PREFERENCE OF BAYESIAN METHODS OVER CLASSICAL METHOD IN ESTIMATING THE SCALE PARAMETER OF INVERSE RAYLEIGH FRECHET DISTRIBUTION

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Abstract

Among the methods of parameter estimation, maximum likelihood approach is the most often used. However, Maximum likelihood function (MLE) is data dependence, and insufficiecncy of data may cause the results obtained from this method to be unreliable. In this case, the Bayesian method, which allows the usage of the prior knowledge on the parameters in the estimation process, is adopted. This research paper aims to study the Bayesian analysis and compared it with maximum likelihood estimator on the scale parameter estimation of Inverse Rayleigh Frechet distribution based on uniform and quasi priors and applied Mean square error MSE criteria as a basis for comparison. In the Bayesian method, the Bayes estimates were obtained under Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and the Precautionary Loss Function (PLF). The performances of these estimators were compared to the Maximum Likelihood Estimates based on simulation study. The results of this analytic simulation show that the quadratic loss function is the preferred estimators since its posseses the lowest mean Square Error (MSE) under uniform prior and quasi prior. Finally, the results show that quadratic loss functions under Uniform prior and Quasi prior outperformed the squared error loss function, the precautionary loss function and Maximum Likelihood estimator across different sample sizes.

Keywords: Bayesian estimator, Bayes theory, Inverse Rayleigh Frechet, Loss functions, Maximum likelihood estimator, Prior distribution, Posterior distribution.

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1.0 Introduction

In the study of probability and statistics, Extreme value theory in a away plays an important role in statistical analysis. The frechet distribution is one of the distributions used to model extreme values. The Inverse Rayleigh-Frechet distribution (IRF) is one of the probability models that can be used to model extreme value distribution. This distribution has α and λ as the shape parameters and β as the scale parameter. The inverse Rayleigh Frechet distribution was developed by Saeed and Muhammad (2020). Since IR has only one parameter and so it does not present extreme flexibility for analyzing different types of lifetime data. Then a new continuous distribution Inverse Rayleigh-Frechet (IR-Frechet) introduced from family of distributions (Inverse Rayleigh family). In their paper, the estimator of the scale parameter was obtained based on maximum likelihood method of estimation. However, there is need to estimate the parameter based on other methods of parameter estimation, since every method of estimation has its advantages and disadvantages. In this paper, we proposed the Bayesian method of parameter estimation of the scale parameter of Inverse Rayleigh frechet distribution. And compared it with the maximum likelihood used in the previous paper based on MSE criteria for basis of comparison.

2. LITERATURE REVIEW

A new family of distribution named Inverse Rayleigh Family of distribution was developed by Saeed and Muhammad (2020), the family is invariably used to generate a new distribution named Inverse Rayleigh Frechet distribution (IR-Frechet). Saeed and Muhammad (2020), in their paper, studied the IR-Frechet Distribution. They derived some of the Statistical measures (properties) of the new generator which includes the moments, quantile and generating functions, entropy measures and order statistics. The Estimation of the model parameters by the maximum likelihood estimation method was done. Eraikhuemen *et al*. (2020) found that the Bayes estimators of the parameter (shape) of exponential inverse exponential distribution putting into consideration, the Jeffrey, Uniform, and gamma prior distributions based on three diffrent loss functions, that is, Quadratic Loss Function, Squared Error Loss Function and Precautionary Loss Function. Overall, their simulation results indicated that bayesian estimation under QLF provides estimator with least MSEs based on all the priors distributions. Eventually, some of the articls in this area of research tends to support Bayesian Method based on Quadratic Loss Function under gamma prior produces the best estimators of the shape parameter compared to estimators of Maximum Likelihood method, irrespective of any chosen parameters values and the assumed sample sizes. Pedro L. *et al* (2019) consider the challenge of parameters estimation of the Fréchet distribution from the frequentist view point and the Bayesian view point respectively. Aliyu and Yahaya (2016) studied the estimation of shape parameter of Generalized Rayleigh distribution with assumption of non-informative prior under squared error, Entropy and Precautionary loss functions. Comparison was also made between the performance of Maximum likelihood Bayesian estimators, and it was concluded that Bayes estimator under the entropy loss function is better than that of squared error loss function, Precautionary loss function and that of likelihood estimation. Abbas and Tang (2015) estimate the Fréchet distribution parameter putting reference priors into consideration. Many authors have made their contributions towards the development of bayesian method, which includes but not limited to Afaq *et al* (2015), Arnold (1983), Terma and Oguntunde (2018), Terna and Angela (2018), *Fatima* and Ahmad* (2017).

