RESTRICTED MULTIVARIATE-GARCH) MODEL OF EXCHANGE RATE, INFLATION RATE AND CRUDE OIL PRICE IN NIGERIA

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Abstract

In this study, the micro- economics in Nigeria were modelled using restricted multivariate Generalized Autoregressive conditional heteroscedasticity (M-GARCH). This strategy was chosen because the restricted technique is smoother than the unconstrained approach. The original series' time plot revealed trend, and the return series' logarithm transformation revealed stationarity. The diagonal VECH and BEKK models were estimate using the return series. However, for both Diagonal VECH and Diagonal BEKK, three conditional variances (GARCH) models and three co-variance models were computed. All of the variance and covariance models were significant at 5%. The Diagonal VECH model's parameters were all significant, although not all of them were positive definite. BEEK MODELS are required because the parameters of the square matrix of the Diagonal VECH model are not positive definite. All parameters in the BEKK model of the currency rate, inflation rate, and crude oil prices are significant and positive definite. Finally, the Diagonal BEKK model is the appropriate model for estimating the variance and co-variances of the exchange rate, inflation rate, and crude oil prices in Nigeria.

Keyword: Restricted, Multivariate GARCH, Diagonal VECH and Diagonal BEKK.

1.0 Introduction

Agricultural items such as cocoa, palm oil, rubber, columbine, and others were the mainstays of the Nigerian economy until oil was discovered in Bayelsa state in 1956. Crude oil accounts for over 80% of Nigeria's earnings and is a major source of money for the country's federal government. It's difficult to model fluctuations among these variable. The price of crude oil on the global marketplace rises or falls, which has a direct impact on the country's economy. The non-oil economy was also influenced by the increase or decrease in foreign or local currency, which increased the cost of products and services in the country. Crude oil price fluctuations are known for having a real impact on macroeconomic indicators such as the currency rate and inflation rates. Here remain theoretic details why crude oil price fluctuation should affect these variables: the crude oil price fluctuation can lead to lower or high request for goods and services. Wiri and Essi (2018)

Volatility between the three variables namely exchange rate, crude oil prices and inflation can be clear as the rate of change in movement above a agreed epoch. It is stated as a percentage variation in monthly variances of the series. The bigger the degree of the variation, the higher the change, the higher the volatility. The high Volatile level of exchange rates in the international market the more the risk on other variables. On the expression of it, fluctuating exchange rates would seem to be dangerous than stable rates meanwhile it is allowed to variation frequently. Etuk, (2012).

2.0 LITERATURE REVIEW

Tuaneh & Wiri (2018) studied the relationship between crude oil prices, exchange rates and inflation rates using the Unrestricted Vector Autoregressive (UVAR) model. In their study, the interactions there exist Granger causality among. The result showed that all variables were stationary at order one 1(1). The inverse root of the vector autoregressive model indicated a steady VAR model. The lag collection standards selected a lag distance of one.
Oluyemi and Essi (2017). Examines the effect of exchange rates on imports and exports using a vector autoregressive model. The result showed VAR model of lag selection criteria discloses that the lag length of order 2 is sufficient for the model based on the (SIC). The Vector autoregressive model result shows that exchange rates have a affirmative effect on imports while it damages exports on the fisrt lag but a affirmative effect on the second lag for both series respectively.

Metsileng *et al.* (2020) used Multivariate GARCH models to analyse the BRICS exchange rate volatility. From January 2008 to January 2018, secondary data was used in the study. All of the variables were determined to be statistically significant using the BEKK-GARCH model. Only Russia and South Africa were statistically significant, according to the diagonal parameter estimations. This means that the conditional variance of Russia and South Africa's exchange rates is influenced by their own prior conditional volatility as well as the conditional variance of other BRICS exchange rates. The BEKK-GARCH model also found that Russia and South Africa had a bidirectional volatility transmission. According to the DCC-GARCH model, the highest volatility persistence was found in Brazil, China, Russia, and South Africa, whereas the lowest volatility persistence was found in India. Except for Russia, all of the BRICS exchange rates demonstrate that the fitted residuals are not normally distributed

. Using Robust GAR-CH modelling, Ijomah and Enewari (2020) investigated volatility transmission between oil price and currency rate. I used the BEKK, DCC, and CCC models to study volatility transmission between series from January 2009 to December 2018. In terms of variances and covariance's, the model estimation procedure behaved similarly. The conditional correlation using the DCC model is negative and very weak, indicating that the crude oil price and the Nigerian exchange rate have a weak negative link. This demonstrates that the volatility of the oil price and the volatility of the exchange rate tend to move in different directions. Both the exchange rate and the price of crude oil show considerable variations in conditional covariances over time, according to the findings.

