On the Efficiency of Calibration Ratio Estimators of Population Mean in Stratified Random Sampling

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Abstract

This paper addressed the problem of the influences of extreme values or outliers on the calibration estimators of population mean in stratified random sampling. Three approaches leading to three classes of calibration ratio estimators were suggested and their properties (biases and MSEs) were derived up to first order approximation using Taylor's series method. Empirical study through was conducted to investigate the efficiency of the proposed estimators over some existing related estimators through simulation studies and the results revealed that the proposed estimators outperformed their counterparts considered in the study.

Keywords: Calibration, Calibration Weights, Population Mean, Efficiency, Simple Random Sampling Without Replacement (SRSWOR).

1. Introduction

Calibration estimation is a technique for modifying the original strata weights by minimization of a given distance measure based on a set of calibration constraints under auxiliary information. Many researchers have worked on calibration estimation using different constraints in survey sampling. Singh, Horn and Yu (1998) were the first to extended calibration approach to a stratified sampling design. Singh (2003), Tracy et al. (2003), Koyuncu and Kadilar (2016) suggested calibration estimators for estimating the population mean in stratified sampling with using different calibration constraints based on auxiliary information. Clement and Enang (2016) applied calibration estimation to ratio-type estimators in stratified sampling. Other researchers who worked in this direction are Arnab and Singh (2005), Sarndal (2007), Kim and Park (2010), Clement, Udofia and Enang(2014), Estavao and Sarndal (2016).

2. Some Existing Calibration Estimators in Stratified Random Sampling

According to Cochran (1977), the traditional estimator of population mean in stratified sampling and its estimated variance are defined as

$$t_0 = \sum_{h=1}^{K} W_h \bar{y}_h \tag{2.1}$$

$$\hat{v}\left(t_0\right) = \sum_{h=1}^{K} W_h^2 \frac{1-f_h}{n_h} s_{yh}^2 \tag{2.2}$$

where $W_h = N_h / N,\ f_h = n_h / N_h, s_{yh}^2 = \left(n_h - 1\right)^{-1} \sum_{h=1}^{K}\left(y_{hi} - \bar{y}_h\right)^2$

Using calibration approach, the weights W_h are modified to obtained new calibration weights Π_h through minimization of the distance measure defined in (2.3) subject to some calibration constraints in order to enhance or improve the efficiency or precision of t_0

$$Z = \sum_{h=1}^{K}\left(\Pi_h - W_h\right)^2 / W_h \phi_h \tag{2.3}$$

Singh (2003) suggested a calibration estimator with two constraints for estimating population mean in stratified sampling. The suggested calibration estimator is given in (2.4).

$$t_1^S = \sum_{h=1}^{K} \Theta_h^S \bar{y}_h \tag{2.4}$$

where Θ_h^S is new calibration weight of stratum K^{th} to be obtained by minimizing (2.3) subject to (2.5)

$$\sum_{h=1}^{K} \Theta_h^S \bar{x}_h = \sum_{h=1}^{K} W_h \bar{X}_h, \quad \sum_{h=1}^{K} \Theta_h^S = \sum_{h=1}^{K} W_h \tag{2.5}$$

By minimizing (2.3) subject to (2.5), Singh (2003) obtained calibration weights and estimator as

$$\Theta_h^S = W_h + \frac{\phi_h W_h \bar{x}_h \sum_{h=1}^{K} W_h \phi_h - W_h \phi_h \sum_{h=}^{K} W_h \phi_h \bar{x}_h}{\sum_{h=}^{K} W \phi_h \sum_{h=}^{K} W \phi_h \bar{x}_h^2 - \left(\sum_{h=}^{K} W \phi_h \bar{x}_h\right)^2}\left(\bar{X} - \sum_{h=}^{K} W_h \bar{x}_h\right) \tag{2.6}$$

$$t_1^S = \sum_{h=1}^{K} W_h \bar{y}_h + \frac{\sum_{h=1}^{K} \phi_h W \sum_{h=1}^{K} \phi_h W_h \bar{x}_h \bar{y}_h - \sum_{h=1}^{K} \phi_h W_h \bar{x}_h \sum_{h=1}^{K} \phi_h W_h \bar{y}_h}{\sum_{h=1}^{K} \phi_h W_h \sum_{h=1}^{K} \phi_h W_h \bar{x}_h^2 - \left(\sum_{h=1}^{K} \phi_h W_h \bar{x}_h\right)^2}\left(\bar{X} - \sum_{h=1}^{K} W_h \bar{x}_h\right) \tag{2.7}$$

