TRANSFER FUNCTION MODELLING OF INFLATION RATE AND IMPORT DUTIES IN NIGERIA

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The goal of this investigation is to use transfer function to model Nigeria's inflation rate and import duties. The two series were collected from the website of Nigeria's Central Bank (for 16 years). The data were checked for stationarity using appropriate transformations at first. The ARIMA (p d q), was used to estimate five models for the input series. The best model was selected based on the minimum information criterion. As a result, the input series was modelled using ARIMA (1,1,0) autoregressive integrated moving average models. The input and output variable was pre whitened. The analysis of cross-correlation function was used to provide a rational for the polynomial representation of the dynamic transfer function models. It was discovered that the calculated noise was autocorrelated. This filled in the gaps in the transfer function model, which was then used to fit the whole thing together. The resulting model was put through a diagnostic test and found to be suitable. As a result, a forecast was made.

Keyword: transfer function model, inflation rate, import duties, *pre-whitened and forecast*

1.0 INTRODUCTION

In Nigeria, increase in prices of goods and services is not new to the Nigerian economy or the rest of the world. Evidence suggests that inflation continues in both developed and developing countries, though at different rates and magnitudes. Price rises in developing countries are higher than in developed nations. Inflation is the gradual rise in the price of goods and services as a result of a large amount of money in circulation being used to exchange a limited number of goods and services. As a result of the rise in foreign prices and the volatility of the international exchange rate, as well as sub-charges from port congestion, storage facilities, marketing agreements, and the distribution network, imported products have become more expensive. With the elimination of the subsidy, the price of importation of goods has increased, leading to increases in the price of most goods, such as transportation fares, which is a living example Navalur and Dewett (2010). Inflation affects imports and exports mostly by changing the currency rate. Inflation causes higher interest rates, which causes the currency to weaken. Higher inflation will have an impact on exports because it will immediately affect the price of commodities like materials, labour etc. The relationship between import duties and inflation in Nigeria rate have been inextricably linked. As the import duties rise, the rate of inflation rises. Inflation primarily impacts imports by affecting the exchange rate. High import results in higher interest rates, which results in a weaker currency. Higher inflation will also influence exports by directly affecting the price of commodities.

Time series models used movement in the data to establish the relationship among the data. It is also important to study the trend that exists between these series to understand the relationship among them. Transfer function models are used to study the relationship between the input process and output process. Box and Jenkins (1976)

2.0 LITERATURE

Anithakumari and Arumugam (2013). In India, their investigated the use of the Transfer function in modelling rubber production. This method was developed to model the series and forecast its values. An input series influenced the output series. The part of the response series in shaping the variable of attention is determined using this method. The Transfer Function ideal is applied to Normal Rubber manufacturing in India in this study. The idea was used to find a model and estimate constraints for rubber manufacture forecasts.

In South Africa, Ntebogang (2015) investigated the relationship between investment and savings. The series was evaluated for stationarity and found to be non-stationary at a high level. At early differences, the variable was stationary. The variable was estimated using five models, and ARIMAX (5, 1, 0) outperformed them all and was utilized for the pre-whitening process. This idea was future used to generate investment projections for the next two years. The error forecast measure offered sufficient data to establish that ARIMAX, gave reliable predictions.

Iwok (2016). Using a transfer function technique, model Naira exchange rates for US Dollars and Swiss Francs. The input and output series were pre-whitened to reduce the bogus correlation effect, and the two series were evaluated for stationarity using appropriate transformation. An ARIMA(1,1,0)) was used to model the input series. The pre-whitened input and output cross-correlation was investigated. The cross-correlation revealed that the dynamic transfer function might be represented by a rational polynomial

Victor-Edema and Essi (2020). studied or the Nigerian current account exchange rate, a transfer function model was proposed. The series was checked for stationarity, and it was found to be non-stationary at the level and lag one. The modified Dickey-Fuller test was employed to confirm the two series' stationarity. The transfer function model was identified using the cross-correlation function, and the input series was modelled using the Box-Jenkins autoregressive integrated moving average approach. Pre-whitening was used in both series. A diagnostic check was performed on the predicted transfer function using noise models.