3.0 METHODOLOGY

3.1 MULTIVARIATE GARCH MODELS

Multivariate GARCH models is an extended case of univariate GARCH models to multivariate process that was significant in modelling the variance, covariance, ARCH and interaction among the variables. Theses model is divided into two-stage restricted and unrestricted. Restricted multivariate GARCH model, this a process of conditional the parameter of the model to variance and co-variance. The unrestricted case involved modelling all the parameters in the model such as variance, covariance, ARCH component and interaction among the variables

3.2 UNRESTRICTED VECH MODEL

These model was presented by Bollerslev,*et al* (1988). In the VECH process, the variance of the past and covariance of the past series is a function of its lag. The unrestricted VECH model can be represented below:

$$\boldsymbol{VECH(H_t)= A + \sum_{i=1}^{p} B_i\, VECH(\mu_{t-i}\mu^{/}_{t-1} + \sum_{i=1}^{q} C_i\, VECH(H_{t-i})} \qquad \textbf{3.1}$$

Where $\boldsymbol{VECH(H_t)}$ is an operator

N=2

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} .= \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} VECH \begin{bmatrix} \varepsilon_{1t-i} \\ \varepsilon_{2t-i} \end{bmatrix} [\varepsilon_{1t-i} \quad \varepsilon_{2t-i}] +$$

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} VECH \begin{bmatrix} h_{11t-i} & h_{12t-i} \\ h_{21t-i} & h_{22t-i} \end{bmatrix} \qquad 3.2$$

3.3 RESTRICTED VECH MODEL

The restriction implies that there are no direct volatility spillovers from one series to another. This reduces the number of parameters to be estimated. The diagonal VECH is represented as,

$$h_{ijt}= C_{il}+A_{ij}\mu_{it-i}\mu^{I}_{jt-1}+B_{ij}h_{ijt-1} \qquad 3.3$$

If N=2

h_{ijt}=$\begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}$, if h_{12t} =h_{21t}, h_{ijt}= $VECH \begin{bmatrix} h_{11t} & h_{12t} \\ 0 & h_{22t} \end{bmatrix}$, the upper diagonal matrix

A_{ij}= $\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}$, b_{ij}= $\begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix}$, μ_{t-i}=$\begin{bmatrix} \varepsilon_{1t-i} \\ \varepsilon_{2t-i} \end{bmatrix}$, μ^{I}_{t-i}=$[\varepsilon_{1t-i} \quad \varepsilon_{2t-i}]$, h_{t-i}= $VECH \begin{bmatrix} h_{11t-i} & h_{12t-i} \\ 0 & h_{22t-i} \end{bmatrix}$, $C = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{12} \end{bmatrix}$

$$\begin{bmatrix} h_{11t} \\ h_{12t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \\ \varepsilon_{1t-1\varepsilon_{2t-1}} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11t-i} & h_{12t-i} \\ 0 & h_{22t-i} \end{bmatrix} \quad 3.4$$

3.4 UNRESTRICTED BEKK MODELS

The BEKK model proposed by Engle and Kroner (1995) addresses the difficultly with VECH of ensuring that the H_t matrix is always positive definite the BEKK model is represented as follow

$$H_t = AA' + B'\varepsilon_{t-1}\varepsilon_{t-1}' B + C' h_{t-1} C \quad 3.5$$

let N=3

In general unrestricted BEKK model can be represented mathematical as follow. Tsay, (2010)

$$\begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} \\ h_{21,t} & h_{22,t} & h_{23,t} \\ h_{31,t} & h_{32,t} & h_{33,t} \end{bmatrix} = \begin{bmatrix} A_{11} \\ A_{21} \\ A_{31} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{31} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{21} & B_{31} \\ B_{12} & B_{22} & B_{32} \\ B_{13} & B_{23} & B_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \\ \varepsilon_{3,t-1} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} & \varepsilon_{3,t-1} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix} +$$

$$\begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{21,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{31,t-1} & h_{32,t-1} & h_{33,t-1} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \quad 3.6$$

