On the Efficiency of Calibration Ratio-Cum-Product Estimators of Population Mean

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Abstract

In this paper, study variable was associated directly with two auxiliary variables in calibration estimator to eliminate or minimize the problem of extreme values or outliers on the calibration estimators of population mean in stratified random sampling. A class of calibration ratio-cum-product estimator was suggested and its bias and MSEas well as efficiency conditions were derived up to second degree approximation using Taylor's series expansion approach. Empirical study was conducted to investigate the efficiency of the proposed estimators over some existing related estimators considered in the study through simulation studies and the results revealed that the proposed estimators outperformed other estimators in the study.

Keywords: Calibration, Calibration Weights, Population Mean, Efficiency, Ratio-cum-product.

1. Introduction

In sample survey, calibration is a commonly used technique for producing new weights in stratified random sampling to enhance the efficiency of the estimators. These calibration weights fulfill some calibration constraints that include auxiliary information. In this direction, several other valued authors have proposed calibration estimators using single auxiliary variable. These authors include Tracy et al. (2003), Singh (2003), Estevao and Sarndal (2006), Sarndal (2007), Kim et al. (2007), Kim and Park (2010), Clement and Enang (2015), Rao et al. (2016), Koyunce and Kadilar (2016). Authors like Rao et al. (2012) and Ozgul (2018) considered two auxiliary variables in studying calibration estimators of population mean.

According to Cochran (1977), the traditional estimator of population mean in stratified sampling is defined as

$$t_0 = \sum_{h=1}^{L} W_h \bar{y}_h \qquad (1.1)$$

Most of the existing calibration estimators targeting modification of strata weights W_h, $h = 1, 2, ..., L$ in (1.1) ignoring strata sample means $\bar{y}_h$ which are easily influenced by extreme values or outliers. This study focused on the use of ratio and product estimation approaches to modify some existing calibration estimators by linking strata sample means with auxiliary information so as to control or minimize the effect of outliers on the estimate.

2. Some Existing Calibration Estimators with Two Auxiliary Variables

Rao *et al* (2012) proposed a calibration estimator with two auxiliary variables for the population mean in the stratified sampling design given by:

$$\bar{y}_{st(R)} = \sum_{h=1}^{L} \Omega_h^R \bar{y}_h \qquad (2.1)$$

where Ω_h^R are the calibration weights due to Rao et al. (2012) which minimized the chi square distance measure Z_R subjected two calibration constraints defined in (2.2)

$$\left.\begin{aligned} Z_R &= \sum_{h=1}^{L} \left(\Omega_h^R - W_h\right)^2 / Q_h W_h \\ s.t.\quad & \sum_{h=1}^{L} \Omega_h^R \bar{x}_{1h} = \bar{X}_1, \sum_{h=1}^{L} \Omega_h^R \bar{x}_{2h} = \bar{X}_2 \end{aligned}\right\} \qquad (2.2)$$

where $\bar{X}_{11} = N_h^{-1} \sum_{i=1}^{N_h} x_{1hi}, \bar{X}_{2h} = N_h^{-1} \sum_{i=1}^{N_h} x_{2hi}$.

The estimator of (2.1) was obtained as

$$\bar{y}_{st(R)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{1(R)} \sum_{h=1}^{L} W_h \left(\bar{X}_{1h} - \bar{x}_{1h} \right) + \hat{\beta}_{2(R)} \sum_{h=1}^{L} W_h \left(\bar{X}_{2h} - \bar{x}_{2h} \right) \tag{2.3}$$

where $\hat{\beta}_{1(R)} = \dfrac{\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_h \sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} - \sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_h \sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)^2 - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right) \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right)}$

$$\hat{\beta}_{2(R)} = \frac{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{y}_h \right) \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right) - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h} \bar{y}_h \right) \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right)}{\left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h} \bar{x}_{2h} \right)^2 - \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{1h}^2 \right) \left(\sum_{h=1}^{L} W_h Q_h \bar{x}_{2h}^2 \right)}$$

