	1	0	0	0	0	0	0	1	1	1	1	0	0	1	1	5	6

	KOTORFFM1	2	3	2	1	1	1	1	11	0	1	1	4	2	4	5	**17**	28
	KOTSRFFM1	1	1	2	1	2	0	2	9	4	3	2	2	1	2	2	**16**	25
	KOTPRFFM1	2	1	1	0	1	0	1	6	4	0	2	0	1	0	0	7	13
	KOTSFFM2	2	2	2	1	1	1	1	10	0	1	0	0	0	0	1	2	12
	KOTPFFM2	2	1	1	1	0	0	0	5	0	0	0	0	0	0	0	0	5
	KOTORFFM2	0	0	0	0	0	1	1	2	0	0	1	0	1	0	0	2	4
1	KOTSRFFM2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
	KOTPRFFM2	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	3	3
	KOTSFVM	0	0	0	1	0	1	1	3	0	1	1	1	1	1	1	6	9
	KOTPFVM	1	1	1	2	0	1	2	8	0	0	0	0	0	0	0	0	8
	KOTORFVM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	KOTOFGM	0	0	0	0	2	1	1	4	0	1	2	2	2	2	2	11	15
	KOTSFGM	0	0	0	0	0	0	0	0	1	1	0	1	1	2	1	7	7
	KOTORFGM	0	1	1	0	0	0	0	2	0	0	0	0	0	0	0	0	2
	KOTSRFGM	0	0	0	1	0	0	0	1	0	1	0	1	0	1	0	3	4
	KOTPRFGM	2	2	2	0	0	0	0	6	0	1	1	0	0	0	0	2	8
	KOTOFHM	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1
	KOTSFHM	1	0	0	0	0	0	0	1	0	3	2	5	3	3	3	19	20
	KOTORFHM	0	0	0	3	2	3	3	11	0	0	0	0	0	0	0	0	11
	KOTSRFHM	2	2	2	1	1	2	0	10	0	1	2	1	2	1	1	8	18
	KOTPRFHM	0	1	0	1	1	1	0	4	0	0	0	1	0	0	0	1	5
	KOTPFAM	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTPRFAM	1	0	1	0	0	0	0	2	2	1	0	0	0	0	0	3	5
	TOTAL	18	15	15	18	12	18	18	114	18	18	16	18	15	18	18	121	235
	OLS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	KOTOFGR	1	0	0	0	1	0	0	2	0	0	1	0	1	0	0	2	4
	KOTOFFM1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTSFFM1	0	3	3	5	5	5	5	**26**	2	0	0	0	0	1	0	3	29
<=1.5	KOTPFFM1	1	0	0	0	0	0	0	1	1	1	3	0	2	1	1	9	10
	KOTORFFM1	2	3	2	1	2	1	1	12	0	1	1	4	3	4	5	**18**	30
	KOTSRFFM1	1	1	2	1	2	0	2	9	4	3	2	2	1	2	2	16	25
	KOTPRFFM1	2	1	1	0	1	0	1	6	4	0	2	0	1	0	0	7	13
	KOTSFFM2	2	5	5	1	6	1	1	21	0	1	0	0	0	0	1	2	23
	KOTPFFM2	2	1	1	1	0	0	0	5	0	0	2	0	2	0	0	4	9
	KOTORFFM2	0	0	0	0	1	1	1	3	0	0	1	0	2	0	0	3	6
	KOTSRFFM2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
	KOTPRFFM2	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	3	3
	KOTSFVM	0	0	0	1	0	1	1	3	0	1	1	1	1	1	1	6	9
	KOTPFVM	1	1	1	2	0	1	2	8	0	0	0	0	0	0	0	0	8
	KOTORFVM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	KOTOFGM	0	0	0	0	2	1	1	4	0	1	2	2	2	2	2	11	15
	KOTSFGM	0	0	0	0	0	0	0	0	1	1	0	1	1	2	1	7	7
	KOTORFGM	0	1	1	0	0	0	0	2	0	0	0	0	0	0	0	0	2
	KOTSRFGM	0	0	0	1	0	0	0	1	0	1	0	1	0	1	0	3	4
	KOTPRFGM	2	2	2	0	0	0	0	6	0	1	1	0	0	0	0	2	8
	KOTOFHM	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1
	KOTSFHM	1	0	0	0	0	0	0	1	0	3	2	5	3	3	3	19	20
	KOTORFHM	0	0	0	3	2	3	3	11	0	0	0	0	0	0	0	0	11
	KOTSRFHM	2	2	2	1	1	2	0	10	0	1	2	1	2	1	1	8	18
	KOTPRFHM	0	1	0	1	1	1	0	4	0	0	0	1	0	0	0	1	5