3.6 RESTRICTED BEKK MODEL

In the Diagonal BEKK model, B and C are $N \times N$ matrix and A is a lower or upper triangular matrix of the parameter. The positive definiteness of the covariance matrix is ensured Taylor, (1986).

$$\begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} = B' = B, \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} = C' = C$$

In general the diagonal BEKK models is represented as follow

$$\begin{bmatrix} h_{11,t} \\ h_{22,t} \\ h_{33,t} \\ h_{12,t} \\ h_{23,t} \\ h_{13,t} \end{bmatrix} = \begin{bmatrix} A_{11} \\ A_{22} \\ A_{33} \\ A_{12} \\ A_{23} \\ A_{13} \end{bmatrix} + \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \\ \varepsilon_{3,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} & \varepsilon_{3,t-1} \end{bmatrix} \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix}$$

$$+ \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{21,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{31,t-1} & h_{32,t-1} & h_{33,t-1} \end{bmatrix} \begin{bmatrix} C_{11} & 0 & 0 \\ 0 & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \quad 3.7$$

$h_{13,t-1}= h_{32,t-1}=h_{12,t-1} =h_{23,t-1}= h_{31,t-1}=h_{32,t-1}$ their respective co-variances of the return series

$\begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \\ \varepsilon_{3,t-1} \end{bmatrix}$ = are the error of the three variable

4.0 Result

4.1 Data Properties:

The data used in this study was obtained from the Central Bank of Nigeria Web site (www.centralbank.org). The time plot of the original series of the exchange rate, inflation rate and crude oil price at the level and first differences is shown in figure (1).