$$\begin{aligned} MSE\left(\bar{y}_{st(R)} \right) = \sum_{h=1}^{L} W_h^2 \theta_h \Big(& S_{yh}^2 + \hat{\beta}_{1(R)}^2 S_{x1h}^2 + \hat{\beta}_{2(R)}^2 S_{x2h}^2 - 2\hat{\beta}_{1(R)} S_{yx1h} \\ & - 2\hat{\beta}_{2(R)} S_{yx2h} + 2\hat{\beta}_{1(R)} \hat{\beta}_{2(R)} S_{x1x2h} \Big) \end{aligned} \tag{2.4}$$

where $S_{yh}^2 = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)^2$, $S_{x1h}^2 = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(x_{1hi} - \bar{X}_{1h} \right)^2$,
$S_{x2h}^2 = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(x_{2hi} - \bar{X}_{2h} \right)^2$, $S_{yx1h} = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right) \left(x_{1hi} - \bar{X}_{1h} \right)$,
$S_{yx2h} = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right) \left(x_{2hi} - \bar{X}_{2h} \right)$, $S_{x1x2h} = \left(N_h - 1 \right)^{-1} \sum_{i=1}^{N_h} \left(x_{1hi} - \bar{X}_{1h} \right) \left(x_{2hi} - \bar{X}_{2h} \right)$

Ozgul (2018) proposed a multivariate calibration estimator for the population mean in the stratified sampling design given by:

$$\bar{y}_{st(O)} = \sum_{h=1}^{L} \Omega_h^O \bar{y}_h \tag{2.5}$$

where Ω_h^O are the calibration weights due to Ozgul (2018) which minimized the chi square distance measure Z_R subjected two calibration constraints defined in (2.6)

$$\left. \begin{aligned} & Z_O = \sum_{h=1}^{L} \left(\Omega_h^O - W_h \right)^2 / Q_h W_h \\ & s.t. \quad \sum_{h=1}^{L} \Omega_h^O = \sum_{h=1}^{L} W_h, \ \sum_{h=1}^{L} \Omega_h^O \hat{R}_h = \sum_{h=1}^{L} W_h R_h \end{aligned} \right\} \tag{2.6}$$

where $\hat{R}_h = \dfrac{\bar{x}_{1h}}{\bar{x}_{2h}}$ and $R_h = \dfrac{\bar{X}_{1h}}{\bar{X}_{2h}}$ are the sample and population ratio of two auxiliary variables.

The estimator of (2.5) and its MSE were given as

$$\bar{y}_{st(O)} = \sum_{h=1}^{L} W_h \bar{y}_h + \hat{\beta}_{1(O)} \sum_{h=1}^{L} W_h \left(R_h - \hat{R}_h \right) \tag{2.7}$$

where $\hat{\beta}_{1(O)} = \dfrac{\sum_{h=1}^{L} W_h Q_h \hat{R}_h \bar{y}_h \sum_{h=1}^{L} W_h Q_h - \sum_{h=1}^{L} W_h Q_h \bar{y}_h \sum_{h=1}^{L} W_h Q_h \hat{R}_h}{\left(\sum_{h=1}^{L} W_h Q_h\right)\left(\sum_{h=1}^{L} W_h Q_h \hat{R}_h^2\right)\left(\sum_{h=1}^{L} W_h Q_h \hat{R}_h\right)^2}$

$$MSE\left(\bar{y}_{st(O)}\right) = \sum_{h=1}^{L} W_h^2 \theta_h \left(S_{yh}^2 + \hat{\beta}_{1(O)}^2 R_h^2 \bar{X}_{1h}^{-2} S_{x1h}^2 + \hat{\beta}_{1(O)}^2 R_h^2 \bar{X}_{2h}^{-2} S_{x2h}^2 - 2\hat{\beta}_{1(O)} R_h \bar{X}_{1h}^{-1} S_{yx1h} + 2\hat{\beta}_{1(O)} R_h \bar{X}_{2h}^{-1} S_{yx2h} - 2\hat{\beta}_{1(O)}^2 R_h^2 \bar{X}_{1h}^{-1} \bar{X}_{2h}^{-1} S_{x1x2h} \right) \tag{2.8}$$

Having study the estimators in section two, it was observed that their first components are functions of strata sample means of the study variable y and consequently can easily be influenced by outliers thereby making the estimators inefficient. To address this problem, concept of ratio estimation was adopted to link the sample mean with auxiliary information so as to eliminate or reduce drastically the influence of extreme values or outliers on the estimators.

