

# MODELLING MONTHLY WIND SPEED IN NORTH WEST NIGERIA: ExAR-FIGARCH AND ExAR-GARCH COMPARATIVE ANALYSIS.

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**Abstract:** Understanding wind speed is the key planning renewable energy projects studying climate patterns and forecasting weather. In our study we explore how monthly wind speeds behave in North West Nigeria using an advanced model known as the Exponential Autoregressive-Fractional Integrated Generalized Autoregressive Conditional Heteroscedasticity (ExAR-FIGARCH). This model not only captures the lingering effects of past wind speeds but also accounts for unpredictable shifts over time. To see how its stacks up, we compare its performance against the more traditional ExAR-GARCH model, which mainly focuses on short-term fluctuations. We estimated two models and results showed the ExAR-FIGARCH model is better based on serial correlation analysis, efficient parameters and measures of accuracy, along with their ability to forecast future values. Our findings suggest that embracing long-memory effects in wind speed analysis could provide better insights into the region's wind energy potential.

Key words: Wind speed, ExpAR-FIGARCH, ExpAR-GARCH model Long Memory, Volatility modelling.

## 1.0 Introductions

Imagine standing outside on a breezy day in North West Nigeria feeling the gentle caress of the wind and then, suddenly, encountering an unexpected gust that reminds you of nature's unpredictability. This natural phenomenon is not just a backdrop to our daily lives, it plays a critical role in renewable energy, agriculture and weather forecasting. Yet understanding and predicting these wind patterns, with all their twists and turns remains a significant challenge.

In this paper we set out to explore the behavior of monthly wind speeds in North West Nigeria using two distinct modeling approaches. On the other hand, we have the ExAR-GARCH model, as trusted method that captures short term volatility. On the other we introduces the ExAR-FIGARCH model which not only handles immediate fluctuations but also accounts for the subtle, long memory effects that influence wind behavior over time. By comparing these models, our goal is to provide a clearer picture of how wind speeds evolve in this region. We aim to offer insights that can help local planners and engineers bitter

harness wind energy and improve weather forecasting, therefore this study is about blending robust statistical methods with real-world observations to better understand a natural force that affect us all.

### 1.1 Literature Review

Understanding and accurately forecasting wind speed is crucial for optimizing wind energy production and ensuring the reliability of the power systems. Traditional models, such as Autoregressive Moving Average (ARMA) by Box & Jenkins (1970) and Generalizes Autoregressive Conditional Heteroskedasticity (GARCH) of Baillie et al (1996), have been employed to capture the temporal dependencies and volatility clustering inherent in wind speed data.

However, these models often fall short in addressing long memory characteristics observed in wind speed time series. To bridge this gap, the Exponential Autoregressive Fractionally Integrated GARCH (ExAR-FIGARCH) model has been introduced by Jibrin et al (2024), offering more nuanced approach to modelling wind speed dynamics. The ARMA-GARCH framework has been widely applied in modelling wind speed due to its capability to handle short term dependencies and volatility clustering, for instance, Liu et al(2011) utilized the ARMA-GARCH to forecast 1 hour mean wind speeds, demonstrating its applicability in wind energy analysis.

Similarly Masseran (2016) highlighted the model's effectiveness in capturing the stochastic nature of wind speed fluctuation. However, these models assume short memory in volatility processes which may not adequately represent the persistence observing the wind speed data.

To address the limitations of traditional models researchers have explored models that account for long memory effects. The ARFIMA-FIGARCH model of Baillie et al(1996) , which combines Autoregressive Fractionally Integrated Moving Average (ARFIMA) with FIGARCH, has been proposed to capture both

long-term dependencies in the mean and volatility of wind speed data. Studies have shown that this approach provides a more accurate representation of wind speed dynamics, especially for long term forecasting horizons (Liu et al 2011).

Building upon the ARFIMA-FIGARCH framework, the ExAR-FIGARCH model introduces exponential autoregressive component to better capture nonlinear patterns and asymmetries in wind speed data. This enhancement allows for a more flexible modelling approach, accommodating sudden changes and extreme values often observed in the wind speed time series. The evaluation of wind speed modelling has progressed from traditional short memory models to advanced framework that incorporate long memory effects and nonlinear components.

## 1.2 Objective of the Study

- (i) To improve wind speed forecasting accuracy.
- (ii) To capture nonlinearity in mean and long memory effects in wind speed volatility.
- (iii) To compare performance with traditional models.
- (iv) To support wind energy planning, management and analyze extreme wind events.
- (v) To contribute to climate and metrological studies.

## 2.0 Methodology

The ExpAR Ozaki(1980) model of order  $p$  denoted by ExpAR( $p$ ) can be defined as:

$$Y_t = C + (\phi_j + \lambda_j e^{-\gamma Y_{t-1}^2}) Y_{t-j} + \varepsilon_t. \quad (1)$$

Many time series exhibits trend, volatility and long memory effect (see Jaiswal et al(2019), Kim et al(2020), Liang et al(2021) and Fameliti and Skintzi(2022)). Besides handling trend, it is clear that the Exp-AR model lack requirements of handling time series with volatility and long memory characteristics.

