On the application of the New Power Topp-Leone Inverse Lomax distribution to the Bladder Cancer Data

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Abstract

We appliedthe New Power Topp-Leone Inverse Lomax (NPTLIL) distribution to the Bladder Cancer dataset. We also compared the NPTLIL distribution with some extensions of the Inverse Lomax distribution in the literature that also used the same dataset. We provided the PDFs and CDFs of the distributions usedasdefined by their respective authors. The model selection criteria used in this current study are the AIC and BIC. The NPTLIL was found to outperform some existingdistributions in the literatures and to be more flexible.We also concluded that the NPRLIL distribution was able to bit others just because of two reasons: the scale parameter of the base line distribution was fixed at 1 and also the family was a combination of two families making it more flexible.

Keywords: Akaike's Information Criteria; Bayesian Information criteria;Inverse Lomax Distribution; Moments; Maximum Likelihood Estimation.

1.0 Introduction

For the last three years, there has been in the literature a lot of extensions of the Inverse Lomax (IL) distribution with the sole aim of achieving flexibility especially by adding a shape parameter to the IL distribution. The IL distribution belongs to the Beta-Type size distributions along sides the Pareto I, the Dagum, Beta distribution of

the second kind (B2), and the generalized beta of the second kind (GB2) as family members (Kleiber&Kotz 2003). The distribution has a lot of applications in actuarial sciences, risk management, and finance. Some of the researchers that studied the IL distribution include the Kleiber (2004), Singh *et al*. (2012), Rahman *et al*. (2013), and Yadav*et al*. (2016).

Rady*et al*. (2016) proposed a three-parameter continuous distribution by name Power Lomax distribution (POLO). POLO distribution accommodate both inverted bathtub and decreasing hazard rate. The authors derived some statistical and reliability properties of the proposed distribution. The estimation of the unknown parameters were done via methods of moments (MOM) and maximum likelihood (ML) and the interval estimation werealso studied. The simulation schemes werecalculated to examine the bias and mean squared error (MSE) of the ML parameter estimators. They also applied the POLO distribution to the remission time of bladder cancer in order to illustrate the usefulness of the proposed distribution in modelling reallife datasets. The comparison showed that the POLO distribution outperformed most well-known extensions of Lomax distribution.

Hassan and Abd-Allah (2019) introduced and studied the Inverse Power Lomax (IPL) distribution. Some statistical properties of the IPL model including the quantile function, moments, skewness, kurtosis, and inverse moments were implemented. The authors considered the parameter estimation for IPL distribution when the available data are in the form of Type-I and Type II censoring schemes. They also performed an intensive simulation study just to evaluate the behavior of the estimators based on their Biases and MSEs. Superiority of the new model over some well-known distributions was illustrated by means of reallife datasets i.e the 72 Guinea pigs data set and the relief times of twenty patients receiving a treatment. The results revealed that the suggested model can produce better fits than some well-known distributions.

Falgore *et al*. (2018) extended the IL distribution with the Odd generalized exponential family by Tahir *et al*. (2015) and named the proposed distribution Odd Generalized Exponential Inverse Lomax (OGEIL) distribution. Theyalso studied some mathematical properties of the OGEIL distribution such as quantile function, moments, order statistics and asymptotic behavior. They estimated the OGEIL distribution's parameters using the method of ML, and finally they illustrated the usefulness of the proposed model by fitting the proposed distribution using a data set of 121 patients being infected with Breast cancer, and compared its performance to

the performance of other distributions in the literature. It was found that the OGEIL distribution is a strong competitor in fitting real life data.