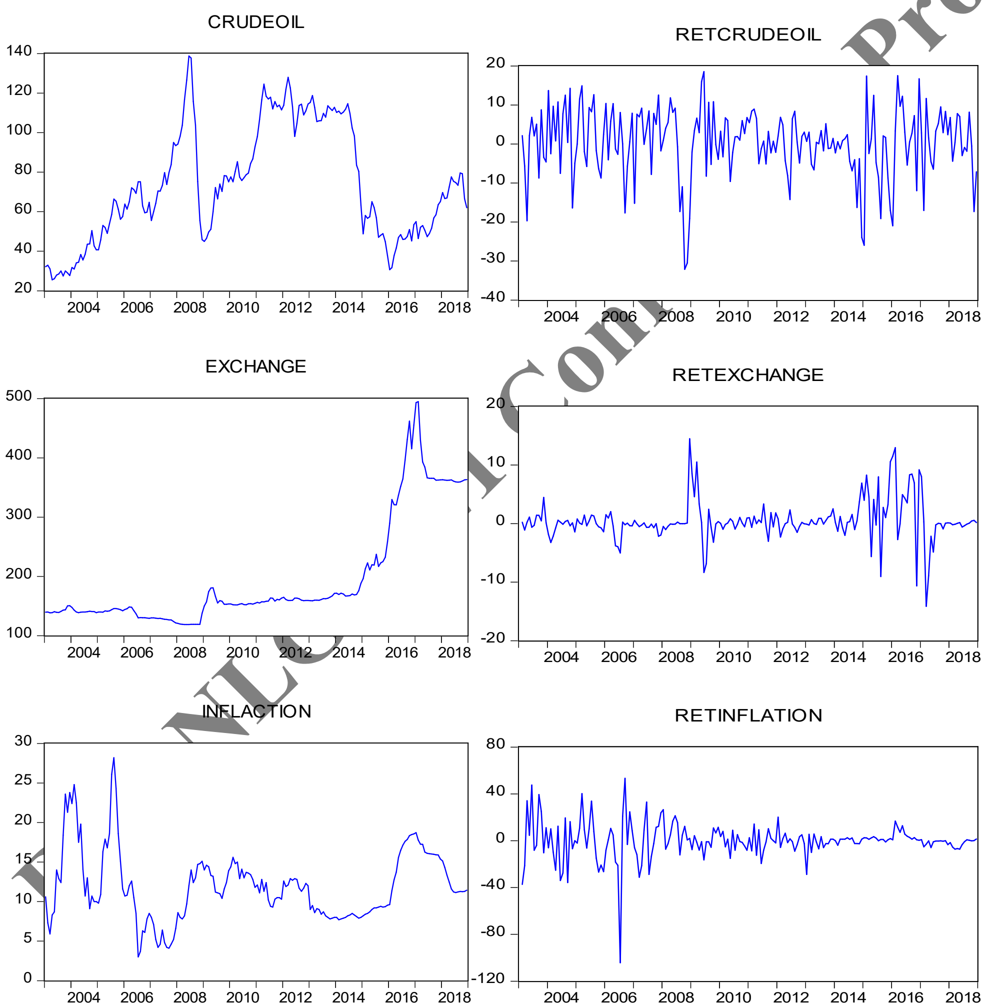

Figure 1.0 The Time Plot of the Original Series and First Differences

4.2 STATIONARITY TEST

The series was stationary at first difference. The probability value of all the variables were tested for stationarity and the three series was stationary at first differences. The probability values are (0.000). Gujarati,(2003.)

Variable	Levels	1st Difference
Exchange rate (EXC_t	-0.39485(0.9063)	-7.4183(0.000)
Inflation rate (INF_t)	-3.3562 (0.0100))	-10.7179 (0.00)
$crude\ oil(crd_t)$	-2.355685(0.156)	-12.47002(000)
1%	-3.46482	
5%	-2.87659	
10%	-2.57487	

Table (4.0) Augmented Dickey-Fuller (ADF) Unit Roots Test

4.3 LAG SELECTION CRITERIA

The criteria to select exact lag length (p) for a multivariate series is resolute by means of the lag distance selection such as(AIC), and (SIC). The best lag length is selected based on minimizing information criteria. Chris brooks, (2008).

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-1900.140	NA	216579.1	20.79934	20.85196	20.82067
1	**-1875.603**	**48.00099***	**182760.2***	**20.62954***	**20.84000***	**20.71485***
2	-1867.410	15.75825	184397.1	20.63837	21.00667	20.78766
3	-1863.783	6.859152	195589.6	20.69708	21.22322	20.91035
4	-1855.454	15.47440	197106.8	20.70441	21.38840	20.98167
5	-1852.975	4.524654	211798.7	20.77568	21.61751	21.11692

6	-1845.530	13.34354	215625.7	20.79268	21.79235	21.19790
7	-1836.326	16.19454	215415.8	20.79045	21.94797	21.25965
8	-1830.527	10.01327	223454.3	20.82544	22.14080	21.35862

Table (4.1) Vector Autoregressive Lag Selection Result

The table above discloses that the lag of order one (p=1) is good for the Multivariate model.

5.0 MODEL ESTIMATION

5.1 MULTIVARIATE GARCH MODELS

The multivariate Garch model is used to estimate covariance and correlation that exist between two or more series. These models are similar to the univariate Garch model, but M-Garch allows the variances and covariance to be time-varying. In this work, we are going to restrict our estimation on the restricted multivariate Garch models giving as follow Diagonal VECH and Diagonal BEKK. This restriction suggests that there are no straight volatility pullovers from one return to another return series. Since the model's strictures are reduced to minimum number for early estimation