3. Proposed Class of Calibration Estimators

Consider estimator defined in (3.1) under stratified sampling

$$\bar{y}_{st}^{AU} = \sum_{h=1}^{L} D_h^{AU} \bar{y}_h \tag{3.1}$$

where D_h^{AU} are new calibrated weights which are chosen such that the chi-square distance measure Z_{AU} is minimum subject to the calibration constraints defined in (3.2)

$$\left.\begin{aligned} &Z_{AU} = \sum_{h=1}^{L}\left(D_h^* - D_h^{AU}\right)^2 / D_h^*\phi_h \\ &st \qquad \sum_{h=1}^{L} D_h^{AU} = \sum_{h=1}^{L} D_h^*,\ \sum_{h=1}^{L} D_h^{AU}\hat{R}_h = \sum_{h=1}^{L} D_h^* R_h \end{aligned}\right\} \tag{3.2}$$

where $D_h^* = W_h \dfrac{\bar{X}_{1h}}{\bar{x}_{1h}} \dfrac{\bar{x}_{2h}}{\bar{X}_{2h}}$

To compute new calibrated weights D_h^{AU}, we defined Lagrange function L_* of the formed;

$$L_* = \sum_{h=1}^{L}\left(D_h^{AU} - D_h^*\right)^2 / D_h^*\phi_h - 2\eta_1\left(\sum_{h=1}^{L} D_h^{AU} - \sum_{h=1}^{L} D_h^*\right) - 2\eta_2\left(\sum_{h=1}^{L} D_h^{AU}\hat{R}_h - \sum_{h=1}^{L} D_h^* R_h\right) \tag{3.3}$$

where η_1 and η_2 are Lagrange's multipliers,

Differentiate partially L_* in (3.3) with respect to D_h^{AU}, η_1, η_2 and equate the results to zero,

$$D_h^{AU} = D_h^* + \eta_1 D_h^*\phi_h + \eta_2 D_h^*\phi_h\hat{R}_h \tag{3.4}$$

$$\sum_{h=1}^{L} D_h^{AU} = \sum_{h=1}^{L} D_h^* \tag{3.5}$$

$$\sum_{h=1}^{L} D_h^{AU}\hat{R}_h = \sum_{h=1}^{L} D_h^* R_h \tag{3.6}$$

Substitute (3.4) into (3.5) and (3.6) and solve the resulting equations simultaneously for η_1, η_2, Thereafter, substitute the results of η_1, η_2 in (3.4), we obtained new calibration weights D_h^{AU} as

$$D_h^{AU} = D_h^* - \frac{Q_3 Q_2}{Q_1 Q_4 - Q_2^2} D_h^*\phi_h + \frac{Q_3 Q_1}{Q_1 Q_4 - Q_2^2} D_h^*\phi_h\hat{R}_h \tag{3.7}$$

where $\begin{aligned} &Q_1 = \sum_{h=1}^{L}\phi_h W_h \bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}, Q_2 = \sum_{h=1}^{L}\phi_h W_h \bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}\hat{R}_h \\ &Q_3 = \sum_{h=1}^{L}\phi_h W_h \bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}R_h - Q_2,\ Q_4 = \sum_{h=1}^{L}\phi_h W_h \bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}\hat{R}_h^2 \end{aligned}$

Substitute (3.7) into (3.1) and simplify, we have;

$$\bar{y}_{st}^{AU}=\sum_{h=1}^{L}W_h\bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}\bar{y}_h+\rho\sum_{h=1}^{L}W_h\bar{X}_{1h}\bar{x}_{2h}\bar{x}_{1h}^{-1}\bar{X}_{2h}^{-1}\left(R_h-\hat{R}_h\right) \tag{3.8}$$

where $$\rho=\frac{\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\bar{y}_h\hat{R}_h-\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\hat{R}_h\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\bar{y}_h}{\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\hat{R}_h^2-\left(\sum_{h=1}^{L}\phi_hW_h\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\hat{R}_h^2\right)^2}$$