Maxwell *et al*. (2019) proposed a new compound distribution called the Marshall-Olkin Inverse Lomax (MOIL) distribution. They extended the IL distribution by adding a new parameter to the existing distribution, leading to greater flexibility in modeling different datasets. Its basic statistical properties were derived and the model parameters were estimated using the method of ML. Theyhowever, applied the MOIL to Cancer Stem Cell data and compared to the Marshall Olkin Flexible Weibull Extension Distribution (MO-FWED), and the Marshall-Olkin exponential Weibull distribution (MO-EWD). The Marshall-Olkin Inverse Lomax distribution provided a better fit than the Marshall Olkin Flexible Weibull Extension Distribution, and the Marshall-Olkin exponential Weibull distribution.

Falgore *et al*. (2019) introduced a four-parameter probability model called Weibull-Inverse Lomax (WIL) distribution with decreasing, increasing and bathtub hazard rate functions. The WIL distribution density function is J-shaped, positively skewed, and J-shaped in reverse. The authors derived some of the statistical characteristics of the WIL distribution including moments, order statistics, entropy, mean, variance, moment generating function, and quantile function. They employed the method of ML estimation to estimate the parameters of the model. The distribution's importance was proved by its implementation to the bladder cancer dataset. Goodness-of-fit of this distribution by various techniques demonstrates that the WIL distribution is empirically better for lifetime application based on the dataset used.

Bantan*et al*. (2019) introduced a new general family of distributions obtained by a subtle combination of two well-established families of distributions: the so-called power Topp–Leone-G and inverse exponential-G families and called it aNew Power Topp-leone Generated family. Its definition is centered around an original cumulative distribution function involving exponential and polynomial functions. They discussed some desirable theoretical properties of the new family in details, with comprehensive results on stochastic ordering, quantile function and related measures, general moments and related measures, and the Shannon entropy. The authors also constructed from the family, a special member by name "IL distribution" as the baseline distribution. The special sub-model is called New Power Topp-Leone Inverse Lomax (NPTLIL) distribution. They applied the method of ML to estimate the unknown model parameters. The authors conducted a simulation study to evaluate

the numerical behavior of the estimates they obtained. Finally, in order to highlight the practical perspectives of the new family, two real-life data sets were analyzed. All the measures considered are favorable to the new model in comparison to four other serious competitors.

The objectives of this paper are to fit the NPTLIL model to the Bladder cancer dataset as most of the extensions of the IL distribution fitted the data, and also to compare the performances of each of the models mentioned above with respect to the Bladder dataset.

3.0 Methodology

Since this study is based on the existing methods already in the literature and our aim is to fit the NPTLIL model by Bantan*et al*. (2019) to Bladder cancer dataset and then compare with some IL models, we are stating the PDFs and the CDFs of those models as they appeared in the literature.

PDFs and CDFs of the Models

The IL distribution as defined by Yadav*et al* (2016) is given by:

$$g_{IL}(x;\sigma,\tau)=\frac{\sigma\tau}{x^2}\left(1+\frac{\tau}{x}\right)^{-(1+\sigma)} \tag{1}$$

$$\text{and } G_{IL}\left(x;\sigma,\tau\right)=\left(1+\frac{\tau}{x}\right)^{-\sigma} \qquad x\geq 0,\sigma,\tau>0 \tag{2}$$

Where σ is the shape parameter and τ is the scale parameter of the distribution respectively.

Bantan*et al.* (2019) assumes the scale parameter to be 1. Therefore, equations (1) and (2) becomes:

$$g_{ILx}(x;\sigma)=\frac{\sigma}{x^2}\left(1+\frac{1}{x}\right)^{-(1+\sigma)} \tag{3}$$

$$\text{and } G_{ILx}\left(x;\sigma\right)=\left(1+\frac{1}{x}\right)^{-\sigma} \qquad x\geq 0,\sigma>0 \tag{4}$$

The PDF and CDF of the POLO distribution as defined by Rady*et al*. (2016) are given in equations (5) and (6) below:

$$f_{POLO}\left(x\right) = \sigma\tau\delta^{\sigma}x^{(\tau-1)}\left(\delta+x^{\tau}\right)^{-(\sigma+1)} \tag{5}$$

and $F_{POLO}\left(x\right)=1-\delta^{\sigma}\left(x^{\tau}+\delta\right)^{-\sigma}$ $\quad x,\sigma,\tau,\delta>0$ (6)