5.2 DIAGONAL VECH MODEL

The trivariate diagonal VECH model of crude oil prices, inflation rate and exchange rate Nigeria were proposed by Bollerslev, et al (1988). The diagonal VECH these variable is represented as

$$\begin{bmatrix} h_{oil.oil.t} & h_{oil.exch.t} & h_{oil.inf.t} \\ h_{exch.oil.t} & h_{exch.exch.t} & h_{exch.inf.t} \\ h_{inf.oilt} & h_{inf.exch.t} & h_{inf.inft} \end{bmatrix} = \begin{bmatrix} h_{oil.oil.t} & h_{oil.exch.t} & h_{oil.inf.t} \\ 0 & h_{exch.exch.t} & h_{exch.inf.t} \\ 0 & 0 & h_{inf.inft} \end{bmatrix},$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} = \begin{bmatrix} 0.344926 & 0.25066 & -0.052731 \\ 0 & 0.39425 & 0.035488 \\ 0 & 0 & -0.070561 \end{bmatrix}$$

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix} = \begin{bmatrix} 0.618068 & -0.191705 & -0.394988 \\ 0 & 0.450900 & 0.861975 \\ 0 & 0 & 1.04710 \end{bmatrix}$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix} = \begin{bmatrix} 7.564377 & -5.226040 & -0.644257 \\ 0 & 1.59821 & 0.230052 \\ 0 & 0 & 2.130646 \end{bmatrix}$$

$$\epsilon_{ti} = \begin{bmatrix} \epsilon_{1t-1} \\ \epsilon_{2t-1} \\ \epsilon_{3t-1} \end{bmatrix}, \boldsymbol{\epsilon_{it-i}^{l}} = [\boldsymbol{\epsilon_{1t-1}} \quad \boldsymbol{\epsilon_{2t-1}} \quad \boldsymbol{\epsilon_{3t-1}}]$$

$$\begin{bmatrix}\epsilon_{1t-1}\\ \epsilon_{2t-1}\\ \epsilon_{3t-1}\end{bmatrix}[\boldsymbol{\epsilon_{1t-1}} \quad \boldsymbol{\epsilon_{2t-1}} \quad \boldsymbol{\epsilon_{3t-1}}]=\begin{bmatrix}\boldsymbol{\epsilon^2_{1t-1}} & \epsilon_{1t-1\epsilon_{2t-1}} & \epsilon_{1t-1\epsilon_{3t-1}}\\ \epsilon_{2t-1}\epsilon_{1t-1} & \boldsymbol{\epsilon^2_{2t-1}} & \epsilon_{2t-1\epsilon_{3t-1}}\\ \epsilon_{3t-1}\epsilon_{1t-1} & \epsilon_{3t-1}\epsilon_{2t-1} & \boldsymbol{\varepsilon^2_{3t-1}}\end{bmatrix}=$$

$$\begin{bmatrix}\boldsymbol{\epsilon^2_{1t-1}} & \epsilon_{1t-1\epsilon_{2t-1}} & \epsilon_{1t-1\epsilon_{3t-1}}\\ \boldsymbol{0} & \boldsymbol{\epsilon^2_{2t-1}} & \epsilon_{2t-1\epsilon_{3t-1}}\\ \boldsymbol{0} & 0 & \boldsymbol{\varepsilon^2_{3t-1}}\end{bmatrix}$$

$$\boldsymbol{VECH(H_{t-i})}=\begin{vmatrix}h_{oil.oil.t-1} & h_{oil.exch.t-1} & h_{oil.inf.t-1}\\ 0 & h_{exch.exch.t-1} & h_{exch.inf.t-1}\\ 0 & 0 & h_{inf.inft-}\end{vmatrix}$$

$$\begin{vmatrix}h_{oil.oil.t} & h_{oil.exch.t} & h_{oil.inf.t}\\ 0 & h_{exch.exch.t} & h_{exch.inf.t}\\ 0 & 0 & h_{inf.inft}\end{vmatrix}=$$

$$\begin{bmatrix}7.564377 & -5.226040 & -0.644257\\ 0 & 1.59821 & 0.230052\\ 0 & 0 & 2.130646\end{bmatrix}+$$