Tracy et al. (2003) suggested calibration estimator with two constraints based on first and second moments of auxiliary variables. The suggested calibration estimator is given by

$$t_2^T = \sum_{h=1}^{K} \Theta_h^T \bar{y}_h \tag{2.8}$$

where Θ_h^T is new calibration weight of stratum K^{th} to be obtained by minimizing (2.3) subject to (2.9)

$$\sum_{h=1}^{K} \Theta_h^T \bar{x}_h = \sum_{h=1}^{K} W_h \bar{X}_h, \quad \sum_{h=1}^{K} \Theta_h^T s_{xh}^2 = \sum_{h=1}^{K} W_h S_{xh}^2 \tag{2.9}$$

Tracy et al. (2003) obtained calibration weights and estimator as

$$\Theta_h^T = W_h + \phi_h W_h \bar{x}_h \frac{\left(\bar{X} - \sum_{h=}^K W_h \bar{x}_h\right)\sum_{h=}^K W_h \phi_h s_{xh}^4 - \left(S_x^2 - \sum_{h=}^K W_h s_{xh}^2\right)\sum_{h=}^K W_h \phi_h \bar{x}_h s_{xh}^2}{\sum_{h=}^K W_h \phi_h s_{xh}^4 \sum_{h=}^K W_h \phi_h \bar{x}_h^2 - \left(\sum_{h=}^K W_h \phi_h \bar{x}_h s_{xh}^2\right)^2}$$

$$-\phi_h W_h s_{xh}^2 \frac{\left(\bar{X} - \sum_{h=}^K W_h \bar{x}_h\right)\sum_{h=}^K W_h \phi_h \bar{x}_h s_{xh}^2 - \left(S_x^2 - \sum_{h=}^K W_h s_{xh}^2\right)\sum_{h=}^K W_h \phi_h \bar{x}_h^2}{\sum_{h=}^K W_h \phi_h s_{xh}^4 \sum_{h=}^K W_h \phi_h \bar{x}_h^2 - \left(\sum_{h=}^K W_h \phi_h \bar{x}_h s_{xh}^2\right)^2}$$

(2.10)

$$t_2^T = \sum_{h=1}^K W_h \bar{y}_h + \frac{\sum_{h=1}^K \phi_h W_h s_{xh}^4 \sum_{h=1}^K \phi_h W_h \bar{x}_h \bar{y}_h - \sum_{h=1}^K \phi_h W_h \bar{x}_h s_{xh}^2 \sum_{h=1}^K \phi_h W_h \bar{y}_h s_{xh}^2}{\sum_{h=1}^K \phi_h W_h \bar{x}_h^2 \sum_{h=1}^K \phi_h W_h s_{xh}^4 - \left(\sum_{h=1}^K \phi_h W_h \bar{x}_h s_{xh}^2\right)^2}\left(\bar{X} - \sum_{h=1}^K W_h \bar{x}_h\right)$$

$$+\left(S_x^2 - \sum_{h=1}^K W_h s_{xh}^2\right)\frac{\sum_{h=1}^K \phi_h W_h \bar{x}_h^2 \sum_{h=1}^K \phi_h W_h \bar{y}_h s_{xh}^2 - \sum_{h=1}^K \phi_h W_h \bar{x}_h s_{xh}^2 \sum_{h=1}^K \phi_h W_h \bar{x}_h \bar{y}_h}{\sum_{h=1}^K \phi_h W_h \bar{x}_h^2 \sum_{h=1}^K \phi_h W_h s_{xh}^4 - \left(\sum_{h=1}^K \phi_h W_h \bar{x}_h s_{xh}^2\right)^2} \quad (2.11)$$