## 2.1 The Proposed ExpAR-FIGARCH Model

The current study assumed that the model in (1)

- a. Failed to account for the volatility and long memory that are present and dwelled in the time series  $\{Y_t\}$ ,  $t = 1, \dots, T$ .
- b. Have residuals  $\{\varepsilon_t\}$ ,  $t = 1, \dots, T$  that are serially correlated and heteroscedastic.
- c. Could not account for the high degree of relationship that exists in volatility of a time series as observed by Gonzaga (2022), Gil-Alana et al (2022) and Aliyu et al(2023) for similar mean models.

Consequently, the current study wants to introduce the ExpAR-FIGARCH hybrid model. The ExpAR( $p$ )-FIGARCH( $u, d, v$ ) is for the study of the nonlinear, volatility and long memory in time series. The  $\varepsilon_t$  in eq.(1) is a stochastic process that can be expressed as:

$$\varepsilon_t = m_t \sigma_t, \quad (2)$$

Where  $E(m_t) = 0$ ,  $Var(m_t) = 1$  and  $\sigma_t$  is positive and changes with respect to time,  $t$ . This implies that the process,  $\{m_t\}$ , is assumed to be serially uncorrelated expressed as:

$$m_t \sim iid(0,1) \quad (3)$$

Thus, the conditional variance  $\sigma_t$  is non-stationary process that changes over time. In view of this, Baillie *et al.*, (1996) introduced the FIGARCH( $u, d, v$ ) model to study  $\sigma_t$  as:

$$\sigma_t^2 = c[1 - \psi(L)]^{-1} + \{1 - [1 - \psi(L)]^{-1}\alpha(B)(1 - L)^d\}\varepsilon_t^2, \quad (4)$$

Now, to develop the hybrid ExpAR-FIGARCH model, from eq.(4), consider the function

$$f(\psi(L)) = [1 - \psi(L)]^{-1}, \tag{5}$$

The Taylor series expansion for eq.(5) is

$$f(\psi(B)) = [1 - \psi(L)]^{-1} = 1 + \psi(L) + (\psi(L))^2 + \dots \tag{6}$$

Let consider part of the expansion in (6)  $[1 - \psi(L)]^{-1} = 1$  and substituting in eq. (4). Then, so that we can have

$$\sigma_t^2 = c + \{1 - \alpha(L)(1 - L)^d\} \varepsilon_t^2. \tag{7}$$

However,  $\alpha(L) = 1 - \alpha(L) - \beta(L)$ (see Lopes, 2008 p.11). Therefore, the FIGARCH( $r,d,v$ ) can be expressed as:

$$\sigma_t^2 = c + \{(\alpha(L) + \beta(L))(1 - L)^d\} \varepsilon_t^2. \tag{8}$$

Similarly, eq.(8) can be expressed as:

$$\sigma_t = [c + \{(\alpha(L) + \beta(L))(1 - L)^d\} \varepsilon_t^2]^{\frac{1}{2}}. \tag{9}$$

Again, substituting eq.(9) in (2), the following is obtained

$$\varepsilon_t = m_t [c + \{(\alpha(L) + \beta(L))(1 - L)^d\} \varepsilon_t^2]^{\frac{1}{2}}. \tag{10}$$

Finally, let  $\varepsilon_t = m_t [c + \{(\alpha(L) + \beta(L))(1 - L)^d\} \varepsilon_t^2]^{\frac{1}{2}}$  in eq. (1) so that the ExpAR( $p$ )-FIGARCH( $u,d,v$ ) can be represented as:

$$Y_t = C + (\phi_j + \lambda_j e^{-\gamma Y_{t-1}^2}) Y_{t-j} + m_t [c + \{(\alpha(L) + \beta(L))(1 - L)^d\} \varepsilon_t^2]^{\frac{1}{2}}. \tag{11}$$

Where  $Y_t$  is dependent variable  $C$  is a constant term,  $\phi_j, \lambda_j$  ( $for j = 1, \dots, p$ ) are unknown parameters to be estimated from  $Y_t$ ,  $\varepsilon_t$  is the error terms that are independent and identically distributed random variables,  $p$  is the order of the model and  $\gamma$  is defined as the scaling parameter.

The  $\alpha(B) = \alpha_1 L^1 + \dots, \alpha_r L^r$  and  $\beta(L) = \beta_1 L^1 + \dots, \beta_v L^v$  are called characteristics polynomial and all their roots are expected to lie in the unit root circle while  $L$  is the lag operator. Where  $\omega > 0, \phi_j \geq 0$ , for  $j = 1, \dots, p, \alpha_k \geq 0$  for  $k = 1, \dots, r, \beta_l \geq 0$  for  $l = 1, \dots, v, d$  is a long- memory parameter.