3.0 METHODOLOGY

3.1 ARIMA Model with Differencing

Many secondary data are always non-stationary in naturel. such data need to difference to make it stationary.The d^{th} difference of a series y_t is symbolized by $\Delta^d\, y_t$ where Δ^d is the difference operator defined by Δ^d=1 – B. If the series $\{\Delta^d\, y_t\}$ follows the model (3.1), then $\{y_t\}$ is said to follow an ARIMA process of order (p, d, q).

$$\varphi(B)(1-B)^d y_t = \theta(B)\varepsilon_t \quad (3.1)$$

3.2 MODELS SELECTION

This a process of selecting the best models using : (i) Akaike information criterion (AIC), (ii) Hannau-Quinn information criterion (HQ) (iii) Schwarz information criterion (SIC). The information criteria and their formula are should below

- AIC = $\ln|\Sigma_r| + \frac{2}{T}MK^2$ (3.2)
- HQ= $\ln|\Sigma_r| + \frac{2InT}{T}MK^2$ (3.3)
- SC = $\ln|\Sigma_r| + \frac{InT}{T}MK^2$ (3.4)

3.3 LINEAR FILTER

A direct filter is a linear process from one input of X_t to output of Y_t mathematically represented as follow

$Y_t= L(X_t)=\sum_{i=-\infty}^{+\infty} \omega_i X_{t-1}=V(B)X_t$ (3.5)

Where t= ………..-1,0,1…………

A linear filter is a procedure that translates the an input data into output data. The process of conversion from one series to another is not instantaneous, it involves all the three phases of the series (past, present and future). The value of input is assumption with different weights (ω_i) on X_{t-1}. The following conditions hold

i. Time invariant at the co-efficient $\{\omega_i\}$ does not depend on time.

ii. Physical possible if ω_i=0

iii. $Y_t= \omega_0 X_t+ \omega_1 X_{t-1}+ \omega_2 X_{t-2}+ \omega_3 X_{t-3} +$……… (3.6)

$Y_t= V(B)X_t$ (3.7)

The deviation of the output variable at time t is denoted as a linear cumulative of input deviation at times past, The operator $V(B)$is the Transfer function of the filter.

iii Stable if $\sum_{i=-\infty}^{+\infty}|\omega_i| < \infty$

this is the stability condition (input and output are finite)

3.4 : RESPONSE FUNCTION

It X_t is a unit response at time $t = 0$

$X_t= \begin{cases} 1 \; t = 0 \\ 0 \; t \neq 0 \end{cases}$

the input $Y_t= \sum_{i=0}^{\infty} \omega_i X_{t-i}= \omega_i$.

STEP RESPONSE FUNCTION.

If X_t is a single step response is

$X_t= \begin{cases} 0 \; t < 0 \\ 1 \geq 0 \end{cases}$

Than the input Y_t is

$Y_t= \sum_{i=0}^{\infty} \omega_i X_{t-i}= \sum_{i=0}^{\infty} \omega_i$ (3.8)

This is also called the step response function

$X_t= \begin{cases} 0t < 0 \\ X \; t \geq 0 \end{cases}$

Hance

$Y_t = \sum_{i=0}^{\infty} \omega_i X_{t-i} = (\sum_{i=0}^{t} \omega_i) X = gX$ (3.9)

3.2.1 DIFFERENCE EQUATIONS

The General models for the representation of a constant dynamic system is the direct differential equation showed below

$(1 + \varphi_1 D + \cdots \dots \dots \dots .. + \varphi_R D^R) Y(t) = \text{g}(1 + \rho_1 D + \cdots \dots \dots \dots .. + \rho_S D^S) \text{X}(t - \tau)$ (3.10)

The above differential equation can parsimoniously represented as linear difference equation as follow

$(1 + \emptyset_1 \nabla + \cdots \dots \dots \dots .. + \emptyset_r \nabla^r) Y_t = \text{g}(1 + \theta_1 \nabla + \cdots \dots \dots \dots .. + \theta_S \nabla^S) X_{t-b}$ (3.11)

This is a transfer function models of order (r,s). this equation can rewrite in term of a backward shift operator $B = 1 - \nabla$

$Y_t = Y_t - Y_{t-1} = (1 - B) Y_t = \nabla$

$(1 - \delta_1 \text{B} - \cdots \dots \dots \dots .. + \delta_r \text{B}^r) Y_t = (\omega_0 - \varphi_1 \text{B} - \cdots \dots \dots \dots .. - \varphi_S \text{B}^S) X_{t-b}$ (3.12)

Or

$\delta(B) Y_t = \omega(B) X_{t-b}$ (3.13)

$\delta(B) Y_t = \omega(B) X_t B^b$ (3.14)

Let $\gamma(B) = \omega(B) B^b$

$\delta(B) Y_t = \gamma(B) X_t$

From equation (3.7) $Y_t = V(B) X_t$

$V(B) = \frac{\gamma(B)}{\delta(B)} = \frac{\omega(B) B^b}{\delta(B)}$ (3.15)

This models is represented by two characteristics polynomial.