	KOTPFAM	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTPRFAM	1	0	1	0	0	0	0	2	2	1	0	0	0	0	0	3	5
	TOTAL	18	21	21	18	24	18	18	138	18	18	20	18	21	18	18	131	269
	OLS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	KOTOFGR	1	0	0	0	1	0	0	2	0	0	1	0	1	0	0	2	4
	KOTOFFM1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTSFFM1	0	3	3	5	5	5	5	**26**	2	0	0	0	0	1	0	3	**29**
	KOTPFFM1	1	0	0	0	0	0	0	1	1	1	3	0	2	1	1	9	10
	KOTORFFM1	2	3	2	1	2	1	1	12	0	1	1	4	3	4	5	18	**30**
	KOTSRFFM1	1	1	2	1	2	0	2	9	4	3	2	2	1	2	2	16	25
	KOTPRFFM1	2	1	1	0	1	0	1	6	4	0	2	0	1	0	0	7	13
	KOTSFFM2	2	5	5	1	6	1	1	21	0	1	0	0	0	0	1	2	23
<=	KOTPFFM2	2	1	1	1	0	0	0	5	0	0	2	0	2	0	0	4	9
2.5	KOTORFFM2	0	0	0	0	1	1	1	3	0	0	1	0	2	0	0	3	6
	KOTSRFFM2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
	KOTPRFFM2	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0	3	3
	KOTSFVM	0	0	0	1	0	1	1	3	0	1	1	1	1	1	1	6	9
	KOTPFVM	1	1	1	2	0	1	2	8	0	0	0	0	0	0	0	0	8
	KOTORFVM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	KOTOFGM	0	0	0	0	2	1	1	4	0	1	2	2	2	2	2	11	15
	KOTSFGM	0	0	0	0	0	0	0	0	1	1	0	1	1	2	1	7	7
	KOTORFGM	0	1	1	0	0	0	0	2	0	0	0	0	0	0	0	0	2
	KOTSRFGM	0	0	0	1	0	0	0	1	0	1	0	1	0	1	0	3	4
	KOTPRFGM	2	2	2	0	0	0	0	6	0	1	1	0	0	0	0	2	8
	KOTOFHM	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1

	KOTSFHM	1	0	0	0	0	0	0	1	0	3	2	5	3	3	3	19	20
	KOTORFHM	0	0	0	3	2	3	3	11	0	0	0	0	0	0	0	0	11
	KOTSRFHM	2	2	2	1	1	2	0	10	0	1	2	1	2	1	1	8	18
	KOTPRFHM	0	1	0	1	1	1	0	4	0	0	0	1	0	0	0	1	5
	KOTPFAM	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1
	KOTPRFAM	1	0	1	0	0	0	0	2	2	1	0	0	0	0	0	3	5
	TOTAL	18	21	21	18	24	18	18	138	18	18	20	18	21	18	18	131	269

Note: The bold value indicate estimators with high number of counts

Figure I: Number of counts at which MSE is minimum (rank =1) for different estimators of original kind, various types and different forms estimators.