In addition, when  $p = u = v = 1$ , the ExpAR(1)-FIGARCH(1,d,1) can be shown as below (12)

$$Y_t = C + (\phi_1 + \lambda_1 e^{-\gamma Y_{t-1}^2}) Y_{t-1} + m_t [\omega + \{(\alpha_1 + \beta_1)(1-L)^d\} \varepsilon_t^2]^{\frac{1}{2}}. \quad (12)$$

## 2.2 The Estimation Procedure of the Hybrid ExpAR-FIGARCH Models

The steps for the estimation of the ExpAR-FIGARCH model are:

- 1) Estimate the long memory parameter,  $d$ , by Geweke and Porter-Hudak (GPH) method.
- 2) Identify the parameters of the ExpAR( $p$ )-FIGARCH( $u, d, v$ ) model.
- 3) Select the best candidates of ExpAR-FIGARCH based on minimum Accuracy measures estimates.
- 4) Test for the adequacy of the chosen ExpAR-FIGARCH model(s).
- 5) Carry out serial correlation analysis on the residuals of ExpAR-FIGARCH model(s).
- 6) Finally, perform the analysis.

The iterative process for fitting the ExpAR, ExpAR-FIGARCH and ExpAR-GARCH models is will be carried out reference to the Box-Jenkins modeling procedure.

## 3.0 Data and Analysis

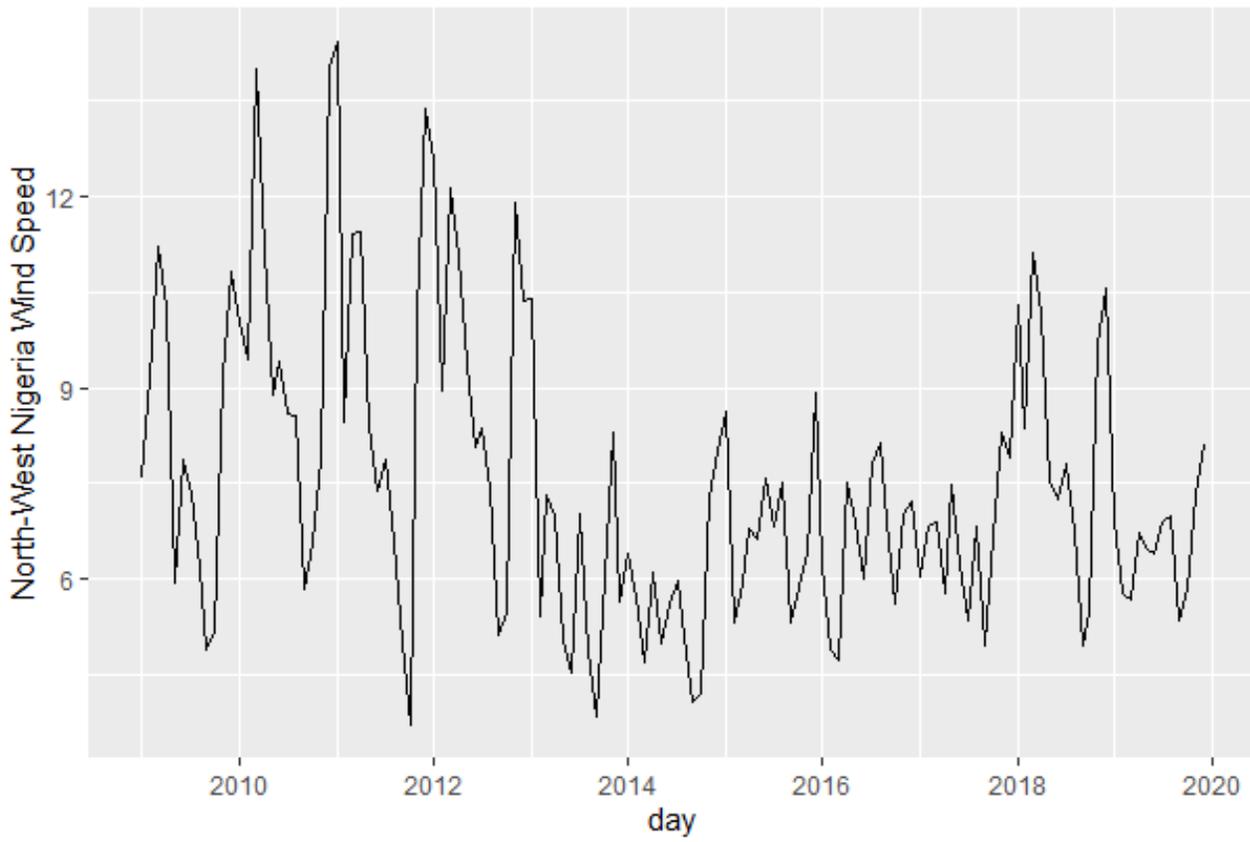
Monthly Nigeria North-West Wind Speed (MNWNWS) is used to determine the best of the class of models considered. The considered models are ExAR, ExAR-GARCH and the proposed hybrid ExAR-FIGARCH model.