Rao *et al*. (2016) introduced coefficient of variation as constraint to obtained new calibration estimator given in (2.12)

$$t_3^{RK} = \sum_{h=1}^K \Theta_h^{RK} \bar{y}_h \quad (2.12)$$

where Θ_h^{RK} is new calibration weight of stratum K^{th} to be obtained by minimizing (2.3) subject to (2.13)

$$\sum_{h=1}^K \Theta_h^{RK}\left(\bar{x}_h + c_{xh}\right) = \sum_{h=1}^K W_h(\bar{X}_h + C_{xh}) \quad (2.13)$$

By minimizing (2.2) subject to (2.13), Rao and Khan (2016) obtained Θ_h^{RK} and t_3^{RK} as

$$\Theta_h^{RK} = W_h + \left(\frac{\sum_{h=1}^K W_h\left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^K W_h\left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^K W_h \phi_h \left(\bar{x}_h + c_{xh}\right)^2}\right)\left(\bar{x}_h + c_{xh}\right) W_h \phi_h \quad (2.14)$$

$$t_3^{RK} = \sum_{h=1}^K W_h \bar{y}_h + \frac{\sum_{h=1}^K \phi_h W_h \bar{y}_h \left(\bar{x}_h + c_{xh}\right)}{\sum_{h=1}^K \phi_h W_h \left(\bar{x}_h + c_{xh}\right)^2}\left(\sum_{h=1}^K W_h\left(\bar{X}_h + C_{xh}\right) - \sum_{h=1}^K W_h\left(\bar{x}_h + c_{xh}\right)\right)$$

(2.15)

Clement (2015) suggested calibration ratio estimator denoted by t_4^C for estimating of population mean under stratified sampling as

$$t_4^C = \sum_{h=1}^{K} \Theta_h^C \hat{R}_h \bar{x}_h \tag{2.16}$$

where Θ_h^C are new calibrated weights which are chosen such that the chi-square distance measure in (2.17) is minimum subject to the calibration constraints defined in (2.17) and $\hat{R}_h = \bar{y}_h \bar{x}_h^{-1}; \bar{x}_h \neq 0$ is the estimate of the ratio $R_h = \bar{Y}_h \bar{X}_h^{-1}; \bar{X} \neq 0,\ d_h = N_h / n_h$

$$L = \sum_{h=1}^{K} d_h \left(\Theta_h^C d_h^{-1} - 1\right)^2 / \phi_h, \quad s.t \ \sum_{h=1}^{K} \Theta_h^C \bar{x}_h = \sum_{h=1}^{K} W_h \bar{X}_h \tag{2.17}$$

The calibration weights Θ_h^C and estimator t_4^C are obtained as

$$\Theta_h^C = d_h + \left(\bar{X} - \sum_{h=1}^{K} d_h \bar{x}_h\right) \phi_h d_h \bar{x}_h / \sum_{h=1}^{K} \phi_h d_h \bar{x}_h^2 \tag{2.18}$$

$$t_4^C = \sum_{h=1}^{K} d_h \hat{R}_h \bar{x}_h + \sum_{h=1}^{K} \phi_h d_h \hat{R}_h \bar{x}_h^2 \left(\bar{X} - \sum_{h=1}^{K} d_h \bar{x}_h\right) / \sum_{h=1}^{K} \phi_h d_h \bar{x}_h^2 \tag{2.19}$$

Having studied the above estimators, we observed that the first components of the estimators are function of sample mean of the study variable which can be easily influenced by outliers thereby leading the estimators to be inefficient.

3. Proposed Calibration Estimators

3.1 First calibration scheme proposed

The first proposed calibration ratio estimator is defined in (3.1) as

$$t_1^{AU} = \sum_{h=1}^{K} \Theta_{1h}^{AU} \bar{y}_h$$

(3.1) where Θ_{1h}^{AU} is proposed calibration weight of K^{th} stratumto be determined using (3.2)

$$\left.\begin{array}{l} \min \quad Z_{AU1} = \sum_{h=1}^{K} (\Theta_{1h}^{AU} - W_h^*)^2 / W_h^* \phi_h \\ s.t \ \sum_{h=1}^{K} \Theta_{1h}^{AU} \bar{x}_h = \sum_{h=1}^{K} W_h^* \bar{X}_h, \ \ \sum_{h=1}^{K} \Theta_{1h}^{AU} = \sum_{h=1}^{K} W_h^* \end{array}\right\} \quad (3.2)$$