$$\begin{vmatrix}0.344926 & 0.25066 & -0.052731\\ 0 & 0.39425 & 0.035488\\ 0 & 0 & -0.070561\end{vmatrix}\begin{bmatrix}\boldsymbol{\epsilon^2_{1t-1}} & \epsilon_{1t-1\epsilon_{2t-1}} & \epsilon_{1t-1\epsilon_{3t-1}}\\ \boldsymbol{0} & \boldsymbol{\epsilon^2_{2t-1}} & \epsilon_{2t-1\epsilon_{3t-1}}\\ \boldsymbol{0} & 0 & \boldsymbol{\varepsilon^2_{3t-1}}\end{bmatrix}+$$

$$\begin{vmatrix}0.618068 & -0.191705 & -0.394988\\ 0 & 0.450900 & 0.861975\\ 0 & 0 & 1.04710\end{vmatrix}\begin{vmatrix}h_{oil.oil.t-1} & h_{oil.exch.t-1} & h_{oil.inf.t-1}\\ 0 & h_{exch.exch.t-1} & h_{exch.inf.t-1}\\ 0 & 0 & h_{inf.inft-1}\end{vmatrix} \qquad 5.1$$

Generally, the multivariate models of crude oil prices, exchange rate and inflation rate are represented above. This equation can be resolved into variances equation (Garch) and the co-variances as follow

5.3 VARIANCES EQUATION

$\boldsymbol{\sigma^2_{oil,t}}$= $\boldsymbol{7.564377 + 0.3449\epsilon^2_{oilt-1}}$+$0.61806\sigma^2_{oil.t-1}$ (GARCH(1,1)) 5.2

$\boldsymbol{\sigma^2_{exch,t}}$= $1598421 + 0.39425\boldsymbol{\epsilon^2_{excht-1}}$+$0.4509\sigma^2_{exch.t-1}$ (GARCH(1,1)) 5.3

$\boldsymbol{\sigma^2_{inf,t}}$=$2.130646 - 0.070561\boldsymbol{\epsilon^2_{infl.t-1}}$+$1.04710\sigma^2_{inf.t-1}$ (GARCH(1,1)) 5.4

5.4 CO-VARIANCES COMPONENT

$\sigma^2_{oil.t}\sigma^2_{excht}$= $-5.226040 + 0.25066\epsilon_{oilt-}\ \epsilon_{excht-1} - 0.191705\sigma^2_{oil.t-1}\sigma^2_{excht-1}$ 5.5

$\sigma^2_{exch.t}\sigma^2_{inf.t}$= $0.230230.035488\epsilon_{inft-}\ \epsilon_{excht-1}\ +0.861975\sigma^2_{exch.t-1}\sigma^2_{infl.t-1}$ 5.6

$$\sigma^2_{inf.t}\sigma^2_{oil.t} = -0.64425 - 0.052731\epsilon_{oilt-1}\epsilon_{inft-1} - 0.394988\sigma^2_{oil.t-1}\sigma^2_{infl.t-1} \quad 5.7$$

Since the parameter of the square matrix are not positive definite there is need for Diagonal BEKK MODELS

5.5 DIAGONAL BEKK MODEL

$$\begin{bmatrix} h_{11,t} \\ h_{22,t} \\ h_{33,t} \\ h_{12,t} \\ h_{23,t} \\ h_{13,t} \end{bmatrix} = \begin{bmatrix} h_{OIL,OIL,t} \\ h_{exch,exch,t} \\ h_{inf,inf,t} \\ h_{oil,exch,,t} \\ h_{exch\ inft} \\ h_{oil,inf,t} \end{bmatrix},$$

$$\begin{bmatrix} A_{11} \\ A_{22} \\ A_{33} \\ A_{12} \\ A_{23} \\ A_{13} \end{bmatrix} = \begin{bmatrix} 6.9304 \\ 1.60 \\ -0.099 \\ -5.947 \\ 0.825 \\ -5.579 \end{bmatrix},$$

$$\begin{bmatrix} B_{11} & o & o \\ o & B_{22} & 0 \\ o & o & B_{33} \end{bmatrix} = \begin{bmatrix} 0.51727 & o & o \\ o & 0.62027 & o \\ o & o & 0.005002 \end{bmatrix},$$

$$\begin{bmatrix} C_{11} & o & o \\ o & C_{22} & o \\ o & o & C_{33} \end{bmatrix} = \begin{bmatrix} -0.82515 & o & o \\ o & 0.661 & o \\ o & o & 0.9907 \end{bmatrix}$$