Table 1 presents the descriptive statistics and normality test for the MNWNWS index. The mean and standard deviation for MNWNWS is 7.51 and 5.06 respectively. The kurtosis, which measures the risk of sudden gust or lulls in MNWNWS is 0.48 indicating wind speed are more with less frequent sudden changes. The skewness and Jarque-Bera statistic for MNWNWS indicate non-normality for the series.

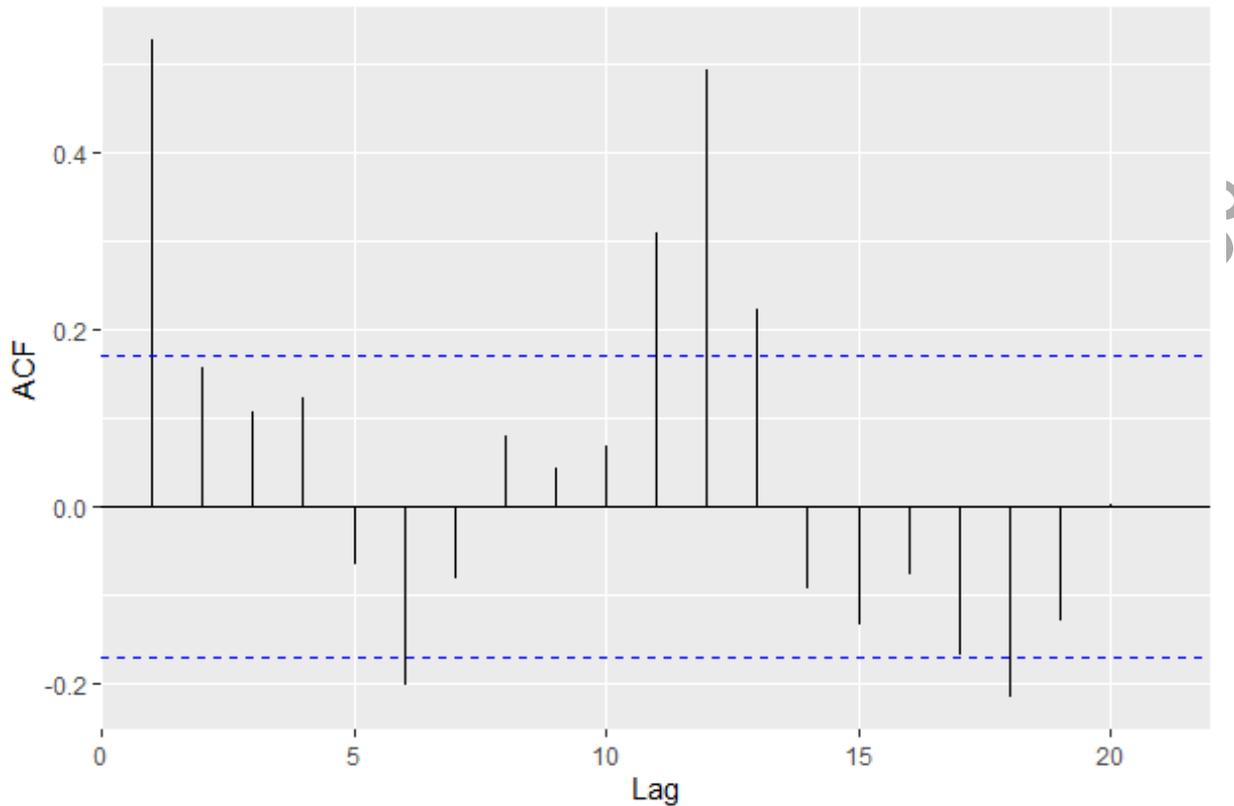
Table 1: Descriptive Statistics for Daily MNWNWS

Statistics	MNWNWS
Minimum	3.74
Maximum	14.39
Mean	7.51
Std.Dev	5.06
Skewness	0.91
Kurtosis	0.48
Jarque-Bera Test	20.18(0.0000)