$V(B) = (v_0 + v_1 B + v_2 B^2 + v_3 B^3 + v_4 B^4 + \dots\dots\dots\dots)$ (3.16)

$\delta(B) = (1 + \delta_1 B + B^2 - \delta_3 B^3 - \delta_4 B^4 - \dots\dots\dots\dots - \delta_r B^r.)$ (3.17)

$\omega(B) = (1 + \omega_1 B + B^2 - \omega_3 B^3 - \omega_4 B^4 - \dots\dots\dots\dots - \omega_r B^r.)$ (3.18)

$(1 + \delta_1 B + B^2 - \delta_3 B^3 - \delta_4 B^4 - \dots\dots\dots\dots - \delta_r B^r.)(v_0 + v_1 B + v_2 B^2 + v_3 B^3 + v_4 B^4 + \dots\dots\dots\dots) =$
$(1 + \omega_1 B + B^2 - \omega_3 B^3 - \omega_4 B^4 - \dots\dots\dots\dots - \omega_r B^r.) B^b$ (3.19)

Equating the coefficient of (applying the principle of mathematical induction) Iwok (2016).

$V_j = 0$ $\quad j < b$

$V_j = \delta_1 V_{j-1} + \delta_2 V_{j-2} + \dots\dots\dots\dots \delta_r V_{j-r} + \omega_0$ $\quad j = b$

$V_j = \delta_1 V_{j-1} + \delta_2 V_{j-2} + \dots\dots\dots\dots \delta_r V_{j-r} + \omega_0$

$V_j = \delta_1 V_{j-1} + \delta_2 V_{j-2} + \dots\dots\dots\dots \delta_r V_{j-r} - \omega_{j-b}$ $\quad J = b + 1, b + 2, \dots \dots \dots \dots . b + s$

$V_j = \delta_1 V_{j-1} + \delta_2 V_{j-2} + \dots\dots\dots\dots \delta_r V_{j-r}$ $\quad j > b + s$

3.2.2 TRANSFER FUNCTION MODELS NOISE

Let X_t and Y_t be two series representing the input and output of a system. In a practical system, the process is infected by error term. It effect is to reducing the output of the system, the disturbances term is denoted as

N_t

The combined transfer function Noise models are denoted as

$(1 - \delta_1 \text{B} - \cdots \dots \dots \dots .. + \delta_r \text{B}^r) Y_t = (\omega_0 - \varphi_1 \text{B} - \cdots \dots \dots \dots .. - \varphi_S \text{B}^S) X_{t-b}$

$$Y_t = \frac{(\omega_0 - \varphi_1 \mathrm{B} - \cdots \ldots\ldots\ldots - \varphi_S \mathrm{B}^S) X_{t-b}}{(1 - \delta_1 \mathrm{B} - \cdots \ldots\ldots\ldots - \delta_r \mathrm{B}^r)} + N_t \qquad (3.20)$$

3.2.3 MODELS IDETIFICATION

The Method of Cross Correlation

This method is used to identify a stochastic process. The technique is the cross-correlation function between the two series.

$$\rho(XY) = \frac{C_{XY}}{S_X S_Y}$$

$$C_{XY} = \frac{1}{n} \sum_{t=1}^{n-1} (X_t - \bar{X})(Y_t - \bar{Y}) \quad K = 0,1,2 \ldots \ldots \ldots \ldots \ldots \ldots$$

II Pre-Whitening

If the process is a stationary process, such that it follows an autoregressive integrated moving average model represented as

$$X_t = \theta X_{t-1} + \ldots\ldots\ldots += \theta_p X_{t-p} + \alpha_t \qquad (3.21)$$

The models is transform to input X_t and output Y_t

$$\alpha_t = X_t - (\theta X_{t-1} + \ldots\ldots\ldots += \theta_p X_{t-p}) \qquad (3.22)$$

In term of the input and rewriting the equation in term of the output

$$\beta_t = Y_t - (\varphi Y_{t-1} + \ldots\ldots\ldots += \varphi_p Y_{t-p}) \qquad (3.23)$$

Iwok (2016).

DATA ANALYSIS

The data used in this paper is a secondary data from (2003-2018). We purpose to build a transfer function process and forecast result of the data. Let X_t denotes the inflation rate and, Y_t the import duties. In our study, $X_t$is the input series and $Y_t$the output series.

