ON EFFICIENT MODIFIED MEDIAN BASED RATIO ESTIMATOR FOR POPULATION MEAN

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Abstract

For the past decade, estimation of population parameters especially mean is one of the challenging aspects in sampling theory and population studies, and it receives wide attention from research community. This paper proposes an efficient exponential median based ratio estimator using Subramani (2016) and Kadilar and Cingi (2003) methods of estimating population mean. The bias and MSE of the proposed estimator are derived up to first order approximation alongside the efficiency conditions. A real life dataset is used to compare the efficiency of the propose estimator with respect to some existing efficient estimators, findings from the empirical analysis show a significant efficiency gain, hence, the estimator is recommended for practical application.

Keywords: Finite population mean, mean square error, median based, and exponential.1.0

1.0 INTRODUCTION

Cochran (1940) was the first to discuss the problem of estimation of population mean when auxiliary variable is present and proposed the classical ratio estimator of population mean which utilizes the ratio of population and sample mean of any supporting variable which is highly positively correlated with variable under study. Sisodia and Dwivedi (1981) proposed ratio-type estimator by the used of the linear combination of mean and coefficient of variation of the auxiliary variable to estimate the population mean of the study variable.

In 1991, Bahl and Tuteja defined exponential ratio and product type estimators of population mean. The main significance of the exponential estimators is that they gives a more efficient result than mean per unit estimator and ratio estimator even if the degree of relationship between the study and auxiliary variable is weak.In recent year (2016) Subramani proposed an efficient estimator of population mean using the median of the study variable in the absence of auxiliary variable. For more details, see the work of Abdullahi and Yahaya (2017), Srija*et.al.* (2016), Srija and Subramani (2018), e.t.c.

2.0 BACKGROUND OF THE STUDY

Consider a finite population $P = (P_1, P_2, ..., P_N)$ of N units, let a sample be drawn using simple random sampling without replacement (SRSWOR), let y_i and x_i represent the values of a study variable Y and auxiliary variable X respectively. The units of the finite population are identifiable in the sense that they are uniquely labeled from 1 to N and the label on each unit is known. Further, suppose in a sample survey problem, we are interested in estimating the population mean $\bar{Y}$ of y, when the population median of study variable is known in presence of auxiliary variable.

The Median based ratio type estimator proposed by Subramani (2016) is given as,

$$\bar{y}_{S-mdeian-based} = \bar{y}\left(\frac{M}{m}\right) \tag{2.1}$$

To the first degree of approximation the mean squared error of $\bar{y}_{S-mdeian-based}$ is given as,

$$MSE\left(\bar{y}_{S-mdeian-based}\right) = V(\bar{y}) + R'^2 V(m) - 2R' cov(\bar{y}, m) \tag{2.2}$$

The exponential median based ratio type estimator is given as,

$$\bar{y}_{\exp-mdeian-based} = \bar{y}\exp\left(\frac{M-m}{M+m}\right) \qquad (2.3)$$

To the first degree of approximation the mean squared error of $\bar{y}_{\exp-mdeian-based}$ is given as,

$$MSE\left(\bar{y}_{\exp-mdeian-based}\right) = \bar{Y}^2\left[C'_{yy} + \frac{1}{4}C'_{mm} - C'_{ym}\right] = V(\bar{y}) + \frac{R'^2 V(m)}{4} - R'c\text{ov}(\bar{y}, m) \qquad (2.4)$$

where, $C'_{yy} = \frac{V(\bar{y})}{\bar{Y}^2}, C'_{xx} = \frac{V(\bar{x})}{\bar{X}^2}, C'_{mm} = \frac{V(m)}{M^2}, C'_{xy} = \frac{Cov\left(\bar{y}, \bar{x}\right)}{\bar{X}\bar{Y}}, C'_{xm} = \frac{Cov\left(\bar{x}, m_d\right)}{\bar{X}M}$, $C'_{ym} = \frac{Cov\left(\bar{y}, m_d\right)}{\bar{Y}M}$

3.0 Proposed Estimator

Motivated by the work of Subramani (2016),we defined a median based ratio estimator of population mean using some known one dimension parameters of auxiliary variable in simple random sampling scheme as,

$$\bar{y}_{uum-based} = \frac{\bar{y}}{2}\left\{\left(\frac{M}{m}\right) + \left(\frac{\bar{X}+\gamma}{\bar{x}+\gamma}\right)^{\alpha}\right\} \qquad (3.1)$$

Where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ are unbiased estimators of the population means $\left(\overline{Y}, \overline{X}\right)$ respectively, where 'γ' is any known one dimension or unit free characteristics positive scalars of the auxiliary variable X and 'α'is a real constant to be determined such that the mean square error of $\bar{y}_{uum-based}$ is minimum.