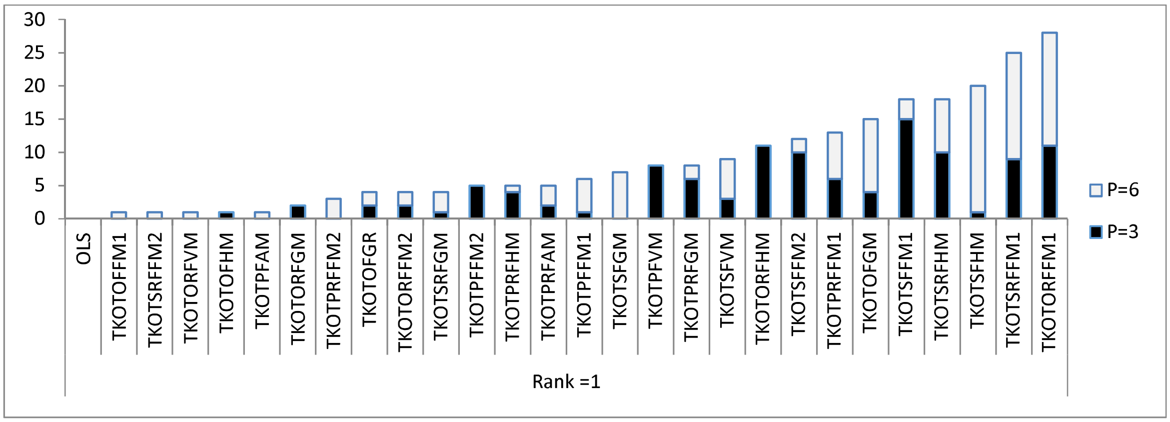
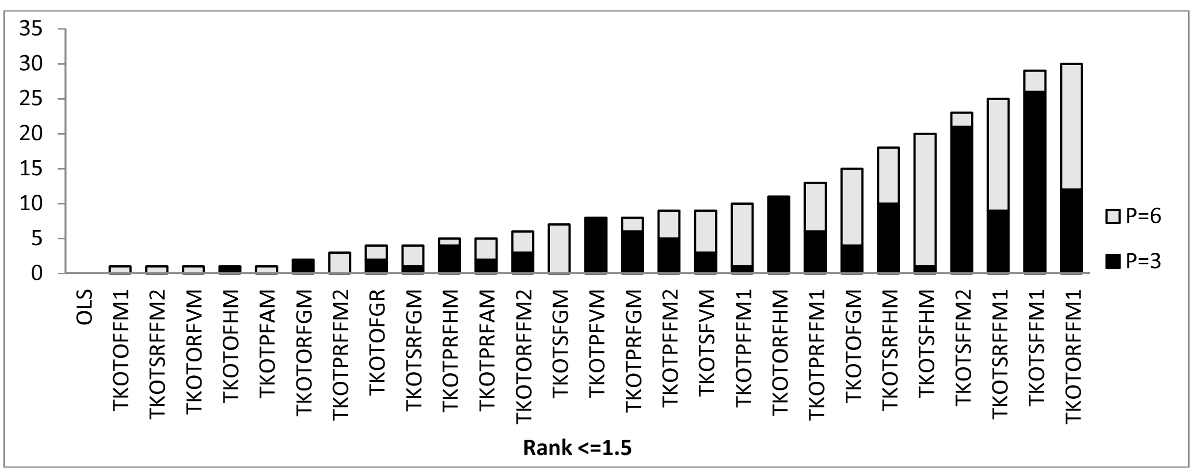


Figure II: Number of counts at which MSE is small (rank <=1.5) for different estimators of original kind, various types and different forms estimators.



The number of times the various ridge estimators ranked between 1 and 25 when counted over the levels of multicollinearity and variance when p =3 and p = 6 at different sample sizes is presented in table III below.

From Table III, the following can be observed:

i. When p=3, the proposed estimator **KORTSFVM** performs best, while estimators **KSRTOFGR, KSRTORFMR, KOTOFGM, KORTSFMR** and **KORTSRFAM** also perform well.
ii. When p=6, the estimator **KORTSFVM** performs best, while the estimator **KSRTORFGM, KOTSFGM, KOTPFVM, KSRTOFGR, KSTOFHM** perform well.
iii. The estimator **KORTSFVM** performs best at both when p=3 and p=6. Hence is considered overall best estimator.

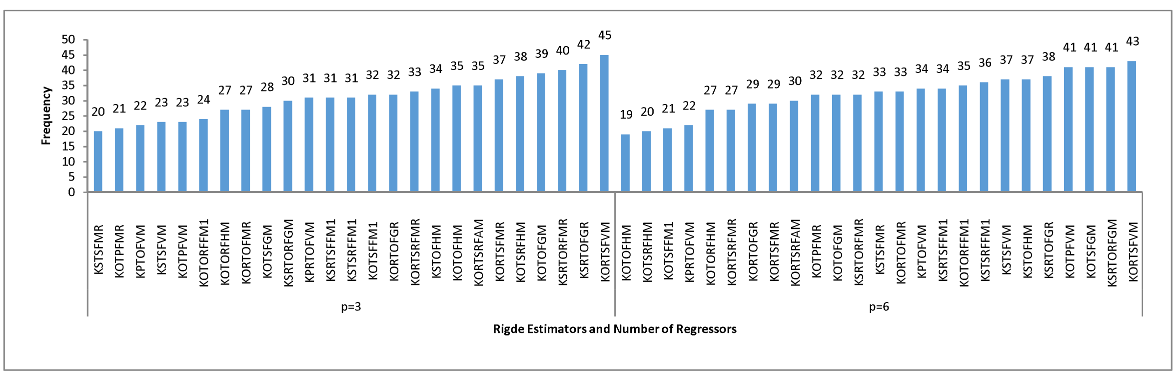
The illustration above is further presented pictorially in Figures III