where $W_h^* = W_h \bar{x}_h^{-1} \bar{X}_h$

To compute new calibration weight Θ_{1h}^{AU}, we define Lagrange function L_1 of the formed;

$$L_1 = \sum_{h=1}^{K} \left(\Theta_{1h}^{AU} - W_h^*\right)^2 / W_h^* \phi_h - 2\lambda_1 \left(\sum_{h=1}^{K} \Theta_{1h}^{AU} \bar{x}_h - \sum_{h=1}^{K} W_h^* \bar{X}_h\right) - 2\lambda_2 \left(\sum_{h=1}^{k} \Theta_{1h}^{AU} - \sum_{h=1}^{K} W_h^*\right) \quad (3.3)$$

Differentiate partially (3.3) with respect to Θ_{1h}^{AU}, λ_1 and λ_2 and equate to zero, we have;

$$\Theta_{1h}^{AU} = W_h^* + \lambda_1 \bar{x}_h W_h^* \phi_h + \lambda_2 W_h^* \phi_h \quad (3.4)$$

$$\sum_{h=1}^{K} \Theta_{1h}^{AU} \bar{x}_h = \sum_{h=1}^{K} W_h^* \bar{X}_h \quad (3.5)$$

$$\sum_{h=1}^{K} \Theta_{1h}^{AU} = \sum_{h=1}^{K} W_h^* \quad (3.6)$$

Substitute (3.4) into (3.5) and (3.6) and solve the resulting equations simultaneously for λ_1, λ_2, Thereafter, substitute the results of λ_1, λ_2 in (3.4), we obtained new calibration weights Θ_{1h}^{AU} as;

$$\Theta_{1h}^{AU} = W_h^* + \left(\phi_h W_h^* \bar{x}_h C_3 C_4 - \phi_h W_h^* C_2 C_4\right) / \left(C_1 C_4 - C_2^2\right) \quad (3.7)$$

where $C_1 = \sum_{h=1}^{K} \phi_h W_h^* \bar{x}_h^2, C_2 = \sum_{h=1}^{K} \phi_h W_h^* \bar{x}_h, C_3 = \sum_{h=1}^{K} W_h^* \bar{X} - \sum_{h=1}^{K} W_h^* \bar{x}_h, C_4 = \sum_{h=1}^{K} W_h^* \phi_h$

Substitute (3.7) into (3.1) and simplify, we have;

$$t_1^{AU} = \sum_{h=1}^{K} W_h \bar{X}_h \bar{x}_h^{-1} \bar{y}_h + \beta \sum_{h=1}^{K} W_h \bar{X}_h \bar{x}_h^{-1} \left(\bar{X}_h - \bar{x}_h\right) \quad (3.8)$$

where $\beta = \dfrac{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{y}_h - \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} \bar{y}_h}{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} - \left(\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h\right)^2}$

To obtain bias and MSE of t_1^{AU}, the following error terms are defined;

$e_0 = \left(\bar{y}_h - \bar{Y}_h\right) / \bar{Y}_h,\ e_1 = \left(\bar{x}_h - \bar{X}_h\right) / \bar{X}_h,\ e_2 = \left(s^2_{xh} - S^2_{xh}\right) / S^2_{xh}$ with expected values defined in (3.9)

$$\left.\begin{aligned} &E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = 0, E\left(e_0^2\right) = \theta_h C^2_{yh}, E\left(e_1^2\right) = \theta_h C^2_{xh}, E\left(e_2^2\right) = \theta_h\left(\lambda_{04h} - 1\right) \\ &E\left(e_0 e_1\right) = \theta_h \rho_{yxh} C_{yh} C_{xh}, E\left(e_0 e_2\right) = \theta_h C_{yh} \lambda_{12h}, E\left(e_1 e_2\right) = \theta_h C_{xh} \lambda_{03h}, \theta_h = \left(1 - f_h\right) n_h^{-1} \end{aligned}\right\} \tag{3.9}$$

where

$$C_{yh} = S_{yh} / \bar{Y}_h, C_{xh} = S_{xh} / \bar{X}_h, \lambda_{rs} = \mu_{rs} / \left(\mu_{20}^{r/2} \mu_{02}^{s/2}\right), \mu_{rs} = \left(N_h - 1\right)^{-1} \sum\nolimits_{i=1}^{N_h} \left(y_{hi} - \bar{Y}\right)^r \left(x_{hi} - \bar{X}\right)^s$$