However, the PDF and CDF of the IPL distribution as outlined by Hassan and Abd-Allah (2019) are given below:

$$f_{IPL}\left(x\right)=\frac{\sigma\tau}{\delta}x^{-(\tau-1)}\left(1+\frac{x^{-\tau}}{\delta}\right)^{-(\sigma+1)} \tag{7}$$

and $F_{IPL}\left(x\right)=\left(1+\frac{x^{-\tau}}{\delta}\right)^{-\sigma}$ $\quad x,\sigma,\tau,\delta>0$ (8)

Also, the PDF and CDF of the OGEIL as defined by Falgore *et al*. (2018) are given below:

$$f_{OGEIL}\left(x\right)=\frac{\sigma\tau\delta\lambda\left(1+\frac{\tau}{x}\right)^{\sigma-1}x^{-2}\exp\left\{-\tau\left(\frac{\left(1+\frac{\lambda}{x}\right)^{-\sigma}}{1-\left(1+\frac{\lambda}{x}\right)^{-\sigma}}\right)\right\}}{\left(1-\exp\left\{-\tau\left(\frac{\left(1+\frac{\lambda}{x}\right)^{-\sigma}}{1-\left(1+\frac{\lambda}{x}\right)^{-\sigma}}\right)\right\}\right)^{1-\sigma}\left(\left(1+\frac{\lambda}{x}\right)^{\sigma}-1\right)^{2}} \tag{9}$$

and $F_{OGEIL}\left(x\right)=\left(1-\exp\left\{-\tau\left(\frac{\left(1+\frac{\lambda}{x}\right)^{-\sigma}}{1-\left(1+\frac{\lambda}{x}\right)^{-\sigma}}\right)\right\}\right)^{\sigma}$ $\quad x,\sigma,\tau,\delta,\lambda>0$ (10)

The PDF and CDF of the MOIL as defined by Maxwell *et al.* (2019) are:

$$f_{MOIL}(x) = \frac{\sigma\tau\delta\left(1+\frac{\tau}{x}\right)^{-(1+\sigma)} x^{-2}}{\left[1-(1-\delta)\left[1-\left(1+\frac{\tau}{x}\right)^{-\sigma}\right]\right]^2} \tag{11}$$

$$\text{and } F_{MOIL}(x) = \frac{\left(1+\frac{\tau}{x}\right)^{-\sigma}}{\left[1-(1-\delta)\left[1-\left(1+\frac{\tau}{x}\right)^{-\sigma}\right]\right]} \qquad x,\sigma,\tau,\delta > 0 \tag{12}$$

So also, the PDF and CDF of the WIL distribution as defined by Falgore *et al*. (2019) are given below:

$$f_{WIL}(x) = \frac{\sigma\tau\delta\lambda x^{-2}\left(1+\frac{\delta}{x}\right)^{-(1+\lambda)} \exp\left\{-\sigma\left[\frac{\left(1+\frac{\delta}{x}\right)^{-\lambda}}{1-\left(1+\frac{\delta}{x}\right)^{-\lambda}}\right]^{\tau}\right\}}{\left[1-\left(1+\frac{\delta}{x}\right)^{-\lambda}\right]^{-(\tau+1)}} \tag{13}$$

$$\text{and } F_{WIL}(x) = 1-\exp\left\{-\sigma\left[\frac{\left(1+\frac{\delta}{x}\right)^{-\lambda}}{1-\left(1+\frac{\delta}{x}\right)^{-\lambda}}\right]^{\tau}\right\} \qquad x,\sigma,\tau,\delta,\lambda > 0 \tag{14}$$