Parameter estimation is important in any probability distribution and as such, many authors adopt various methods of estimation. The classical estimation methods such as the Maximum likelihood estimation (MLE), Method of moment (MM), Least Square Estimation (LSE) and Weighted Least Square stimation (WLSE), maximum product spacing estimator (MPS), percentile estimator (PE), Cramér-von-Mises estimator (CME), Anderson-Darling estimator (ADE) and L-moment (LME) estimator etc. are frequently used for parameter estimation each one of them with its advantages and disadvantages. However, the most used among the methods of parameter estimation is Maximum likelihood estimation method. However, Maximum likelihood function (MLE) depends on data, when data is insufficient, MLE may not be reliable. In this paper, we intend to use Bayesian method of estimation to obtain the best estimator of the model and compared it with the maximum likelihood method based on mean square error MSE criteria.

3. Materials and Methods

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In this section, we considered the scale parameter estimation of Inverse Rayleigh Frechet distribution based on Bayesian approach. The prior distributions considered in estimating the posterior distribution of the scale parameter are uniform prior and quasi prior. These prior distribution were used to derived the posterior distribution of the scale parameter. Three loss functions, thus the Square Error Loss Function SELF, Quadratic Loss Function QLF, Precautionary Loss Function PLF were employed to derive the estimators through which the best estimator is selected based on mean square error (MSE) criteria. The best estimator according to the MSE criterion is the estimator with the small MSEs, the estimator with the smallest estimate is considered to be the best estimator of the scale parameter. The steps is under listed in subsection three (3)

3.1 Maximum Likelihood Estimation

The PDF of IR-Frechet is expressed as:

$$f(x;\,\theta,\alpha,\beta) = 2\theta\alpha^{-3}\beta^{-3}x^{3(\alpha+1)}e^{2\beta x^{-\alpha}}\left(1-\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}\right)\exp\left\{-\theta\left(\frac{1}{\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}}-1\right)^2\right\}; \qquad (1)$$

$$0 < x < \infty; \quad \alpha,\beta,\theta > 0$$

Where, θ is the scale parameter, α is the location Parameter and β is the shape parameter.

And its Hazard function is

$$h(x;\,\theta,\alpha,\beta) = \frac{2\theta\alpha^{-3}\beta^{-3}x^{3(\alpha+1)}e^{2\beta x^{-\alpha}}\left(1-\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}\right)\exp\left\{-\theta\left(\frac{1}{\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}}-1\right)^2\right\}}{\left[1-\exp\left\{-\theta\left(\frac{1}{\alpha\beta x^{-\alpha-1}e^{-\beta x^{-\alpha}}}-1\right)^2\right\}\right]} \qquad (2)$$

Suppose $(X_1, X_2, \ldots, X_n)$ are random variables from Inverse Rayleigh Frechet distribution with parameters $\theta, \alpha\ and\ \beta$. Then the MLE of θ (i.e $\hat{\theta}$) is derived as follows:

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\,\theta,\alpha,\beta) \qquad (3)$$

By substituting (1) in (3)

$$L(\theta) = (2\theta\alpha^{-3}\beta^{-3})^n \prod_{i=1}^{n}\left\{x_i^{3(\alpha+1)}\left(1-\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\right)\right\} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2+2\beta\sum_{i=1}^{n}x_i^{-\alpha}} \qquad (4)$$

This implies that,

$$\therefore\ \hat{\theta}_{MLE} = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2} \qquad (5)$$

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3.2 Bayesian Estimation

To estimate the scale parameter of the Inverse Rayleigh Frechet distribution, this study will adopt the Bayesian approach of estimation in which the posterior distributions will be obtained under the assumption of Uniform and Quasi prior.

- Uniform prior is defined as:

$$\pi(\theta) \propto 1; \quad where\ \theta\ is\ the\ parameter\ and\ 0 < \theta < \infty \tag{6}$$

- Quasi prior is defined as:

$$\pi(\theta) = \frac{1}{\theta^c}; \quad where\ \theta\ is\ the\ parameter, c\ is\ constant\ and\ 0 < \theta < \infty \tag{7}$$

Posterior distribution under Uniform prior

Posterior distribution of the scale parameter (θ) based on uniform prior can be obtained by substituting equation (4) and (6) in equation (8)