Figure 1: show time series plot of MNWNS

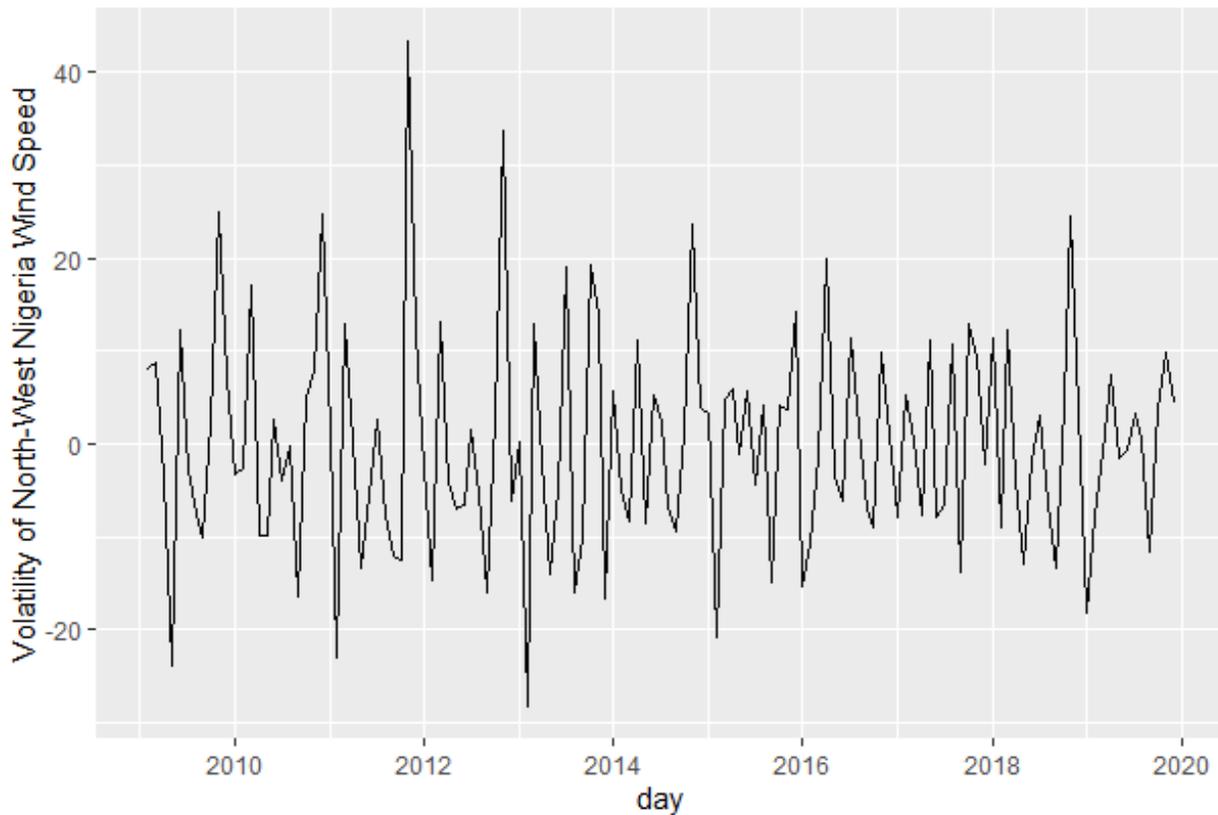


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*Figure 2: The ACF of the MNWNWS*

The time series plot of the Monthly North-West Nigeria are shown in Fig. 1. The time series graphs exhibit unstable trend behavior. The ACF of the MNWNWS is shown in Fig. 2. The autocorrelations indicates a slow decay which is evidence of long memory process. Therefore on the average, the MNWNWS are not stationary. Having said this, the trend behavior observed in MNWNWS series indicates the ExAR is a candidate model study time series data with this type of attributes.

Figure 3: The volatility plot of the MNWNWS



The volatility plot of the MNWNWS is shown in Fig. 3. The plot indicates evidence of volatility clustering indicating that the series is volatile. The observed volatility shows that the volatility models such as GARCH and FIGARCH are other candidate models to study the MNWNWS series.

### 3.1 ExAR(p) Model Estimation and Diagnostic tests

The ExAR modeling based on model estimation and diagnostic analysis are carried out in this section. The estimation of ExAR(1) and ExAR(2) model for the MNWNWS and serial correlation analysis results are displayed in Table 2 and 3.

All the parameters in the ExAR models estimated using the MNWNWS are insignificant due to large standard errors of the parameters. The serial correlation analysis results show that the residuals

of the estimated mean models, ExAR(1) and ExAR(2) are heteroscedastic and non-normal. This is because the p-values are less than the 0.01 significance level. This suggests the two models are inadequate to be considered for studying the MNWNWS series as they failed to reduce or eliminate the noise signals in the MNWNWS so that the models can be considered for forecast.

Table 2: ExAR(1) Models Estimation and Diagnostic Analysis Of Monthly North-West Wind Speed (MNWNWS)

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
c	6.3692	0.9168	6.947	0.0000
$\lambda_1$	0.1250	0.1031	1.212	0.2278
$\phi_1$	3.0929	0.8092	3.822	0.0002
ARCH-LM Test= 13.375(0.0003) and Jarque-Bera Test = 83.06(0.0088)				

Table 3: ExAR(2) Models Estimation and Diagnostic Analysis of Monthly North-West Wind Speed (MNWNWS)

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
c	6.6489	0.4812	13.819	0.0000
$\lambda_1$	0.3750	1.6347	0.229	0.8190
$\lambda_2$	0.5000	2.3379	0.214	0.8310
$\phi_1$	24.3046	743.1532	0.033	0.9740
$\phi_2$	-23.7429	742.8946	-0.032	0.9750
ARCH-LM Test= 13.19(0.0003) and Jarque-Bera Test = 4.8593(0.0018)				

Statistical fact that explained the heterogeneity and non-normality in models residual are high noise signals, size of volatility, outliers and volatility clustering as seen in Figure 3.

The GARCH and FIGARCH known as volatility and long memory volatility models respectively could be joined with the ExAR models to form hybrid model. This could help in eliminating the observed unwanted signals. It would also assists in improving the fitting of the ExAR model to the MNWNWS data. Having said this, an analyses by considering hybrid model of ExAR-GARCH and ExAR-FIGARCH models would be carried out and discussed in next section.