4. Properties of the Proposed Estimator

To obtain the MSE of $\bar{y}_{st}^{AU}$, we defined the following error terms

$\bar{y}_h=\bar{Y}_h\left(1+e_0\right),\bar{x}_{1h}=\bar{X}_{1h}\left(1+e_1\right),\bar{x}_{2h}=\bar{X}_{2h}\left(1+e_2\right)$ such that $e_i\left(i=0,1,2\right)$ are error term with expected values given in (4.1)

$$\left.\begin{aligned}&E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=0,E\left(e_0^2\right)=\theta_hC_{yh}^2,E\left(e_1^2\right)=\theta_hC_{x1h}^2,E\left(e_2^2\right)=\theta_hC_{x2h}^2\\&E\left(e_0e_1\right)=\rho_{yx1h}C_{yh}C_{x1h},E\left(e_0e_2\right)=\theta_h\rho_{yx2h}C_{yh}C_{x2h},E\left(e_1e_2\right)=\theta_h\rho_{x1x2h}C_{x1h}C_{x2h},\theta_h=n_h^{-1}-N_h^{-1}\end{aligned}\right\} \tag{4.1}$$

where C_{yh},C_{x1h},C_{x2h} are coefficients of variation of y_h,x_{1h},x_{2h} and $\rho_{yx1h},\rho_{yx2h},\rho_{x1x2h}$ are correlation coefficients of $\left(y_h,x_{1h}\right),\left(y_h,x_{2h}\right),\left(x_{1h},x_{2h}\right)$ respectively.

Let $Z_h=\frac{\bar{X}_{1h}\bar{x}_{2h}\bar{y}_h}{\bar{x}_{1h}\bar{X}_{2h}}+\rho\frac{\bar{X}_{1h}\bar{x}_{2h}}{\bar{x}_{1h}\bar{X}_{2h}}\left(R_h-\hat{R}_h\right)$ in (3.8), then we have

$$\bar{y}_{st}^{AU}=\sum_{h=1}^{L}W_hZ_h \tag{4.2}$$

$$MSE\left(\bar{y}_{st}^{AU}\right)=\sum_{h=1}^{L}W_h^2MSE\left(Z_h\right) \tag{4.3}$$

Express Z_h in terms of e_i, we obtain

$$Z_h = \bar{Y}_h (1+e_0)(1+e_2)(1+e_1)^{-1} + R_h\rho(1+e_2)(1+e_1)^{-1}\left(1-(1+e_1)(1+e_2)^{-1}\right) \quad (4.4)$$

Simplify (4.4) up to second degree approximation, we have

$$Z_h - \bar{Y}_h = \bar{Y}_h\left(e_0 - e_1 + e_2 + e_1^2 - e_0e_1 + e_0e_2 - e_1e_2\right) + R_h\rho\left(e_2 - e_1 - e_2^2 + e_1^2\right) \quad (4.5)$$

Square both sides of (4.5), simplify up to second degree approximation and apply the results of (4.1), we obtained the MSE of $\bar{y}_{st}^{AU}$ was obtained as

$$MSE\left(\bar{y}_{st}^{AU}\right) = \sum_{h=1}^{L} W_h^2\theta_h\left(S_{yh}^2 + \left(R_{1h} + \rho R_h\bar{X}_{1h}^{-1}\right)^2 S_{x1h}^2 + \left(R_{2h} + \rho R\bar{X}_{2h}^{-1}\right)^2 S_{x2h}^2 - 2\left(R_{1h} + \rho R_h\bar{X}_{1h}^{-1}\right)\right.$$
$$\left. - S_{yx1h} + 2\left(R_{2h} + \rho R_h\bar{X}_{2h}^{-1}\right)S_{yx2h} - 2\left(R_{1h}R_{2h} + \rho^2R_h^2\bar{X}_{1h}^{-1}\bar{X}_{2h}^{-1} + \rho R_hR_{1h}\bar{X}_{2h}^{-1} + \rho R_hR_{2h}\bar{X}_{1h}^{-1}\right)S_{x1x2h}\right)$$
(4.6)

where $R_{1h} = \bar{Y}_h / \bar{X}_{1h}, R_{2h} = \bar{Y}_h / \bar{X}_{2h},$

The estimated MSE of the proposed estimator $\bar{y}_{st}^{AU}$ denoted by $mse\left(t_2^{AU}\right)$ was obtained as