$\pi(\theta / x) =$

$$\frac{\left(2\theta\alpha^{-3}\beta^{-3}\right)^n \prod_{i=1}^{n}\left\{x_i^{3(\alpha+1)}\left(1-\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\right)\right\} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2+2\beta\sum_{i=1}^{n}x_i^{-\alpha}} * 1}{\int_0^\infty \left(2\theta\alpha^{-3}\beta^{-3}\right)^n \prod_{i=1}^{n}\left\{x_i^{3(\alpha+1)}\left(1-\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\right)\right\} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2+2\beta\sum_{i=1}^{n}x_i^{-\alpha}} d\theta} \tag{8}$$

$$\therefore \pi(\theta / x) = \frac{\theta^n e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2}\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^{n+1}}{\Gamma(n+1)} \tag{9}$$

Posterior distribution under Quasi prior

Posterior distribution of the scale parameter (θ) based on Quasi prior can be obtain by substituting equation (4) and (7) in equation (8)

$\pi(\theta / x) =$

$$\frac{\left(2\theta\alpha^{-3}\beta^{-3}\right)^n \prod_{i=1}^{n}\left\{x_i^{3(\alpha+1)}\left(1-\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\right)\right\} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2+2\beta\sum_{i=1}^{n}x_i^{-\alpha}} * \frac{1}{\theta^c}}{\int_0^\infty \left(2\theta\alpha^{-3}\beta^{-3}\right)^n \prod_{i=1}^{n}\left\{x_i^{3(\alpha+1)}\left(1-\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}\right)\right\} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2+2\beta\sum_{i=1}^{n}x_i^{-\alpha}} * \frac{1}{\theta^c} d\theta} \tag{10}$$

$$\therefore \pi(\theta / x) = \frac{\theta^{n-c} e^{-\theta\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2}\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^{n-c+1}}{\Gamma(n-c+1)} \tag{11}$$

Loss function

To obtain the Bayes estimators of the parameter (scale) of Inverse Rayleigh Frechet distribution from the posterior distributions obtained in equation (10) and (12) under the SELF, QLF and PLF.

- Squared Error Loss Function (SELF) is defined as:

$$L\left(\theta, \hat{\theta}_{SELF}\right) = \left(\theta - \hat{\theta}_{SELF}\right)^2 \tag{12}$$

Where $\hat{\theta}_{SELF}$ denotes the Bayes estimator under SELF and is given by:

$$\hat{\theta}_{SELF} = E(\theta / x) = \int_0^\infty \theta\, \pi(\theta / x)\, d\theta \tag{13}$$

- Quadratic Loss Function (QLF) is defined as:

$$L\left(\theta, \theta_{QLF}\right) = \left(\frac{\theta - \theta_{QLF}}{\theta}\right)^2 \tag{14}$$

Where θ_{QLF} denotes the Bayes estimator under QLF and is given by:

$$\theta_{QLF} = \frac{E(\theta^{-1}/x)}{E(\theta^{-2}/x)} = \frac{\int_0^\infty \theta^{-1}\, \pi(\theta / x)\, d\theta}{\int_0^\infty \theta^{-2}\, \pi(\theta / x)\, d\theta} \tag{15}$$

- Precautionary Loss Function (PLF) is defined as:

$$L(\theta_{PLF}, \theta) = \frac{(\theta_{PLF} - \theta)^2}{\theta} \tag{16}$$

Where θ_{PLF} denotes the Bayes estimator under PLF and is given by:

$$\theta_{PLF} = \{E(\theta^2 / x)\}^{\frac{1}{2}} = \left\{\int_0^\infty \theta^2\, \pi(\theta / x)\, d\theta\right\}^{\frac{1}{2}} \tag{17}$$

Bayesian Estimation under different loss function based on Uniform prior

The Bayes estimator under SELF based on uniform prior can be obtained from equation (9) and (13) as:

$$\therefore \hat{\theta}_{SELF} = \frac{(n+1)}{\left(\sum_{i=1}^n \left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2\right)} \tag{18}$$

Under QLF based on uniform prior, the Bayes estimator can be obtained from equation (9) and (15)

$$\text{Where, } E(\theta^{-1}/x) = \frac{\left(\sum_{i=1}^n \left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2\right)}{n} \tag{19}$$

$$\text{And } E(\theta^{-2}/x) = \frac{\left(\sum_{i=1}^n \left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2\right)^2}{n(n-1)} \tag{20}$$

By substituting equation (19) and (20) in equation (15)

$$\therefore \theta_{QLF} = \frac{(n-1)}{\left(\sum_{i=1}^n \left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2\right)} \tag{21}$$

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Under PLF based on uniform prior, the Bayes estimator can be obtained from equation (9) and (17)