### 3.2 Hybrid ExAR-GARCH and ExAR-FIGARCH Modeling

This section discusses the estimation and diagnostic tests of hybrid model; ExAR-GARCH and ExAR-FIGARCH using the MNWNWS. Before estimating the hybrid models that involved the long memory volatility model, FIGARCH, it is important to investigate the presence of long memory in the MNWNWS.

Table 4: Long Memory Parameter Estimation

Data	Volatility
MNWNWS	-0.3565

The long memory in the volatility of MNWNWS was further estimated and is displayed in Table 4. The Geweke and Porter-Hudak (GPH) long memory estimation method produced the fractional differencing value to be -0.3565 for the volatility of MNWNWS. This value confirmed the long memory attributes in the original series and the volatility of MNWNWS and indicating the suitability of considering the FIGARCH model. The results of the parameters estimation of ExAR-GARCH models using the MNWNWS is shown in Table 5 and 6 and ExAR-FIGARCH in Table 7 and 8. The hybrid models estimated are assumed to be normal and student  $t$  distribution because of the heterodasticity of the ExAR models,

Table 5: ExAR(1)-GARCH(1,1) Models Estimation and Diagnostic Analysis of Monthly North-West

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
c	6.3692	0.9168	6.947	0.0000
$\lambda_1$	0.1250	0.1031	1.212	0.2278
$\phi_1$	3.0929	0.8092	3.822	0.0002
GARCH(1,1) Components with $m_t$ assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0712	0.0003	-248.9392	0.0000
$\alpha_1$	0.0470	0.0002	266.5832	0.0000
$\beta_1$	0.9306	0.0010	963.6716	0.0000
ARCH-LM Test= 1.7003(0.1922) and Jarque-Bera Test = 8.1281(0.0172)				
GARCH(1,1) Components with $m_t$ assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.1204	0.0003	346.3377	0.0000
$\alpha_1$	0.0357	0.0002	189.1099	0.0000
$\beta_1$	0.9296	0.0010	965.8108	0.0000
v	6.7199	0.0026	2598.1585	0.0000
ARCH-LM Test= 1.7003(0.1922) and Jarque-Bera Test = 8.1281(0.0172)				

Note: standard errors in parenthesis, p-values are in square brackets and the Q(.) is a Box-Ljung type of Portmanteau test.

Table 6: ExAR(2)-GARCH(1,1) Models Estimation and Diagnostic Analysis of Monthly North-West Wind Speed (MNWNWS)

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$c$	6.6489	0.4812	13.819	0.0000
$\lambda_1$	0.3750	1.6347	0.229	0.8190
$\lambda_2$	0.5000	2.3379	0.214	0.8310
$\phi_1$	24.3046	743.1532	0.033	0.9740
$\phi_2$	-23.7429	742.8946	-0.032	0.9750
GARCH(1,1) Components with $m_t$ assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0704	0.0003	265.8304	0.0000
$\alpha_1$	0.0455	0.0002	213.3960	0.0000
$\beta_1$	0.9322	0.0010	964.0272	0.0000
ARCH-LM Test=1.5656(0.2108) and Jarque-Bera Test = 8.6517(0.0132)				
GARCH(1,1) Components with $m_t$ assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.1246	0.0004	353.4190	0.0000
$\alpha_1$	0.0347	0.0002	186.2043	0.0000
$\beta_1$	0.9299	0.0010	962.5185	0.0000
$v$	6.3706	0.0025	2523.2105	0.0000
ARCH-LM Test=1.5656(0.2108) and Jarque-Bera Test = 8.6517(0.0132)				

Note: standard errors in parenthesis, p-values are in square brackets and the Q (.) is a Box-Ljung type of Portmanteau test.

The estimation of ExAR(1)-FIGARCH models for the wind speed series and serial correlation analysis results are displayed in Table 7-8

Table 7: ExAR(1)-FIGARCH(1,1) Models Estimation and Diagnostic Analysis of Monthly North-West Wind Speed (MNWNWS)

ExAR(1) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
c	6.3692	0.9168	6.947	0.0000
$\lambda_1$	0.1250	0.1031	1.212	0.2278
$\phi_1$	3.0929	0.8092	3.822	0.0002
FIGARCH(1,1) Components with $m_t$ assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0130	0.0001	113.9807	0.0000
$\alpha_1$	0.3255	0.0006	567.8667	0.0000
$\beta_1$	0.8060	0.0009	897.9479	0.0000
$d_v$	0.6348	0.0008	794.7497	0.0000
ARCH-LM Test= 0.6834(0.4084) and Jarque-Bera Test = 1.065(0.5871)				
FIGARCH(1,1) Components with $m_t$ assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0180	0.0001	133.9983	0.0000
$\alpha_1$	0.4476	0.0007	667.7989	0.0000
$\beta_1$	0.8123	0.0010	902.3381	0.0000
$d_v$	0.5852	0.0008	765.1760	0.0000
v	6.4414	0.0025	2534.7062	0.0000
ARCH-LM Test= 0.6777(0.4104) and Jarque-Bera Test = 1.2714(0.5296)				

Note: standard errors in parenthesis, p-values are in square brackets and the Q (.) is a Box-Ljung type of Portmanteau test.