$$mse\left(\bar{y}_{st}^{AU}\right) = \sum_{h=1}^{K} W_h^2\theta_h\Lambda_{yh}^2 \quad (4.7)$$

where

$$\Lambda_h^2 = \sum_{i=1}^{n_h} e_{hi}^2,\ e_{hi} = \left(y_{hi} - \bar{y}_h\right) - \left(\hat{R}_{1h} + \rho\hat{R}_h\bar{x}_{1h}\right)\left(x_{1hi} - \bar{x}_{1h}\right) + \left(\hat{R}_{2h} + \rho\hat{R}_h\bar{x}_{2h}\right)\left(x_{2hi} - \bar{x}_{2h}\right)$$
$$\hat{R}_{1h} = \bar{y}_h / \bar{x}_{1h}, \hat{R}_{2h} = \bar{y}_h / \bar{x}_{2h}, \hat{R}_h = \bar{x}_{1h} / \bar{x}_{2h}$$

5. Theoretical Efficiency Comparison

In this section, efficiency conditions of the proposed estimator over Rao et al. (2012) $\bar{y}_{st(R)}$ and Ozgul (2018) $\bar{y}_{st(O)}$ estimators were established.

(i) By comparison, $MSE\left(\bar{y}_{st(R)}\right) - MSE\left(\bar{y}_{st}^{AU}\right) > 0$, if (5.1) is satisfied.

$$\sum_{h=1}^{L}\left(A_{1h}^2 - A_{2h}^2\right) > 2\sum_{h=1}^{L}\left(A_{3h} + A_{4h} - A_{5h}\right) \quad (5.1)$$

where $A_{1h}=\hat{\beta}_{1(R)}S_{x1h}+\hat{\beta}_{2(R)}S_{x2h}, A_{2h}=\left(R_{1h}+\rho R_h\overline{X}_{1h}^{-1}\right)S_{x1h}-\left(R_{2h}+\rho R_h\overline{X}_{2h}^{-1}\right)S_{x2h}$

,

$A_{3h}=\left(\hat{\beta}_{1(R)}-\left(R_{1h}+\rho R_h\overline{X}_{1h}^{-1}\right)\right)\rho_{yx1h}S_{yh}S_{x1h}, A_{4h}=\left(\hat{\beta}_{2(R)}+\left(R_{2h}+\rho R_h\overline{X}_{2h}^{-1}\right)\right)\rho_{yx2h}S_{yh}S_{x2h}$

, $A_{5h}=\left(\hat{\beta}_{1(R)}\hat{\beta}_{2(R)}-\left(R_{1h}+\rho R_h\overline{X}_{1h}^{-1}\right)\left(R_{2h}+\rho R_h\overline{X}_{2h}^{-1}\right)\right)\left(1-\rho_{x1x2h}\right)S_{x1h}S_{x2h}$.

(ii) By comparison, $MSE\left(\overline{y}_{st(O)}\right)-MSE\left(\overline{y}_{st}^{AU}\right)>0$, if (5.2) is satisfied.

$$\sum_{h=1}^{L}\left(B_{1h}^2-B_{2h}^2\right)>-2\sum_{h=1}^{L}\left(B_{3h}-B_{4h}-B_{5h}\right) \tag{5.2}$$

where

$B_{1h}=\hat{\beta}_{1(O)}R_h\left(\overline{X}_{1h}^{-1}S_{x1h}-\overline{X}_{2h}^{-1}S_{x2h}\right), B_{2h}=\left(R_{1h}+\rho R_h\overline{X}_{1h}^{-1}\right)S_{x1h}-\left(R_{2h}+\rho R_h\overline{X}_{2h}^{-1}\right)S_{x2h}$,

$B_{3h}=\left(\left(\hat{\beta}_{1(O)}-\rho\right)R_h\overline{X}_{1h}^{-1}-R_{1h}\right)\rho_{yx1h}S_{yh}S_{x1h}, B_{4h}=\left(\left(\hat{\beta}_{1(O)}-\rho\right)R_h\overline{X}_{2h}^{-1}-R_{2h}\right)\rho_{yx2h}S_{yh}S_{x2h}$

, $B_{5h}=\left(\hat{\beta}_{1(O)}^2R_h^2\overline{X}_{1h}^{-1}\overline{X}_{2h}^{-1}-\left(R_{1h}+\rho R_h\overline{X}_{1h}^{-1}\right)\left(R_{2h}+\rho R_h\overline{X}_{2h}^{-1}\right)\right)\left(1-\rho_{x1x2h}\right)S_{x1h}S_{x2h}$.