Express (3.8) in terms of $e_i, i = 0,1,2$ and simplify up to second degree approximation, we obtained (3.10) as

$$t_1^{AU} - \sum\nolimits_{h=1}^{K} W_h \bar{Y}_h = \sum\nolimits_{h=1}^{K} W_h \left(\bar{Y}_h \left(e_0 - e_1 + e_1^2 - e_0 e_1\right) - \beta \bar{X}_h \left(e_1 - e_1^2\right)\right) \tag{3.10}$$

By taking expectation of (3.10) and apply the results of (3.9), we obtained the bias of t_1^{AU} as

$$Bias\left(t_1^{AU}\right) = \sum\nolimits_{h=1}^{K} W_h \theta_h \left(\left(\bar{Y}_h + \beta \bar{X}_h\right) C^2_{xh} - \bar{Y}_h \rho_{yxh} C_{yh} C_{xh}\right) \tag{3.11}$$

By squaring (3.10), take expectation of and apply the results of (3.9), we obtained the MSE of t_1^{AU} as

$$MSE\left(t_1^{AU}\right) = \sum\nolimits_{h=1}^{K} W_h^2 \theta_h \left(S^2_{yh} + \left(\beta + R_h\right)^2 S^2_{xh} - 2\left(\beta + R_h\right) S_{yxh}\right) \tag{3.12}$$

The estimated MSE of the proposed estimator t_1^{AU} denoted by $m\hat{s}e\left(t_1^{AU}\right)$ was obtained as

$$m\hat{s}e\left(t_1^{AU}\right) = \sum\nolimits_{h=1}^{K} W_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \frac{1}{\left(n_h - 1\right)} \sum\nolimits_{i=1}^{n_h} \left(\left(y_{hi} - \bar{y}_h\right) - \left(\hat{R}_h + \beta\right)\left(x_{hi} - \bar{x}_h\right)\right)^2 \tag{3.13}$$

3.2 Second calibration scheme proposed

The second proposed calibration ratio estimator is defined in (3.14) as

$$t_2^{AU} = \sum_{h=1}^{K} \Theta_{2h}^{AU} \bar{y}_h \qquad (3.14)$$

where Θ_{2h}^{AU} is proposed calibration weight of K^{th} stratumto be determined using (3.15)

$$\left.\begin{aligned} &\min \quad Z_{AU2} = \sum_{h=1}^{K} \left(\Theta_{2h}^{AU} - W_h^*\right) / W_h^* \phi_h \\ &s.t \ \sum_{h=1}^{K} \Theta_{2h}^{AU} \bar{x}_h = \sum_{h=1}^{K} W_h^* \bar{X}_h, \ \sum_{h=1}^{h} \Theta_{2h}^{AU} s_{xh}^2 = \sum_{h=1}^{K} W_h^* S_{xh}^2 \end{aligned}\right\} \qquad (3.15)$$

Using similar approach in section (3.1), we obtained

$$\Theta_{2h}^{AU} = W_h^* + \frac{D_3 D_4 - D_5 D_2}{D_1 D_4 - D_2^2} \phi_h W_h^* \bar{x}_h + \frac{D_1 D_5 - D_2 D_3}{D_1 D_4 - D_2^2} \phi_h W_h^* s_{xh}^2 \qquad (3.16)$$

where
$$D_1 = \sum_{h=1}^{K} \phi_h W_h^* \bar{x}_h^2, D_2 = \sum_{h=1}^{K} \phi_h W_h^* s_{xh}^2 \bar{x}_h, D_3 = \sum_{h=1}^{K} W_h^* \bar{X}_h - \sum_{h=1}^{K} W_h^* \bar{x}_h,$$
$$D_4 = \sum_{h=1}^{K} \phi_h W_h^* s_{xh}^4, D_5 = \sum_{h=1}^{K} W_h^* S_{xh}^2 - \sum_{h=1}^{K} W_h^* s_{xh}^2$$