Table 8 ExAR(2)-FIGARCH(1,1) Models Estimation and Diagnostic Analysis of Monthly North-West Wind Speed (MNWNWS)

ExAR(2) Components				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
c	6.6489	0.4812	13.819	0.0000
$\lambda_1$	0.3750	1.6347	0.229	0.8190
$\lambda_2$	0.5000	2.3379	0.214	0.8310
$\phi_1$	24.3046	743.1532	0.033	0.9740
$\phi_2$	-23.7429	742.8946	-0.032	0.9750
FIGARCH(1,1) Components with $m_t$ assumed to be normal distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0261	0.0002	161.7473	0.0000
$\alpha_1$	0.1458	0.0004	381.0774	0.0000
$\beta_1$	0.8339	0.0009	910.8783	0.0000
$d_v$	0.8684	0.0009	928.8897	0.0000
ARCH-LM Test=0.5061(0.4768) and Jarque-Bera Test = 2.0656(0.3560)				
FIGARCH(1,1) Components with $m_t$ assumed to be Student-t distribution				
Parameters	Estimate	Std. Errors	t-value	Pr(> t )
$\Omega$	0.0185	0.0001	135.4982	0.0000
$\alpha_1$	0.4525	0.0007	672.7026	0.0000
$\beta_1$	0.7870	0.0009	887.4412	0.0000
$d_v$	0.5598	0.0008	746.0056	0.0000
$v$	7.8478	0.0028	2796.2552	0.0000
ARCH-LM Test=0.5269(0.4679) and Jarque-Bera Test = 2.264(0.3224)				

Note: standard errors in parenthesis, p-values are in square brackets and the Q(.) is a Box-Ljung type of Portmanteau test.

All parameters of the hybrid models specifically the ExAR-FIGARCH that were assumed to be Student-t-distribution comes with smaller standard errors than parameters of the ExAR models. This indicates the goodness-of-fit of the hybrid models to the MNWNWS and adequacy of the hybrid

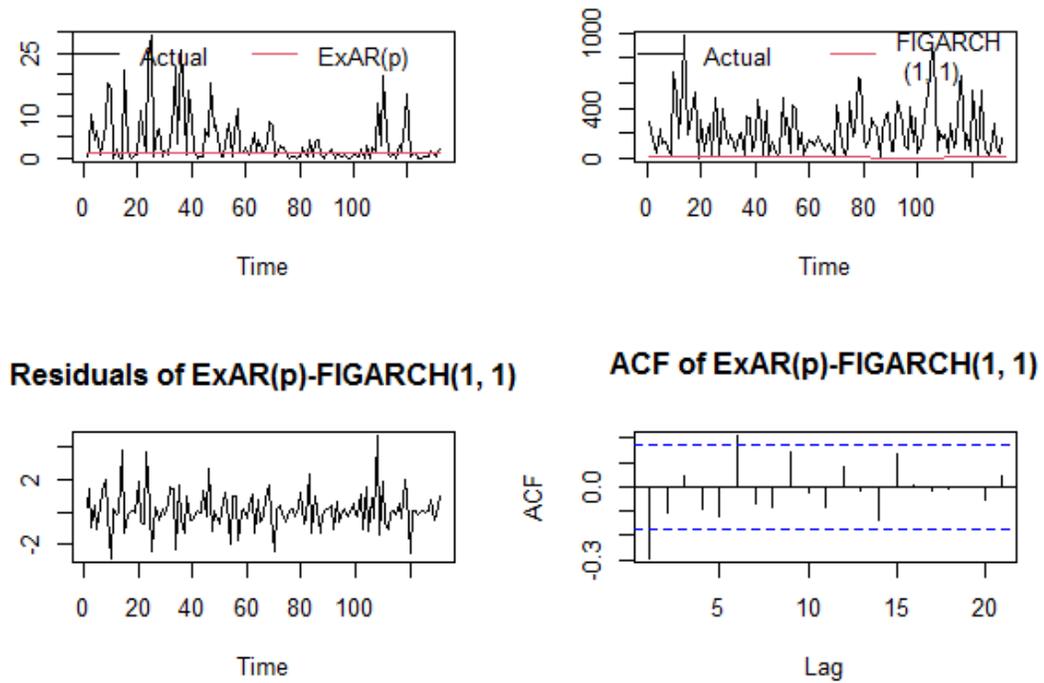
ExAR-FIGARCH models. Also, the p-values of the serial correlation analysis of the hybrid models are larger compare to the mean models, the ExAR. This results shows evidence of improvement in model fitting as a results of introducing the FIGARCH model to the ExAR models.