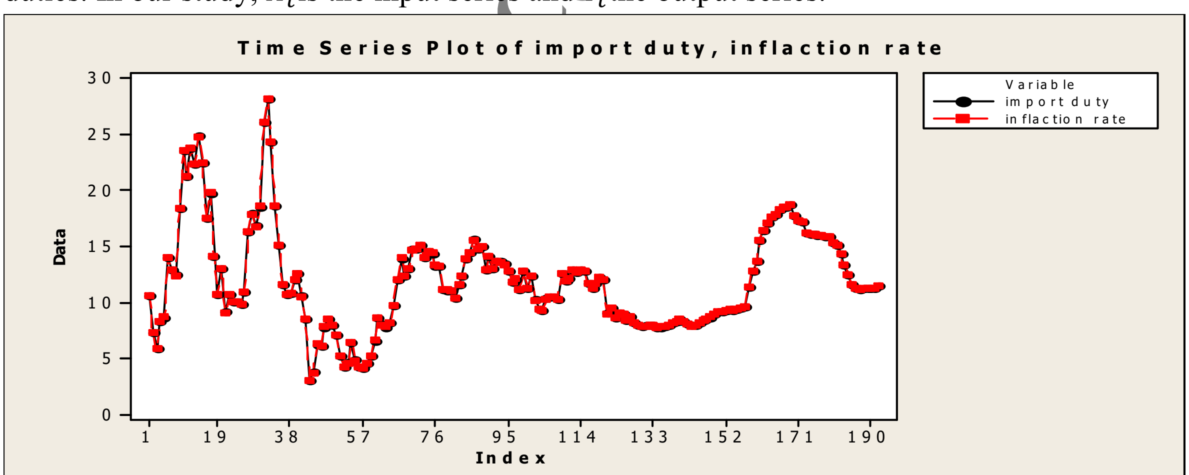

FIGURE 4.1: The PLOT OF THE TWO SERIES

From the figure above, we notice that the series is not stationary at the level. This required conversion to make it stationary. The data were stationary at first differences.

4.2: PEARSON CORRELATION OF THE TWO SERIES

The relationship between input series inflation rate and output series import duties series gives 1.00.

4.3 MODEL IDENTIFICATION FOR THE INPUT SERIES (INFLATION RATE)

The ARIMA (p,d,q) is used to model the input series. ARIMA model estimated with AIC and SC value shown below.

Models	A I C	S C
ARMA(1.1.0)	**3.93**	**3.981**
ARMA(2,1,0)	3.9557	4.0006
ARMA(3,1,0)	3.9533	4.004
ARMA(4,1,0)	3.953	4.0005
ARMA(4,1,0)	3.951	4.0127

Table (1.1) Selection ARMA (p, q) models

Five models were assessed with (AIC) and (SC) are in table 1.1. The best proces is with minimise coefficient and its ARIMA (1,1,0) with AIC (3.93) SC (3.981) showed table 1.0. resulting in the model.