$$t_2^{AU} = \sum_{h=1}^{K} W_h \bar{X}_h \bar{x}_h^{-1} \bar{y}_h + \omega_1 \sum_{h=1}^{K} W_h \bar{X}_h \bar{x}_h^{-1} \left(\bar{X}_h - \bar{x}_h\right) + \omega_2 \sum_{h=1}^{K} W_h \bar{X}_h \bar{x}_h^{-1} \left(S_{xh}^2 - s_{xh}^2\right) \qquad (3.17)$$

where

$$\omega_1 = \frac{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} s_{xh}^4 \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{y}_h - \sum_{h=1}^{K} \phi_h W_h \bar{X}_h s_{xh}^2 \sum_{h=1}^{K} \phi_h W \bar{X}_h \bar{x}_h^{-1} s_{xh}^2 \bar{y}_h}{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{y}_h \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} s_{xh}^4 - \left(\sum_{h=1}^{K} \phi_h W_h \bar{X}_h s_{xh}^2\right)^2}$$

$$\omega_2 = \frac{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} \bar{y}_h s_{xh}^2 - \sum_{h=1}^{K} \phi_h W_h \bar{X}_h s_{xh}^2 \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{y}_h}{\sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{y}_h \sum_{h=1}^{K} \phi_h W_h \bar{X}_h \bar{x}_h^{-1} s_{xh}^4 - \left(\sum_{h=1}^{K} \phi_h W_h \bar{X}_h s_{xh}^2\right)^2}$$

The bias and MSE t_2^{AU} were obtained as

$$Bias\left(t_2^{AU}\right)=\sum W_h\theta_h\left(\bar{Y}_h\left(C_{xh}^2-\rho_{yxh}C_{yh}C_{xh}\right)+\omega_1\bar{X}_hC_{xh}^2+\omega_2S_{xh}^2C_{xh}\lambda_{03h}\right)$$

(3.18)

$$MSE\left(t_2^{AU}\right)=\sum_{h=1}^{K}W_h^2\theta_h\left(S_{yh}^2+\left(\omega_1+R_h\right)^2S_{xh}^2-2\left(\omega_1+R_h\right)S_{yxh}\right.$$
$$\left.+\omega_2^2S_{xh}^4\left(\lambda_{04h}-1\right)-2\omega_2S_{xh}^2\left(S_{yh}\lambda_{12h}-\left(R_h+\omega_1\right)S_{xh}\lambda_{03h}\right)\right)$$

(3.19)

The estimated MSE of the proposed estimator t_2^{AU} denoted by $m\hat{s}e\left(t_2^{AU}\right)$ was obtained as

$$m\hat{s}e\left(t_2^{AU}\right)=\sum_{h=1}^{K}W_h^2\frac{1-f_h}{n_h\left(n_h-1\right)}\sum_{i=1}^{n_h}\left(\left(y_{hi}-\bar{y}_h\right)-\left(\omega_1+\hat{R}_h\right)\left(x_{hi}-\bar{x}_h\right)-\omega_2\left(\left(x_{hi}-\bar{x}\right)^2-s_{xh}^2\right)\right)^2$$

(3.20)

3.3 Third calibration scheme proposed

The third proposed calibration ratio estimator is defined in (3.21) as

$$t_3^{AU}=\sum_{h=1}^{K}\Theta_{3h}^{AU}\bar{y}_h$$

(3.21)

where Θ_{3h}^{AU} is proposed calibration weight of K^{th} stratumto be determined using (3.22)

$$\left.\begin{aligned}&\min\ Z_{AU3}=\sum_{h=1}^{K}\left(\Theta_{3h}^{AU}-W_h^*\right)/W_h^*\phi_h\\&s.t.\ \sum_{h=1}^{K}\Theta_{3h}^{AU}\left(\bar{x}_h+c_{xh}\right)=\sum_{h=1}^{K}W_h^*\left(\bar{X}_h+C_{xh}\right)\end{aligned}\right\}$$

(3.22)