Table 9: Diagnostic Analysis of the Hybrid Models

Candidate Models	Residuals as Normal Distribution		Residuals as Student-t-Distribution	
	ARCH-LM Test	Jarque-Bera – Test	ARCH-LM Test	Jarque-Bera - Test
MNWNWS				
ExAR(1)-GARCH(1,1)	2.6256(0.6391)	7.9557(0.000)	2.7391(0.2691)	8.0482(0.0000)
ExAR(2)-GARCH(1,1)	2.4201(0.4872)	7.8201(0.000)	2.9682(0.1721)	7.1119(0.000)
ExAR(1)-FIGARCH(1,1)	2.3022(0.3391)	7.0363(0.000)	1.9429(0.4859)	9.1167(0.0000)
ExAR(2)-FIGARCH(1,1)	2.1042(0.3251)	7.9341(0.000)	1.1410(0.3785)	9.2431(0.000)

The ARCH-LM test which is a serial correlation analysis investigates the homoscedasticity of residuals of a time series models. The p-values of the ARCH-LM test of the hybrid models in Table 6 are larger than the p-values of the ExAR mean models as shown in Table 7 and 8. This results shows evidence of improvement in model fitting as a results of introducing the FIGARCH model to the ExAR models. Also, the residuals of the ExAR(1)-FIGARCH(1,1) is homocedastic and therefore the model could be consider in producing reliable forecasts. However, Results of the Jarque-Bera test show evidence of non-normality in all the mean and hybrid model residuals due to zero p-values of Jarque-Bera test statistics.

Figure 4: Diagnostic Plots of ExAR(1)-FIGARCH(1,1) Fitted to MNWNWS



The diagnostic plots and in-sample forecasts for the ExAR(1)-FIGARCH(1,1) are shown in Figure 4. The in sample forecast plot (right top panel) show that the actual volatility coincide with the fitted or observed volatilities for the ExAR(1)-FIGARCH(1,1) model.

The Mean Error (MSE), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are used in the literature to evaluate performance (see Hyndman and Athanasopolous(2013) and Papailias and Dias (2017)).

*Table 10: Forecast Accuracy Measures for Hybrid Models*

MNWNWS						
Candidate Models	Residuals as Normal Distribution			Residuals as Student-t-Distribution		
	MSE	RMSE	MAE	MSE	RMSE	MAE
ExAR(1)-GARCH(1,1)	7.0389	2.6531	1.4868	3.4612	1.8604	0.7708
ExAR(2)-GARCH(1,1)	9.5168	3.0849	1.3511	5.0389	2.2447	1.1911
ExAR(1)-FIGARCH(1,1)	4.2351	2.0579	0.7401	1.0368	1.0182	0.3714
ExAR(2)-FIGARCH(1,1)	5.1721	2.2742	0.9849	2.1113	1.4530	0.5731

The forecast accuracy measures results of MNWNWS using the hybrid ExAR-GARCH and ExAR-FIGARCH models are shown in Table 10 Compared to the candidates hybrid model, The ExAR-FIGARCH model produces a better forecast performace with minimum accuracy measures. The MSE, RMSE and Mean Absolute Error (MAE) are used in the literature to evaluate performance (see Hyndman and Athanasopolous(2013) and Papailias and Dias (2015)). The forecast accuracy measures results of MNWNWS using the hybrid ExAR-GARCH and ExAR-FIGARCH models are shown in Table 10. Compared to the hybrid ExAR- GARCH models, the ExAR-FIGARCH model produces the minimum and better forecast performances based on the assumptions that their residuals are Student-t-distributed. Again, the ExAR(1)-FIGARCH(1,1) for the MNWNWS produces accuracy measures; MSE, RMSE and MAE equal 1.0368, 1.0182 and 0.3714 respectively. In view

of this, the ExAR(1)- FIGARCH(1,1) is chosen as the best model. Consequently, the

chosen model which is equivalent to the hybrid ExpAR( $I$ )-FIGARCH( $I, d_v, I$ ) model in

eq.(13) has estimated parameters  $C = 6.3692, \phi_1 = 3.0929$

$\lambda_1 = -0.1250, \omega = 0.0391, \alpha_1 = 0.0, \beta_1 = 0.87$  and  $d_v = 0.87$  as shown in Table 7

.Therefore, the fitted model for MNWNWS is

$$Y_t = 6.3692 + (3.09 + 0.13e^{-6.37Y_{t-1}^2})Y_{t-1} + m_t [0.04 + \{(0.00 + 0.87(1-L)^{0.87}\} \varepsilon_t^2)^{\frac{1}{2}}] \quad (13)$$

#### 4.0 Summary and Conclusion

Accurately modelling wind speed is essential for optimizing wind energy production, forecasting extreme weather events and ensuring infrastructure resilience. The statistical properties of wind speed data such as skewness, kurtosis and normality tests, provide valuable insights that influence models selection. The ExAR-FIGARCH model, which accommodates long memory effects, volatility clustering, and asymmetric, is a powerful tool for wind speed forecasting. Future research should focus on real world application of ExAR-FIGARCH models, validating their performance across different regions and times scales.

#### 6.0 References

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