$$(1-0.163B)(1-B)X_t = \alpha_t \quad (4.1)$$

$$X_t - 1.163X_{t-1} + 0.163X_{t-2} = \alpha_t \quad (4.2)$$

4.4 PRE-WHITENING OF X_t AND Y_t

The pre-whitened models for the input series is

$$\widehat{\alpha_t} = X_t - 1.163X_{t-1} + 0.163X_{t-2} \quad (4.5)$$

The pre-whitened models for the output series is

$$\widehat{\beta_t} = Y_t - 1.163Y_{t-1} + 0.163Y_{t-2} \quad (4.6)$$

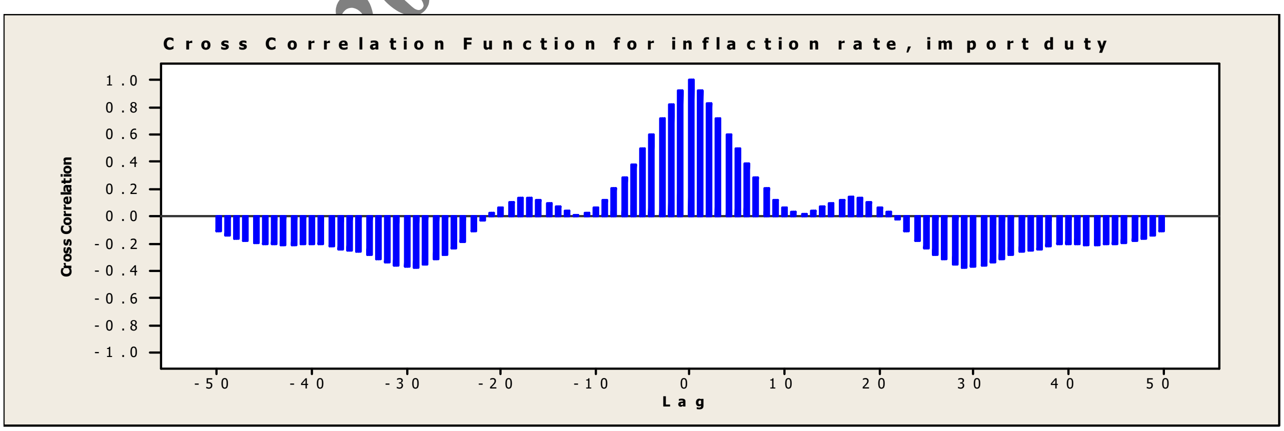

Figure 4.2: CROSS CORRELATION AFTER PRE-WHITENING

The cross-correlation between the pre-whitening series shown above is statistically significant at lag 1 to 6.

4.5 ESTIMATION OF IMPULSE RESPNSES FUNCTION V_j

From the estimated model the order of **(r, b, s)=(2,2,2)**

$(1+\delta_1 B+\delta_2 B^2)(v_0+v_1 B+v_2 B^2+v_3 B^3+v_4 B^4+\ldots\ldots\ldots\ldots)=(\omega_0-\omega_1 B-\omega_2 B^2.)B^2$ (4.7)

$V_j=0$ if $J<2$

$V_0=V_1=0$

$V_2=w_0=0.824$, $V_3=0.717$, $V_4=0.602, V_5=0.496, V_6=0.38$

$V_2=w_0$ $J=B=2,$ $w_0=0.824$ (1)

$\boldsymbol{V_3}=\delta_1 V_2-W_1$ $j=b+1$ $\mathbf{0.717}=0.824\delta_1-W_1$ (2)

$\boldsymbol{V_4}=\delta_1 \boldsymbol{V_3}+\delta_2 V_2-W_2$ $j=b+2$ $\mathbf{0.602}=\mathbf{0.717\delta_1}+\mathbf{0.824\delta_2}-W_2$ (3)

$j>b+s$

$\boldsymbol{V_5}=\delta_1 \boldsymbol{V_4}+\delta_2 V_3$ $0.496=0.602\delta_1+0.717\delta_2$ (4)

$\boldsymbol{V_6}=\delta_1 \boldsymbol{V_5}+\delta_2 V_4$ $0.38=0.496\delta_1+0.602\delta_2$ (5)

From the above equation, we can solve for $(\delta_1, \delta_2$ $and\ w_1, w_2)$

$\delta_1=3.85$, $\delta_2=-2.54$, $w_1=2.455, w_2=0.068$

$(1-3.58B+2.54B^2)X_t=(0.824-2.58B+0.068B^2)Y_{t-2}$ (4.8)

Or

When $V(B)=\frac{(0.824-2.58B+0.068B^2)B^2}{(1-3.58B+2.54B^2)}$ (4.9)