3.1 bias and Mean Square Error

To obtain the bias and mean square error (MSE) of the proposed estimator $\bar{y}_{uum-based}$ in (3.1), we write

Let $\Omega_{\bar{y}} = \frac{(\bar{y} - \bar{Y})}{\bar{Y}}$, $\Omega_{\bar{x}} = \frac{(\bar{x} - \bar{X})}{\bar{X}}$ and $\Omega_m = \frac{(m - M)}{M}$

Such that $E(\Omega_{\bar{y}}) = E(\Omega_{\bar{x}}) = 0$ and $E(\Omega_m) = \frac{bias(m)}{M}$

$$E(\Omega_{\bar{y}}^2) = \frac{Var(\bar{y})}{\bar{Y}^2} = \left(\frac{1-f}{n}\right)C_y^2 = C_{yy}^! , \quad E(\Omega_{\bar{x}}^2) = \frac{Var(\bar{x})}{\bar{X}^2} = \left(\frac{1-f}{n}\right)C_x^2 = C_{xx}^!$$

$$E(\Omega_m^2) = \frac{Var(m)}{M^2} = C_{mm}^! , \quad E(\Omega_{\bar{x}}\Omega_{\bar{y}}) = \frac{Cov(\bar{x},\bar{y})}{\bar{X}\bar{Y}} = \left(\frac{1-f}{n}\right)\rho_{xy}C_xC_y = C_{xy}^!$$

Where, $E(\Omega_{\bar{y}}\Omega_m) = \frac{Cov(\bar{y},m)}{\bar{Y}M} = C_{ym}^!$, $\quad E(\Omega_m\Omega_{\bar{x}}) = \frac{Cov(\bar{x},m)}{\bar{X}M} = C_{xm}^!$

And expanding (3.1) in terms of '$\Omega_{\bar{x}}$'s, we have

$$\bar{y}_{uum-based} = \frac{\bar{Y}}{2}(1+\Omega_{\bar{y}})\left\{(1+\Omega_m)^{-1} + (1+\lambda\Omega_{\bar{x}})^{-\alpha}\right\} \quad (3.2)$$

where $\lambda = \frac{\bar{X}}{\bar{X}+\gamma}$

We assume that $|\Omega_m| < 1$, $|\lambda\Omega_{\bar{x}}| < 1$ so that the expression, $(1+\Omega_m)^{-1}$, $(1+\lambda\Omega_{\bar{x}})^{-\alpha}$ can be expanded to a convergent infinite series using binomial theorem.

$$(1+\Omega_m)^{-1} = 1 - \Omega_m + \Omega_m^2 \quad , \quad (1+\lambda\Omega_{\bar{x}})^{-\alpha} = 1 - \alpha\lambda\Omega_{\bar{x}} + \alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^2}{2}$$

We also assume that the contribution of terms involving powers in $\Omega_m, \Omega_{\bar{y}}, \Omega_{\bar{y}}$ higher than the second is negligible, being of order $1/n^v$, where $v > 1$. Thus, from the above expression we write to a first order of approximation. . Hence from (3.2) we have.

$$\bar{y}_{uum-based} = \bar{Y}(1+\Omega_{\bar{y}})\left\{\left(1-\Omega_m+\Omega_m^{\ 2}\right)+\left(1-\alpha\lambda\Omega_{\bar{x}}+\alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^{\ 2}}{2}\right)\right\} \tag{3.3}$$

$$= \bar{Y}\left\{\begin{array}{l}\left(1-\Omega_m+\Omega_m^{\ 2}+\Omega_{\bar{y}}-\Omega_m\Omega_{\bar{y}}+\Omega_m^{\ 2}\Omega_{\bar{y}}\right)\\ +\left(1-\alpha\lambda\Omega_{\bar{x}}+\alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^{\ 2}}{2}+\Omega_{\bar{y}}-\alpha\lambda\Omega_{\bar{x}}\Omega_{\bar{y}}+\alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^{\ 2}\Omega_{\bar{y}}}{2}\right)\end{array}\right\} \tag{3.4}$$

$$\bar{y}_{uum-based} - \bar{Y} = \frac{\bar{Y}}{2}\left\{\begin{array}{l}\left(-\Omega_m+\Omega_m^{\ 2}+\Omega_{\bar{y}}-\Omega_m\Omega_{\bar{y}}+\Omega_m^{\ 2}\Omega_{\bar{y}}\right)\\ +\left(-\alpha\lambda\Omega_{\bar{x}}+\alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^{\ 2}}{2}+\Omega_{\bar{y}}-\alpha\lambda\Omega_{\bar{x}}\Omega_{\bar{y}}+\alpha(\alpha+1)\frac{\lambda^2\Omega_{\bar{x}}^{\ 2}\Omega_{\bar{y}}}{2}\right)\end{array}\right\} \tag{3.5}$$