The PDF and CDF of the NPTLIL distribution as defined Bantan*et al*. (2019) are given below:

$$f_{NPTLIL}(x) = \frac{2\sigma\tau\delta x^{-2}\left(1+x^{-1}\right)^{\delta-1}\exp\left\{\sigma\tau\left[1-\left(1+x^{-1}\right)^{\delta}\right]\right\}\left\{1-\exp\left\{\tau\left[1-\left(1+x^{-1}\right)^{\delta}\right]\right\}\right\}}{\left\{2-\exp\left\{\tau\left[1-\left(1+x^{-1}\right)^{\delta}\right]\right\}\right\}^{\sigma-1}}$$

(15)

and

$$F_{NPTLIL}(x) = \exp\left\{\sigma\tau\left[1-\left(1+x^{-1}\right)^{\delta}\right]\right\}\left\{2-\exp\left\{\tau\left[1-\left(1+x^{-1}\right)^{\delta}\right]\right\}\right\}^{\sigma},\ x,\sigma,\tau,\delta>0$$

(16)

3.1 The Model Selection Criteria for the Analysis

Log-Likelihood Function

Let $X_1, X_2, X_3, X_4, ...X_n$ have a joint density function $f(X_1, X_2, X_3, X_4, ...X_n \mid \Omega)$ then, given that $X_1 = x_1, X_2 = x_2, X_3 = x_3, X_4 = x_4, ...X_n = x_n$ is observed, then the function ofΩ defined by

$$L(\Omega) = L(x_1, x_2, x_3, x_4, ...x_n) = f(x_1, x_2, x_3, x_4, ...x_n \mid \Omega) \quad (17)$$

Is the likelihood function. Taking the natural logarithm of equation (17) will give the log-likelihood function.

Akaike's Information Criteria AIC)

The AIC tries to select the model that most adequately describes an unknown, high dimensional reality. This means that reality is never in the set of candidate models that are being considered.

$$AIC = 2par - 2\log(L) \quad (18)$$

Bayesian Information Criteria (BIC)

BIC tries to find the TRUE model among the set of candidates.

$$BIC = \log(n)par - 2\log(L) \quad (19)$$

Where par is the number of the parameters in the model, L is likelihood function, and n is the sample size.

4.0 Results and Discussion

Table 1: Summary of the Bladder Cancer Dataset

Minimum	1st quartile	Median	Mean	3rd Quartile	Maximum
0.080	3.348	6.395	9.366	11.838	79.050

Table 2: MLEs for the Models

Models	σ	τ	δ	λ
NPTLIL	**0.2465**	**0.0932**	**142.0638**	**0.1293**
OGEIL	**0.8749**	**2.5967**	**1.4153**	**15.8219**
POLO	**2.0701**	**1.4276**	**34.8626**	**-**
WIL	**38.6899**	**0.0932**	**4.1773**	**15.7365**
INPL	**0.6136**	**2.1147**	**0.0098**	**-**
MOIL	**2.1024**	**0.6054**	**1.1804**	**-**
IL	**-**	**-**	**4.1170**	**1.0000**

Table 3: The Values of the –L, AIC, and BIC

Models	-L	AIC	BIC	Rank
NPTLIL	199.28495	404.5699	413.1260	1
WIL	353.4222	714.8443	726.2520	2
INPL	409.71405	825.4301	833.9862	3
POLO	409.7400	825.4800	834.7000	4
OGEIL	412.9428	833.8856	845.2930	5
IL	426.8402	855.6804	858.5240	6
MOIL	488.114	982.228	990.7841	7

Discussions

It is clear from Table 3 that the NPTLIL is the best model. The AIC and BIC values of the NPTLIL are the lowest. We therefore, recommend it as the best model based on the dataset.

Conclusions

Based on the model selection criteria in Table 3, NPTLIL is the best. The reason behind its superiority may be due to; setting the scale parameter of the baseline (IL) distribution to 1; and It's a combination of two families of distributions.

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