Similarly, we obtained new Θ_{h3}^{AU} and the proposed estimator t_3^{AU} as;

$$\Theta_{3h}^{AU}=W_h^*+\left(\sum_{h=1}^{K}W_h^*\left(\bar{X}_h+C_{xh}\right)-\sum_{h=1}^{K}\left(\bar{x}_h+c_{xh}\right)\right)\left(\bar{x}_h+c_{xh}\right)W_h^*\phi_h/\sum_{h=1}^{K}W_h^*\phi_h\left(\bar{x}_h+c_{xh}\right)^2$$

(3.23)

$$t_3^{AU}=\sum_{h=1}^{K}W_h\frac{\bar{X}_h}{\bar{x}_h}\bar{y}_h+\nu\sum_{h=1}^{K}W_h\frac{\bar{X}_h}{\bar{x}_h}\left(\left(\bar{X}_h+C_{xh}\right)-\left(\bar{x}_h+c_{xh}\right)\right)$$

(3.24)

The bias and MSE t_3^{AU} were obtained as

$$Bias\left(t_3^{AU}\right)=\sum_{h=1}^{K}W_h\theta_h\left(\bar{Y}_h\left(C_{xh}^2-\rho_{yxh}C_{yh}C_{xh}\right)+\nu\bar{X}_hC_{xh}^2\right.$$
$$\left.+8^{-1}\nu C_{xh}\left(\lambda_{04h}-1\right)-\nu C_{xh}^2\left(2C_{xh}-\lambda_{03h}\right)\right)$$

(3.25)

$$MSE\left(t_3^{AU}\right)=\sum_{h=1}^{K}W_h^2\theta_h\left(S_{yh}^2+\left(R_h+\nu-\nu\bar{X}^{-1}C_{xh}\right)^2S_{xh}^2-2\left(R_h+\nu-\nu\bar{X}^{-1}C_{xh}\right)S_{yxh}\right.$$
$$\left.+4^{-1}\bar{X}^{-2}\nu^2\left(\lambda_{04h}-1\right)S_{xh}^2+\nu\bar{X}^{-1}\left(R_h+\nu-\nu\bar{X}^{-1}C_{xh}\right)\lambda_{03h}S_{xh}^2-\nu\bar{X}^{-1}S_{yh}S_{xh}\lambda_{12h}\right)$$

(3.26)

The estimated MSE of the proposed estimator t_3^{AU} denoted by $m\hat{s}e\left(t_3^{AU}\right)$ was obtained as

$$m\hat{s}e\left(t_3^{AU}\right)=\sum_{h=1}^{K}\frac{W_h^2}{n_h}\frac{1-f_h}{\left(n_h-1\right)}\sum_{i=1}^{n_h}\left(\left(y_{hi}-\bar{y}_h\right)-\left(\hat{R}_h+\nu-\nu c_x\bar{x}_h^{-1}\right)\left(x_{hi}-\bar{x}_h\right)-\frac{\nu}{2\bar{x}}\left(\frac{\left(x_{hi}-\bar{x}_h\right)^2}{s_{xh}}-s_{xh}\right)\right)^2$$

(3.27)

where $\nu=\dfrac{\sum_{h=1}^{K}W_h\phi_h\bar{X}_h\bar{x}_h^{-1}\left(\bar{x}_h+c_{xh}\right)\bar{y}_h}{\sum_{h=1}^{K}W_h\phi_h\bar{X}_h\bar{x}_h^{-1}\left(\bar{x}_h+c_{xh}\right)^2}$

4. Empirical Study

In this section, simulation study was conducted to examine the superiority of the proposed estimators over other estimators considered in the study.

Data of size 1000 units were generated for study Populations stratified into 3 non-overlapping heterogeneous using functions defined in Table 1 and samples of size1 20, 30 and 50 were selected 10,000 times by method SRSWOR from each stratum respectively. The precision (PRE) of the considered estimators were computed using (4.1).

$$PRE\left(\hat{\theta}_i\right)=\left(\operatorname{var}\left(\theta\right)/\operatorname{var}\left(\theta_l\right)\right)\times 100 \qquad (4.1)$$