Taking the expectation of both side of (3.5), we obtained the bias of $(\bar{y}_{uum-based})$ to the first degree of approximation as,

$$bias(\bar{y}_{uum-based}) = \frac{\bar{Y}}{2}\left\{\left(-C^!_m+C^!_{mm}-C^!_{my}\right)+\left(\alpha(\alpha+1)\frac{\lambda^2 C^!_{xx}}{2}-\alpha\lambda C^!_{xy}\right)\right\} \tag{3.6}$$

Squaring both side of the equation (3.5), and neglecting the terms of $\Omega's$ having power greater than two we have,

$$(\bar{y}_{uum-based}-\bar{Y})^2 = \bar{Y}^2\left\{\Omega_{\bar{y}}^{\ 2}-\Omega_{\bar{y}}\Omega_m-\alpha\lambda\Omega_{\bar{x}}\Omega_{\bar{y}}+\frac{\alpha\lambda\Omega_{\bar{x}}\Omega_m}{2}+\frac{\Omega_m^{\ 2}}{4}+\frac{\alpha^2\lambda^2\Omega_{\bar{x}}^{\ 2}}{4}\right\} \tag{3.7}$$

Taking the expectation of both sides of (3.7), we get the MSE of $\bar{y}_{uum-based}$ as,

$$MSE(\bar{y}_{uum-based}) = \bar{Y}^2 \left\{ \frac{Var(\bar{y})}{\bar{Y}} - \frac{Cov(m,\bar{y})}{M\bar{Y}} - \alpha\lambda \frac{Cov(\bar{x},\bar{y})}{\bar{X}\bar{Y}} + \frac{\alpha\lambda Cov(\bar{x},M)}{2\bar{X}M} + \frac{Var(m)}{4M} + \frac{\alpha^2\lambda^2 Var(\bar{x})}{4\bar{X}^2} \right\} \quad (3.8)$$

$$MSE(\bar{y}_{uum-based}) = \bar{Y}^2 \left\{ C^!_{yy} - C^!_{my} - \alpha\lambda C^!_{xy} + \frac{\alpha\lambda C^!_{xm}}{2} + \frac{C^!_{mm}}{4} + \frac{\alpha^2\lambda^2 C^!_{xx}}{4} \right\} \quad (3.9)$$

The minimum $MSE(\bar{y}_{uum-based})$ is obtain for the optimal value of α which is

$$\frac{\partial MSE(\bar{y}_{uum-based})}{\partial\alpha} = \left\{ \frac{\lambda C^!_{xm}}{2} - \lambda C^!_{xy} + \frac{\alpha\lambda^2 C^!_{xx}}{2} \right\} = 0 \quad (3.10)$$

$$\Rightarrow \alpha = \frac{2C^!_{xy} - C^!_{xm}}{\lambda C^!_{xx}} \quad (3.11)$$

Therefore, the minimum MSE of the proposed estimator is obtained by substituting (3.11) in (3.9)

$$MSE(\bar{y}_{uum-based})_{\min} = \bar{Y}^2 \left\{ C^!_{yy} - C^!_{my} + \frac{C^!_{mm}}{4} + \frac{C^!_{xm}C^!_{xy}}{C^!_{xx}} - \frac{C^{!\,2}_{xy}}{C^!_{xx}} - \frac{\left(C^!_{xm}\right)^2}{4C^!_{xx}} \right\} \quad (3.12)$$

4. Theoretical Efficiency Comparison of Proposed Estimators with Existing Estimators

4.1. The usual unbiased estimator

The proposed estimator is more efficient than mean per unit estimator in SRSWOR if $MSE(\bar{y}_{uum-based}) \le MSE(\bar{y})$,i.e

$$\Rightarrow C^!_{my} + \alpha\lambda C^!_{xy} \le \frac{\alpha\lambda C^!_{xm}}{2} + \frac{C^!_{mm}}{4} + \frac{\alpha^2\lambda^2 C^!_{xx}}{4}$$

4.2. The usual ratio estimator.

The proposed estimator is more efficient than the usual ratio estimator if
$MSE(\bar{y}_{uum-based1}) \leq MSE(\bar{y}_r)$

$$\frac{\alpha\lambda RR^{'}Cov(m,\bar{x})}{2}+\frac{R^{'2}Var(m)}{4}-R^{'}Cov(\bar{y},m)\leq -2RCov(\bar{y},\bar{x})+R^{2}Var(\bar{x})+\alpha\lambda RCov(\bar{y},\bar{x})-\frac{\alpha^{2}\lambda^{2}R^{2}Var(\bar{x})}{4}$$