6. Empirical Study

In this section, simulation study was conducted to examine the superiority of the proposed estimators over other estimators considered in the study.Data of size 1000 units were generated for study population stratified into 3 non-overlapping heterogeneous groups as 200, 300 and 500 using function defined in Table 1. Samples of sizes 20, 30 and 50 were selected 10,000 times by method SRSWOR from each stratum respectively. The efficiency (MSE) of the considered estimators were computed using (6.1).

$$MSE\left(\theta_l\right)=\frac{1}{10000}\sum_{j=1}^{10000}\left(\theta_l-\overline{Y}\right)^2,\ \theta_l=t_0,\overline{y}_{st(R)},\overline{y}_{st(O)},\overline{y}_{st}^{AU} \tag{6.1}$$

Table1: Populations used for Empirical Study

Auxiliary variables x_1, x_2	**Study variable** y
$x_{1h} \sim \exp(\lambda_h), \lambda_1 = 0.2,$ $\lambda_2 = 0.3,\ \lambda_3 = 0.1$ $x_{2h} \sim chsq(\theta_h), \theta_1 = 3,$ $\theta_2 = 1, \theta_3 = 2, h = 1,2,3$	$Model\ I : y_{hi} = x_{1hi}^2 - x_{2hi}^2 + \xi_{hi},$ $Model\ II : y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^2 - x_{2hi}^2 + \xi_{hi}$ $Model\ III : y_{hi} = x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$ $Model\ IV : y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$ $Model\ V : y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^2 - x_{2hi}^2 + x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$ Where $\xi_h \sim N(0,1), h = 1,2,3$

Table 2: MSE of the Proposed and Some Existing Estimators using Model I

Estimators	$Model\ II : y_{hi} = x_{1hi}^2 - x_{2hi}^2 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \overline{x}_{1h}^{-1}$	$\phi_h = \overline{x}_{2h}^{-1}$	$\hat{R}^{-1} = \overline{x}_{2h} / \overline{x}_{1h}$
Sample mean t_0	862.2489	862.2489	862.2489	862.2489
Rao et al. (2012) $\overline{y}_{st(R)}$	0.02478455	0.02577657	0.02639959	0.02481833
Ozgul (2018) $\overline{y}_{st(O)}$	0.01909836	0.01913633	0.01922508	0.01912832
Proposed Estimator				
$\overline{y}_{st}^{AU}$	0.01028432	0.01029803	0.0102672	0.0103518

Table 3: MSE of the Proposed and Some Existing Estimators using Model II

Estimators	$Model\ II: y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^2 - x_{2hi}^2 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \bar{x}_{1h}^{-1}$	$\phi_h = \bar{x}_{2h}^{-1}$	$\hat{R}^{-1} = \bar{x}_{2h} / \bar{x}_{1h}$
Sample mean t_0	785.4979	785.4979	785.4979	785.4979
Rao et al. (2012) $\bar{y}_{st(R)}$	1.15761	1.077685	1.062445	1.199028
Ozgul (2018) $\bar{y}_{st(O)}$	0.4972489	0.4214986	0.4239928	0.4997682
Proposed Estimator				
$\bar{y}_{st}^{AU}$	0.07521642	0.07571417	0.0738931	0.08207184

Table 4: MSE of the Proposed and Some Existing Estimators using Model III

Estimators	$Model\ III: y_{hi} = x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \bar{x}_{1h}^{-1}$	$\phi_h = \bar{x}_{2h}^{-1}$	$\hat{R}^{-1} = \bar{x}_{2h} / \bar{x}_{1h}$
Sample mean t_0	110467.8	110467.8	110467.8	110467.8
Rao et al. (2012) $\bar{y}_{st(R)}$	92855.79	82636.46	77046.91	102803.5
Ozgul (2018) $\bar{y}_{st(O)}$	93236.47	82393.55	83688.8	91973.38
Proposed Estimator				
$\bar{y}_{st}^{AU}$	16154.64	16182.75	16193.92	16689.41