Now, from equation (18)

$$E(\theta^2/x) = \frac{(n+1)(n+2)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2} \tag{22}$$

By substituting equation (23) in (18)

$$\therefore \theta_{PLF} = \frac{[(n+1)(n+2)]^{\frac{1}{2}}}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)} \tag{23}$$

Bayesian Estimation under different loss function based on Quasi prior

The Bayes estimator under SELF based on Quasi prior can be obtained from equation (11) and (13) as:

$$\therefore \hat{\theta}_{SELF} = \frac{(n-c+1)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)} \tag{24}$$

The Bayes estimator under QLF based on Quasi prior can be obtained from equation (11) and (15)

Where, $$E(\theta^{-1}/x) = \frac{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)}{(n-c)} \tag{25}$$

And $$E(\theta^{-2}/x) = \frac{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2}{(n-c)(n-c-1)} \tag{26}$$

By substituting equation (25) and (26) in equation (15)

$$\therefore \theta_{QLF} = \frac{(n-c-1)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)} \tag{27}$$

The Bayes estimator under PLF based on Quasi prior can be obtained from equation (11) and (17)

Now, from equation (17)

$$E(\theta^2/x) = \frac{(n-c+1)(n-c+2)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2} \tag{28}$$

By substituting equation (28) in (17)

$$\therefore \theta_{PLF} = \frac{[(n-c+1)(n-c+2)]^{\frac{1}{2}}}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)} \tag{29}$$

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Posterior Risk under three Loss Functions

The posterior risk of the Bayes estimator under SELF is defined as:

$$\pi(\theta_{self}) = E(\theta^2) - \left(E(\theta)\right)^2 \tag{30}$$

Posterior Risk under SELF based on uniform prior

Where, $E(\theta^2) = \frac{(n+1)(n+2)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2}$ (31)

Therefore the Bayes risk corresponding to the Bayes estimator under SELF based on on uniform prior can be derived by substituting equation (18) and (31) in equation (30)

$$\therefore \pi(\theta_{self}) = \frac{(n+1)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2} \tag{32}$$

Posterior Risk under SELF based on Quasi prior

Where, $E(\theta^2) = \frac{(n-c+1)(n-c+2)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2}$ (33)

Therefore the Bayes risk corresponding to the Bayes estimator under SELF based on on Quasi prior can be derived by substituting equation (24) and (33) in equation (30)

$$\therefore \pi(\theta_{self}) = \frac{(n-c+1)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1}e^{-\beta x_i^{-\alpha}}}-1\right)^2\right)^2} \tag{34}$$

Posterior risk under Quadratic Loss Function (QLF)

The Posterior risk of the Bayes estimator under QLF is given as:

$$\pi(\theta_{QLF}) = 1 - \frac{\left(E(\theta^{-1})\right)^2}{E(\theta^{-2})} \tag{35}$$

Posterior Risk under QLF based on Uniform prior

The posterior risk under QLF based on uniform prior can be obtain by substituting equation (19) and (20) in equation (35)

$$\therefore \pi(\theta_{QLF}) = \frac{1}{n} \tag{36}$$

Posterior Risk under QLF based on Quasi prior

The posterior risk under QLF based on Quasi prior can be obtain by substituting equation (25) and (26) in equation (35)

$$\therefore \pi(\theta_{QLF}) = \frac{1}{(n-c)} \tag{37}$$

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Posterior risk under Precautionary Loss Function (PLF)

The Posterior risk of the Bayes estimator under QLF is given as:

$$\pi(\theta_{PLF}) = 2\{\theta_{PLF} - E(\theta)\} \tag{38}$$

Posterior risk under PLF based on uniform prior

The Posterior risk under PLF based on uniform prior is obtained by substituting equation (18) and (23) in equation (38)

$$\therefore \pi(\theta_{PLF}) = 2\left\{\frac{[(n+1)(n+2)]^{\frac{1}{2}} - (n+1)}{\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2}\right\} \tag{39}$$

Posterior risk under PLF based on Quasi prior

The Posterior risk under PLF based on Quasi prior is obtained by substituting equation (24) and (29) in equation (38)

$$\therefore \pi(\theta_{PLF}) = 2\left\{\frac{[(n-c+1)(n-c+2)]^{\frac{1}{2}} - (n-c+1)}{\sum_{i=1}^{n}\left(\frac{1}{\alpha\beta x_i^{-\alpha-1} e^{-\beta x_i^{-\alpha}}} - 1\right)^2}\right\} \tag{40}$$

Simulation Study

simulation was carried out and comparison was made in order to check the performance of the different estimators focusing on biases and mean square errors under single replication to generate random samples of sizes n = (25, 35, 75 and 125) from the IR-Frechet. Under the following parameter values; $\theta = 0.5$ assuming α, β and c are known, given $\alpha = \beta = 1$, and $c = 0.5$. The MSE: $MSE = \frac{1}{n}E(\hat{\theta} - \theta)^2$ was adopted as the yardstick to ascertain relative performances of the estimators.