Table III: Number of times the overall results existing and proposed estimators produced minimum MSE when ranked between 1 and 25

P=3									P=6								
	SAMPLE SIZES									**SAMPLE SIZES**							
ESTIMATORS	**10**	**20**	**30**	**50**	**100**	**150**	**250**	**T**	**ESTIMATORS**	**10**	**20**	**30**	**50**	**100**	**150**	**250**	**T**
KSTSFMR	2	3	2	3	3	3	4	20	KOTOFHM	2	0	1	2	4	5	5	19
KOTPFMR	1	3	3	3	3	4	4	21	KOTSRFHM	2	4	2	3	3	3	3	20
KPTOFVM	0	4	3	3	4	4	4	22	KOTSFFM1	3	4	1	4	2	4	3	21
KOTPFVM	1	4	3	3	4	4	4	23	**KPRTOFVM**	4	2	3	3	8	1	1	22
KSTSFVM	1	4	3	3	4	4	4	23	**KORTSRFMR**	3	4	4	5	2	5	4	27
KOTORFFM1	3	4	4	2	5	2	4	24	KOTORFHM	2	4	5	5	3	4	4	27
KOTORFHM	1	2	3	5	5	5	6	27	**KORTOFGR**	5	5	5	4	4	4	2	29
KOTSFGM	4	2	2	5	5	5	5	28	KORTSRFAM	4	4	2	5	4	6	5	30
KSTSRFFM1	6	5	4	3	3	4	6	31	**KOTPFMR**	0	2	4	5	6	7	8	32

KPRTOFVM	6	5	4	3	3	4	6	31	**KSTSFMR**	1	3	4	5	7	6	7	33
KORTOFGR	6	5	5	5	4	3	4	32	**KORTOFMR**	2	5	4	5	5	6	6	33
KOTSFFM1	2	4	4	5	5	6	6	32	**KSRTSFFM1**	7	6	5	5	4	5	2	34
KORTSRFMR	4	4	5	2	6	6	6	33	**KPTOFVM**	1	6	5	6	8	7	1	34
KSTOFHM	5	3	2	5	4	8	7	34	KOTORFFM1	2	3	6	6	6	6	6	35
KOTOFHM	0	3	4	6	6	8	8	35	**KSTSRFFM1**	7	6	5	5	4	5	4	36
KORTSRFAM	4	4	4	5	6	6	6	35	KSTOFHM	2	5	2	7	7	8	6	37
KOTSRFHM	4	5	5	5	7	7	5	38	**KSRTOFGR**	5	7	7	4	4	4	7	38
KOTOFGM	0	4	6	6	7	8	8	39	**KOTPFVM**	1	6	5	6	8	7	8	41

Figure III: Bar Chart showing the frequency of overall counts of ridge estimators that ranked between 1 and 25



Conclusion and Recommendations

This study discussed the implications of the presence of multicollinearity in estimating the parameters in a linear regression model and identified the ridge regression as a panacea to this problem. The proposed and existing estimators were examined in the light of the newly introduced concept of various kinds into the existing classification of various types and different forms. The preferred estimators resulting from these classifications were identified in the light of the various kind newly introduced. For Original Kind, when p=3, the proposed estimator **KOTSFFM1** performed best. When p=6, the estimators **KOTSFHM** and **KOTORFFM1** are preferred having produced the highest number of times MSE is minimum when ranked over the different levels of multicollinearity and variance. Also for Square Root of Original Kind, when p=3 and p=6, the proposed estimator **KSTSFVM** is preferred having produced the highest number of times MSE is minimum when ranked over the different levels of multicollinearity and variance. Similarly, for Pth Root of Original Kind, when p=3 and p=6, the proposed estimators **KPTORFFM2** and **KPTORFFM1** are preferred having produced the highest number of times MSE is minimum when ranked over the different levels of multicollinearity and variance.