Table 5: MSE of the Proposed and Some Existing Estimators using Model IV

Estimators	$Model\ IV: y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \overline{x}_{1h}^{-1}$	$\phi_h = \overline{x}_{2h}^{-1}$	$\hat{R}^{-1} = \overline{x}_{2h} / \overline{x}_{1h}$
Sample mean t_0	144959.9	144959.9	144959.9	144959.9
Rao et al. (2012) $\overline{y}_{st(R)}$	62065.45	58852.74	55033.82	68462.58
Ozgul (2018) $\overline{y}_{st(O)}$	79285.91	73171.6	75042.07	77094.59
Proposed Estimator				
$\overline{y}_{st}^{AU}$	17835.05	17880.81	17829.94	18688.79

Table 6: MSE of the Proposed and Some Existing Estimators using Model V

Estimators	$Model\ V: y_{hi} = x_{1hi} - x_{2hi} + x_{1hi}^2 - x_{2hi}^2 + x_{1hi}^3 - x_{2hi}^3 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \overline{x}_{1h}^{-1}$	$\phi_h = \overline{x}_{2h}^{-1}$	$\hat{R}^{-1} = \overline{x}_{2h} / \overline{x}_{1h}$
Sample mean t_0	163874.4	163874.4	163874.4	163874.4
Rao et al. (2012) $\overline{y}_{st(R)}$	67718.66	64265.72	60063.5	74721.08
Ozgul (2018) $\overline{y}_{st(O)}$	86491.56	79871.64	81933.86	84075.58
Proposed Estimator				
$\overline{y}_{st}^{AU}$	19253.6	19306.89	19244.1	20212.49

Tables 2, 3, 4, 5 and 6 show MSEs of the proposed and some existing estimators using models I, II, III, IV and V respectively. The result revealed that the proposed estimator has minimum MSE compared to conventional and other related estimators considered in the study. This implies that the proposed estimator is more efficient in estimation of population mean than other related estimators considered in this study.

5. Conclusion

In Conclusion, based on the empirical study conducted in this study, the proposed estimator demonstrated high level of efficiency over other estimators in the study and therefore has higher chances of producing estimates that are closer to the true values of population means than other estimators.

References

Clement, E. P., and Enang, E. I. (2015) Calibration approach alternative ratio estimator for population mean in stratified sampling. *International Journal of Statistics and Economics,* 16(1):83-93.

Cochran, W.G. (1977). *Sampling Technique*. 3rd edn. Wiley Eastern Limited, New York.

Estavao, V. M. and Samdal, C. E. (2016). Survey estimates by calibration on complete auxiliary information. *International Statistical Review, 74: 127-147.*

Kim, J.K and Park, M. (2010). Calibration estimation in survey sampling. *International Statistical Review.* 78(1), 21 -29.

Kim, J. M., Sungur, E. A., and Heo, T. Y. (2007). Calibration approach estimators in stratified sampling. *Stat. Probability. Lett.* 77: 99–103.

Koyuncu, N., and Kadilar, C. (2016), Calibration Weighting in Stratified Random Sampling. *Communications in Statistics-Simulation Computation* 45:2267-75.

Ozgul N. (2018). New calibration estimator based on two auxiliary variables in stratified sampling. Communication in Statistics-Theory and Methods, 1-12

Rao, D., Khan, M. G. and Khan, S. (2012). Mathematical programming on multivariate calibration estimation in stratified sampling. *World Academy of Science, Engineering and Technology*, 72, 12-27

Rao, D. K., Tekabu, T. and Khan, M. G. M. (2016). New calibration estimators in stratified sampling. *Asia-Pacific World Congress on Computer Science and Engineering*, 66-69

Sarndal, C. E. (2007). The calibration approach in survey theory and practice. *Survey Methodology.* 33: 99-119.

Singh, S. (2003): *Advanced Sapling theory with applications*. Dorrdrechi: Kluwer Academic Publisher.

Tracy, D.S., Singh, S., Arnab, R. (2003). Note on calibration in stratified and double sampling. *Survey Methodology.* 29: 99-104