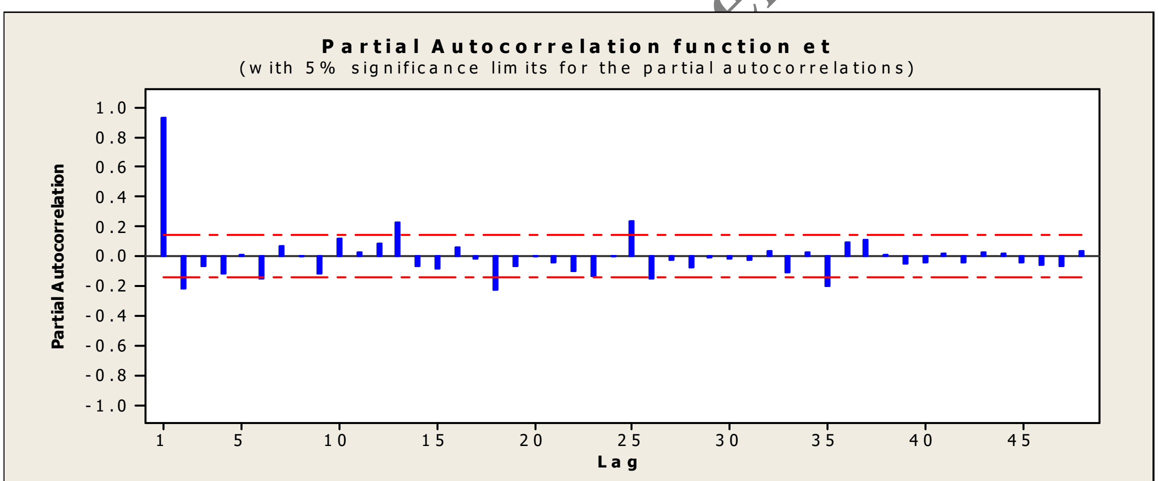

Figure 4.3: IDENTIFICATION OF NOISE MODEL

A spike in lag 1 of the partial autocorrelation function indicates an autoregressive model of order one AR(1).

$(1-\emptyset_1 B)\varepsilon_t=\alpha_t$ (4.10)

$(1-1.163B+0.163B^2)N_t=\varepsilon_t$ (4.11)

$(1-0.92B)(1-1.163B+0.163B^2)N_t=\alpha_t$ (4.12)

Since the last coefficient is small, N_t might be represented as a second order autoregressive model

$(1-2.083B+1.23B^2)N_t=\alpha_t$ (4.13)

The Transfer Function - Noise Model of Inflation rate and import Duty

$X_t=\frac{(\omega_0-\varphi_1 \mathrm{B}-\cdots\ldots\ldots-\varphi_S \mathrm{B}^S)Y_{t-b}}{(1-\delta_1 \mathrm{B}-\cdots\ldots\ldots-\delta_r \mathrm{B}^r)}+N_t$ (4.14)

$$X_t = \frac{(0.824 - 2.58B + .068B^2)Y_{t-2}}{(1 - 3.58B + 2.54B^2)} + \left(\frac{1}{1 - 2.083B + .23B^2}\right)\alpha_t \quad (4.15)$$

Generally, the transfer function white noise of inflation rate and import duties in Nigeria is represented in equation above

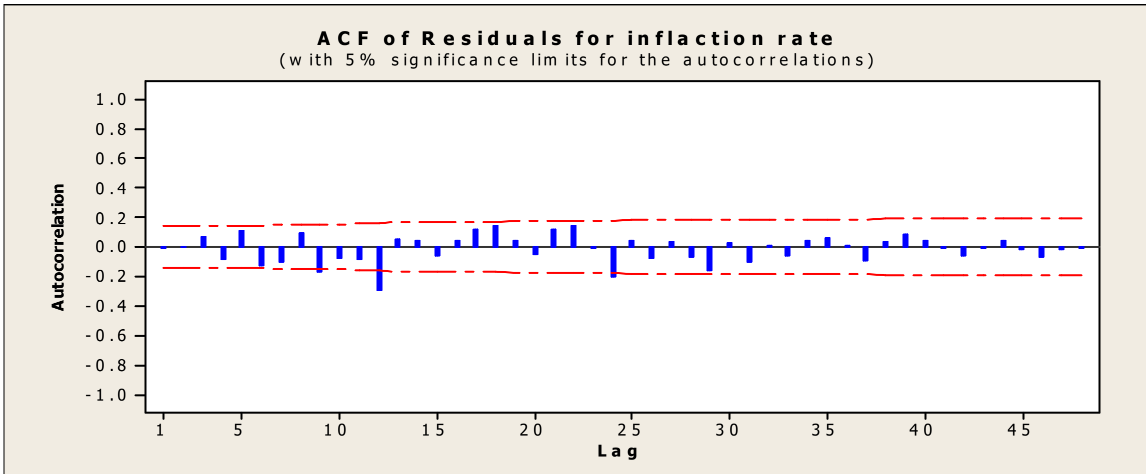

Figure 4.4: DIAGNOSIS

The residual autocorrelation function revealed the sufficiency in the model. since all the spikes are not significant at 5%.

5.0 CONCLUSION

This paper examined the monthly inflation rate and import duties in Nigeria The data for both series span from January 2003 to December 2018. The study applied transfer function to models series. taking the inflation rate as the input and import duties as the out variable. The input variable was modelled using an autoregressive integrated moving average process. Five models were estimated for input series and the best model was ARIMA (1,1,0) based on information criteria. The input and output series was pre-whitening and the resulting equation was used in modelling the dynamic transfer function models of the variables. The transfer model was tested for adequacy using the residual autocorrelation function. The model was found to be appropriate. The forecast was made based on the model.

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