Recommendation

This study was restricted to simulation study. However, findings in this study can be practicalized with a real life data with high confidence. The recommended biasing ridge parameters with their estimators are presented as follows:

The recommended best/most efficient biasing parameters from the study are as follows:

$$\hat{k}_i = \left(\frac{Max(\alpha_i^2)}{\hat{\sigma}^2} \right) \qquad \text{KOTORFFM1} \qquad (17)$$

Where $\hat{\sigma}^2$ is the Mean Square Error from the OLS regression and $\hat{\alpha}_i$ is the regression coefficient from the OLS regression,

$$\hat{k}_i = \sqrt{\left(\frac{Max(\alpha_i^2)}{\hat{\sigma}^2}\right)} \quad \text{KSTORFFM1} \quad (18)$$

$$\hat{k}_i = \sqrt{\left(\frac{[Max(\hat{\alpha}_i)]^2}{\hat{\sigma}^2}\right)} \quad \text{KSTORFFM2} \quad (19)$$

$$\hat{k}_i = Max\left[\sqrt[p]{\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i}\right)}\right] \quad \text{KPTOFVM} \quad (20)$$

Where $\hat{\sigma}^2$ is the Mean Square Error from the OLS regression and $\hat{\alpha}_i$ is the regression coefficient from the OLS regression, and p is the number of parameter.

REFERENCES

Alkhamisi, M., Khalaf, G. and Shukur, G. (2006). Some modifications for choosing ridge parameters.Communications in Statistics- Theory and Methods, 35(11), 2005-2020.

Alkhamisi, M. and Shukur, G. (2008). Developing Ridge Parameters for SUR Model. Communications in Statistics – Theory and Methods, 37(4), 544-564.

Duzan, H.M.B and Shariff, S.M (2015). Ridge Regression for Solving the Multicollinearity Problem: Review of Methods and Models. Journal of Applied Sciences, 15: 392-404.

Green, W.H. (1993). Econometric Analysis, Mac-Millan, New York. (7th Edition).

Gujarati, D.N. (1995) Basic Econometrics, Mcgraw-Hill, New York. (3rd Edition).

Gibbons, D. G. (1981). A Simulation Study of Some Ridge Estimators. Journal of the American Statistical Association, 76, 131-139.

Hoerl, A.E. and Kennard, R.W. (1970). Ridge regression: Biased Estimation for Non-Orthogonal Problems. Technometrics, 12, 55-67.

Hoerl, A. E., Kennard, R. W. and Baldwin, K. F. (1975). Ridge Regression: Some Simulation. Communications In Statistics 4 (2), 105-123.

Hocking, R. R., Speed, F. M., and Lynn, M. J. (1976). A Class of Biased Estimators in Linear Regression. Technometrics, 18(4), 55-67.

Judge, G.G. (1988). Introduction to Theory and Practice of Econometrics, John Willy and Sons, New York.

Khalaf, G. and Iguernane, M. (2016).Multicollinearity and A Ridge Parameter Estimation Approach. Journal of Modern Applied Statistical Methods. 15(2), 25.

Khalaf, G. and Shukur, G. (2005). Choosing ridge parameters for regression problems. Communications in Statistics- Theory and Methods, 34, 1177-1182.

Kibria, B. M. G. (2003). Performance of Some New Ridge Regression Estimators. Communications in Statistics - Simulation and Computation, 32, 419-435.

Kibria, B. M. G. and Banik, S. (2016). Some Ridge Regression Estimators and Their Performances. Journal of Modern Applied Statistical Methods. 15(1), 206-238.

Lawless, J. F. and Wang, P. (1976). A simulation study of ridge and other regression estimators. Communications in Statistics A, 5, 307-323.

Lukman, A. F. and Ayinde, K. (2015): Review and Classification of the Ridge Parameter Estimation Techniques. Hacettepe Journal of Mathematics and Statistics 46 (5), 953-967

Mansson, K., Shukur, G. and Kibria, B. M. G. (2010). A simulation study of some ridge regression estimators under different distributional assumptions. Communications in Statistics-Simulations and Computations, 39(8), 1639 – 1670.

McDonald, G. C. and Galarneau, D. I. (1975). A Monte Carlo evaluation of some ridge-type estimators. Journal of the American Statistical Association, 70, 407-416.

Montgomery D. C, Peck E.A and Vini G.G (2001). Introduction to Linear Regression Analysis, 3rd Edition

Muniz, G. and Kibria, B. M. G. (2009). On Some Ridge Regression Estimators: An empirical comparison. Communications in Statistics – Simulation andComputation, 38(3), 621-630.

Saleh, A. K. Md. E. and Kibria, B. M. G. (1993). Performances of some new preliminary test ridge regression estimators and their properties. Communications in Statistics-Theory and Methods, 22, 2747-2764.