$$\begin{bmatrix} h_{11,t} \\ h_{22,t} \\ h_{33,t} \\ h_{12,t} \\ h_{23,t} \\ h_{13,t} \end{bmatrix} = \begin{bmatrix} 6.9304 \\ 1.60 \\ -0.099 \\ -5.947 \\ 0.825 \\ -5.579 \end{bmatrix} + \begin{bmatrix} 0.51727 & 0 & 0 \\ 0 & 0.62027 & 0 \\ 0 & 0 & 0.005002 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \\ \varepsilon_{3,t-1} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} & \varepsilon_{3,t-1} \end{bmatrix} \begin{bmatrix} 0.51727 & 0 & 0 \\ 0 & 0.62027 & 0 \\ 0 & 0 & 0.005002 \end{bmatrix}$$

$$+ \begin{bmatrix} -0.8251 & 0 & 0 \\ 0 & 0.6861 & 0 \\ 0 & 0 & 0.989 \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} & h_{13,t-1} \\ h_{21,t-1} & h_{22,t-1} & h_{23,t-1} \\ h_{31,t-1} & h_{32,t-1} & h_{33,t-1} \end{bmatrix} \begin{bmatrix} -0.8251 & 0 & 0 \\ 0 & 0.6861 & 0 \\ 0 & 0 & 0.989 \end{bmatrix}$$

5.8

5.6 VARIANCES EQUATION

$\boldsymbol{\sigma^2_{oil,t}} = \mathbf{6.930 + 0.2690}\boldsymbol{\epsilon^2_{oilt-1}} + 0.6808\sigma^2_{oil.t-1}$ (GARCH(1,1)) 5.9

$\boldsymbol{\sigma^2_{exch,t}} = 1.680 + 0.384\boldsymbol{\epsilon^2_{excht-1}} + \mathbf{0.4707}\sigma^2_{exch.t-1}$ (GARCH(1,1)) 5.10

$\boldsymbol{\sigma^2_{inf,t}} = -0.0909 + 0.000025\boldsymbol{\epsilon^2_{infl.t-1}} + \mathbf{0.97829}\sigma^2_{inf.t-1}$ (GARCH(1,1)) 5.11

5.7 CO-VARIANCES COMPONENT

$\sigma^2_{oil.t}\sigma^2_{excht} = -5.947 + 0.3217\epsilon_{oilt-}\ \epsilon_{excht-1} - 0.51617\sigma^2_{oil.t-1}\sigma^2_{excht}$ 5.12

$\sigma^2_{exch.t}\sigma^2_{inf.t} = -5.571 + 0.0002594\epsilon_{inft-1}\epsilon_{excht-1} - 0.81614\sigma^2_{exch.t-1}\sigma^2_{infl.t-1}$ 5.13

$\sigma^2_{inf.t}\sigma^2_{oil.t} = 0.825 + 0.0031\epsilon_{oil}\quad \epsilon_{inft-1} + 0.6785\sigma^2_{oil.t-1}\sigma^2_{infl.t-1}$ 5.14

6.0 Conclusion

This section seeks to deduce the key result derived from the estimate of all the M-GARCH models. The study examined the monthly exchange rate, inflation rate and crude oil prices in Nigeria. The data span from January 2003 to December 2018. The paper applied Multivariate specification of generalized autoregressive conditional heteroscedasticity (M-GARCH) models, in modelling variance and co-variance component that capture the feature of a financial series, such as volatility clustering, risk in the financial market and leverage effect. However, three models were estimated for variance and CO-VARIANCE from the VECH and BEKK process. The parameter of the VECH models does not satisfy the necessary and sufficient conditions for asymmetric and symmetric process , but the GARCH models and interaction among the return series all satisfy the covariance stationary condition that $\alpha + \beta < 1$. Furthermore, the coefficient of the BEKK models satisfies the necessary and sufficient condition for asymmetric and symmetric models (the matrix is positive definite). This implies that, increase in risk will lead to an increase in the conditional variance. This implied an increase in the mean return of the variables in international market.

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