Table1: Populations used for Empirical Study

Auxiliary variable x	**Study variable** y
$x_h \sim \exp\left(\lambda_h\right), \lambda_1 = 0.2,$ $\lambda_2 = 0.3,\ \lambda_3 = 0.1$	$Model\ I: y_{hi} = x_{hi}^2 + \xi_{hi},\ \xi_h \sim N\left(0,1\right)$
	$Model\ II: y_{hi} = x_{hi}^3 + \xi_{hi},\ \xi_h \sim N\left(0,1\right)$
	$Model\ III: y_{hi} = x_{hi}^3 + \xi_{hi},\ \xi_h \sim N\left(0,1\right)$

Table 2: PRE of the Proposed and Some Existing Estimators using Model I

Estimators	$Model\ I: y_{hi} = x_{hi}^2 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \bar{x}_h^{-1}$	$\phi_h = s_{xh}^{-2}$	$\phi_h = \left(\bar{x}_h + c_{xh}\right)^{-1}$
Sample mean t_0	100	100	100	100
Singh (2003) t_1^S	0.4618129	0.4654986	0.6853281	0.4987068
Tracy et al. (2003) t_2^T	156.8282	200.249	273.1943	186.2513
Rao et al. (2016) t_3^{RK}	19.6508	19.6508	19.6508	19.6508
Clement (2015) t_4^C	101.9259	38.5834	13.1371	53.1198
Proposed Estimators				
t_1^{AU}	125.2144	124.3564	123.5926	124.6711
t_2^{AU}	107.5511	97.32242	103.2859	103.6819
t_3^{AU}	357.4837	357.4837	357.4837	357.4837

Table 3: PRE of the Proposed and Some Existing Estimators using Model II

Estimators	$Model\ II: y_{hi} = x_{hi}^3 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \bar{x}_h^{-1}$	$\phi_h = s_{xh}^{-2}$	$\phi_h = \left(\bar{x}_h + c_{xh}\right)^{-1}$
Sample mean t_0	100	100	100	100
Singh (2003) t_1^S	2.016083	2.034654	3.066547	2.184256
Tracy et al. (2003) t_2^T	130.4708	151.0057	185.5967	145.2709
Rao et al. (2016) t_3^{RK}	33.14468	33.14468	33.14468	33.14468
Clement (2015) t_4^C	3.1859	3.5999	0.5086	5.4713
Proposed Estimators				
t_1^{AU}	143.7675	143.2237	142.9408	143.4072
t_2^{AU}	585.3195	578.88	638.4995	593.1084
t_3^{AU}	133.3531	133.3531	133.3531	133.3531

Table 4: PRE of the Proposed and Some Existing Estimators using Model III

Estimators	$Model\ III: y_{hi} = x_{hi}^4 + \xi_{hi}$			
	$\phi_h = 1$	$\phi_h = \overline{x}_h^{-1}$	$\phi_h = s_{xh}^{-2}$	$\phi_h = \left(\overline{x}_h + c_{xh}\right)^{-1}$
Sample mean t_0	100	100	100	100
Singh (2003) t_1^S	7.946145	8.016244	12.11434	8.609822
Tracy et al. (2003) t_2^T	120.6025	133.9832	157.4484	130.6219
Rao et al. (2016) t_3^{RK}	39.69877	39.69877	39.69877	39.69877
Clement (2015) t_4^C	3.3394	14.7189	0.7782	22.10691
Proposed Estimators				
t_1^{AU}	132.0035	131.7362	131.6606	131.8184
t_2^{AU}	445.8137	445.5222	444.817	446.1108
t_3^{AU}	99.95038	99.95038	99.95038	99.95038

Tables 2, 3 and 4 showed PREs of the proposed and some existing estimators using model I, II and III. The result revealed that all the proposed estimators have higher PRE compared to their counterparts, that is, t_1^{AU} vs t_1^S, t_2^{AU} versus t_2^T and t_3^{AU} versus t_3^{RK}. This implies that the proposed estimators are more efficient in estimation of population mean than other related estimators considered in this study.

5. Conclusion

Considering the results obtained from the empirical study on the efficiency of the proposed calibration estimators over some exists related estimators considered in the study, it was obtained that the proposed estimators have higher PREs compared to other estimators considered in all the numerical computations carried out in the study, hence, the proposed estimators demonstrated high level of efficiency over other estimators. In conclusion, the proposed estimators have higher chances of producing estimates that are closer to the true values of population mean than other estimators.

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