Table1: Estimates, Biases and MSEs for $\hat{\theta}$ under Uniform and Quasi prior across different loss functions

Sample size (n)	Measures	c	MLE	Uniform prior			Quasi prior		
				SELF	QLF	PLF	SELF	QLF	PLF
25	Estimates	0.5	0.4295439	0.431410	0.411093	0.412242	0.472998	0.459113	0.47247
	Bias	0.5	0.245427	0.253734	0.231808	0.282516	0.224483	0.224016	0.397015
	MSE	0.5	0.0123	0.0123	0.0105	0.0133	0.0133	0.0106	0.0145
35	Estimates	0.5	0.4398	0.4310	0.4134	0.4354	0.4398	0.4222	0.4442
	Bias	0.5	0.201042	0.201990	0.1787211	0.2059311	0.2296612	0.5479883	0.229389
	MSE	0.5	0.03208037	0.03299695	0.008172967	0.008773097	0.01199666	0.01039056	0.01216446

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75	**Estimates**	0.5	0.454926	0.454394	0.6173099	0.6214502	0.4874026	0.7049898	0.014214
	Bias	0.5	0.21235	0.2119801	0.2166346	0.249824	0.2377381	0.2110102	0.237659
	MSE	0.5	0.03830469	0.03881541	0.005796209	0.005991898	0.01412125	0.01102416	0.01421446
125	**Estimates**	0.5	0.4549263	0.454394	0.4517309	0.46214502	0.4874026	0.457049898	0.4873195
	Bias	0.5	0.21235	0.2119801	0.212347	0.24982475	0.2377381	0.21101021	0.2376594
	MSE	0.5	0.03830469	0.03881541	0.005796209	0.005991898	0.01412125	0.01024164	0.01421446

Table 1. Shows the results obtained from the simulation. The Bayes estimates, biases, mean square error (MSE) were obtained under different sample sizes across the different estimators (maximum likelihood, SELF, QLF, PLF under uniform and quasi prior). where $\alpha = 0.5$ assuming β, λ and c are known, given $\beta = \lambda = 1$ and $c = 0.5$. It is observed that, at sample size 25, the QLF has the smallest MSE of 0.0105 and 0.0106 under extended Jeffery and Quasi prior among other MSE from other estimators. Also at sample size 35, the QLF has the smallest MSE of 0.0052 and 0.0052 under extended Jeffery and Quasi prior among other MSE from other estimators. Similarly, the same apply to sample size 75 and sample size 125.

4. Results and Interpretation

MLE and Bayesian estimation of parameter (scale) θ of inverse Rayleigh Frechet (IR-Frechet) distribution is considered. Under the assumption of Uniform and Quasi prior with three (3) loss functions consisting Quadratic Loss Function, Squared Error Loss Function, and Precautionary Loss Function. Derivation of posterior distributions and Bayes estimators are obtained based on the priors and its loss functions respectively. Simulation was conducted in order to compare the performances of the estimators based on Mean Square Error as a performance measure. The performances of the MLEs and the Bayes estimators were conducted the result that follows indicate that the Bayesian estimator of the parameter (scale) θ under uniform and quasi prior based on quadratic loss function is better than the maximum likelihood estimate in terms of Mean Square Error. The results shows that quadratic loss functions under Uniform prior and Quasi prior outperformed the squared error loss function and the precautionary loss function across different sample sizes. The result also reveals that the Bayesian estimates of the scale parameter θ under uniform and quasi prior based on quadratic loss function is better than the maximum likelihood estimate.

5. Conclusion

The results of the comparison between Maximum likelihood and Bayesian methods of estimating the parameter (scale) θ of the inverse Rayleigh Frechet (IR-Frechet) distribution shows that quadratic loss functions (QLF) under Uniform prior and quasi prior outperformed the squared error loss function (SELF) and the precautionary loss function (PLF) across different sample sizes. Finally, the results reveal that the Bayesian estimates of the scale parameter θ under the uniform and quasi prior based on quadratic loss function is better than the maximum likelihood Estimates.

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