4.3. The linear regression estimator

The proposed estimator is more efficient than linear regression estimator if
$MSE(\bar{y}_{uum-based1}) \leq MSE(\bar{y}_{regression})$,i.e

$$\left\{-R^{'}Cov(\bar{y},m)-\alpha\lambda RCov(\bar{y},\bar{x})+\frac{\alpha\lambda RR^{'}Cov(m,\bar{x})}{2}+\frac{Var(m)R^{'2}}{4}+\frac{\alpha^{2}\lambda^{2}Var(\bar{x})R^{2}}{4}\right\}\geq\left(\frac{(Cov(\bar{y},\bar{x}))^{2}}{Var(\bar{x})}\right)$$

4.5. Subramani (2016) proposed estimator

The proposed estimator is more efficient than median based ratio estimator, if
$MSE(\bar{y}_{uum-based1}) \leq MSE(\bar{y}_{median-based})$,i.e

$$\frac{3C^{!}_{mm}}{4}-C^{!}_{ym}\geq -\alpha\lambda C^{!}_{xy}+\frac{\alpha\lambda C^{!}_{xm}}{2}+\frac{\alpha^{2}\lambda^{2}C^{!}_{xx}}{4}$$

4.6. Bahl and Tutejamedian basedestimator

The proposed estimator is more efficient than exponential median based ratio estimator in if

$MSE(\bar{y}_{uum-based1}) \leq MSE(\bar{y}_{uum-based})$,i.e

$$C^{!}_{xy}\geq\frac{C^{!}_{xm}}{2}+\frac{\alpha\lambda C^{!}_{xx}}{4}$$

5.0 Numerical Comparison

In this section we see the merit of the proposed estimator $\bar{y}_{uum-based}$ over $\bar{y}$, $\bar{y}_r$, $\bar{y}_{linear-regression}$ and $\bar{y}_{\exp-median-based}$ estimators.

Dataset: The populations considered in this study are real-life dataset and is taken from Singh and Chaudhary (1986). The population pertain to estimate the area of cultivation under wheat in the year 1974 (*Y*) and the auxiliary variable is the cultivated areas under wheat in 1971 (*X*). the data set is also used by Srija andSubramani(2018).

Table 1 below summarized the dataset for the study and the corresponding functions to be used for numerical analysis.

Table 1: Summary of the Dataset

Parameter		Parameter		Parameter		Parameter	
N	34	R	4.0999	$Cov(\bar{y},m)$	90236.294	C'_{ym}	0.1372841
N	3	R'	1.1158	$Cov(\bar{y},\bar{x})$	15061.401	C'_{yx}	0.0841917
$\bar{Y}$	856.42	$V(\bar{y})$	163356.41	$Cov(\bar{x},m)$	18342.18	C'_{xm}	0.1144118
$\bar{X}$	208.88	$V(\bar{x})$	6884.45	C'_{yy}	0.222726	$bias(m)$	-19.77774
$\bar{M}$	747.72	$V(m)$	101518.77	C'_{xx}	0.1577848	$bias(m)/M$	- 0.0257690 4
M	767.50	ρ	0.4491	C'_{mm}	0.1723414		

Table 2 below gives the MSE and PRE of the existing and proposed estimators with respect to mean per unit, usual ratio and linear regression estimators respectively.

Table 2: MSE and PRE

MSE/ Variance of some selected existing estimators and that of proposed estimator		PRE with respect to mean per unit estimator ($\bar{y}$)	PRE with respect to usual ratio estimator ($\bar{y}_r$)	PRE with respect to linear regression estimator ($\bar{y}_{linear-reg}$)
Estimator	MSE	PRE	PRE	PRE
$\bar{y}$	163356.4	100	-	-
$\bar{y}_r$	155583	-	100	-
$\bar{y}_{linear-regression}$	130408.93	-	-	100
$\bar{y}_{\exp-median-based}$	94267.18	173.29	165.04	138.34
$\bar{y}_{uum-based} = \bar{y}_{pr}$	90882.08	179.7455	171.19	143.49

Discussion

Result from Table 2 reveals that the proposed estimator has the least mean square error compared to some existing estimators and it also show significant efficiency gain in respect of percentage relative efficiency.

Conclusion

In this paper, we have proposed efficient median based ratio estimator with some known parameters of auxiliary variable and their linear combinations. The theoretical efficiencies of the proposed median based ratio estimators are obtained, empirical study is carried out to assessed the efficiency with respect to mean per unit estimator, ratio estimator and some of the modified ratio estimators. Result from the numerical comparison shows significant efficiency gain in the proposed estimator with respect to the existing estimators. Hence, the proposed estimator is recommended for the use in practice when the efficiency condition is satisfied.

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