MODELLING RETURN UNPREDICTABILITY WITH THE ODD GENERALIZED EXPONENTIAL LAPLACE DISTRIBUTION: A SIMULATION STUDY

David, R. O.[1], Ishaq, A. I.[1], Nnamani, C. N.[1], Umar, A. A.[1], Zakari, Y.[1], and Obalowu, J.[2]
[1]Department of Statistics, Ahmadu Bello University, Zaria
[2]Department of Statistics, University of Ilorin, Ilorin.

Corresponding author: rodavid@abu.edu.ng ; +2348037557300

ABSTRACT

The accurate modelling of return unpredictability remains a pivotal challenge in financial econometrics. Traditional models often assume a normal distribution for error terms, which fails to capture the leptokurtic and skewed nature of financial returns. This paper introduces the odd generalized exponential Laplace distribution (OGELAD) as an error distribution tailored for modelling asset return unpredictability. The proposed distribution addresses the limitations of conventional error distributions such as normal (NORM), skew normal (SNORM), normal inverse Gaussian (NIG), and skew generalized error distribution (SGED) in capturing key characteristics of financial returns, such as asymmetry and heavy tails. Using simulated data, the study evaluates the performance of the OGELAD within symmetric and asymmetric volatility models, demonstrating its effectiveness in modelling and forecasting return volatility. Diagnostic tests confirm that all error distributions, including the OGELAD, successfully eliminate ARCH effects from residuals, ensuring robust model performance. Notably, the positive and significant asymmetry parameter in the selected model highlights that positive shocks exert a smaller influence on volatility compared to negative shocks of the same magnitude. This finding underscores the relevance of the proposed distribution in capturing leverage effects observed in financial data. The OGELAD distribution consistently outperformed existing distributions in modelling and forecasting volatility, showcasing its potential for broader applications. It can be

extended to multivariate settings for portfolio risk management and applied to high-frequency financial data to test its robustness under varying market conditions.

1. INTRODUCTION

One prominent feature of financial time series is that they are mostly characterized by sudden changes which often result in the unpredictability of asset returns. This sudden change otherwise referred to as volatility has significant impact on risk control, asset pricing, and portfolio optimization in the financial markets. Engle (1982) with the introduction of the Autoregressive Conditional Heteroscedastic (ARCH) model, pioneered the study of conditional heteroscedasticity of asset returns. Engle (1982) model expressed conditional variance of returns as a weighted average of previous innovations, making it suitable for describing volatility clustering. Notwithstanding the success of ARCH model, it has faced criticism due to various weaknesses including difficulties in parameter estimation and the assumption of equal effects for both negative and positive shocks on volatility, among others. This led to the development of other heteroscedastic models such as the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model introduced by Bollerslev (1986), the Exponential GARCH (EGARCH) model developed by Nelson (1991). The EGARCH is the first in the family of asymmetric GARCH models, enabling the measurement of asymmetric effects between negative and positive returns. Other volatility models include the Integrated GARCH (IGARCH) by Engle & Bollerslev (1986), the Nonlinear GARCH (NGARCH) model by Bollerslev (1986), the ARCH-in-Mean (ARCH-M) model by Engle, Lilien & Robins (1987), the Asymmetric Power ARCH (APARCH) by Ding, Granger & Engle (1993), the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model by Glosten, Jagannathan & Runkle (1993), the Threshold GARCH (TGARCH) by Zakoian (1994), the Quadratic GARCH (QGARCH) model by Sentana (1995), and recently Zero-Drift GARCH (ZD-GARCH) by Li, Zhang, Zhu & Ling (2018), among others.

The modelling of asset returns has evolved significantly with the advent of volatility models like ARCH and GARCH. These models initially assumed normality, which was later extended to accommodate distributions like Student's-t and GED to address leptokurtosis. Recent advancements have explored other skewed distributions, highlighting the need for more flexible error structures. Some of the empirical studies that have investigated the use of both normal and non-normal error distributions within the framework of GARCH models are hereby discussed. Hsieh (1989) used the heteroscedastic (ARCH and GARCH) models with some nonnormal innovations in modelling the exchange rate of five currencies (Canadian dollar, Swiss franc, Deutsche mark, Japanese yen, and British pound) and observed that the GARCH model with the selected nonnormal innovations explained a large part of the nonlinearities for the Canadian dollar, Swiss franc and the Deutsche mark currencies. Only one of the selected nonnormal innovations fit the Japanese currency while none fits the British pound. Hansen & Lunde (2005) compared multiple ARCH-type of models with Student's-t distributed error in modelling exchange rate. They found that GARCH (1,1) outperformed other sophisticated models in actual out-of-sample forecast. Atoi (2014) studied the volatility in the Nigeria stock market using GARCH models with normal, Student's-t and GED distributed error. Upon fitting, they found the specification of the volatility models with normal distributed error is inadequate. In particular, TGARCH (1,1)-GED is selected as the best model in fitting the volatility in the Nigeria stock market and PGARCH (1,1) with Student's-t error is chosen as the best forecasting model. Asemota & Ekejiuba (2017) conducted an analysis of equity return volatility in six banks using GARCH models. They identified that, for two of the banks displaying ARCH effects, the

EGARCH (1,1) and CGARCH (1,1) models with a Student's-t distribution outperformed other GARCH models. The study recommended the utilization of different GARCH model variants and alternative error distributions to enhance the robustness of results when modelling stock market volatility. In a separate study, Iwada, Omoyeni & Temitope (2018) fitted symmetric and asymmetric GARCH models with normal, Student's-t and GED distributed errors to daily stock prices of Access and Fidelity Banks in Nigeria. They found asymmetric GARCH models; PGARCH (1,1)-GED and EGARCH (1,1)-GED as the best fitted models for Access and Fidelity Banks stock returns respectively and PGARCH (1,1) with GED error is selected as the best out-of-sample forecasting model for the two returns. Gyamerah & Abaitey (2022) modelled the volatility of bitcoin using GARCH models with Student's-t distribution and normal inverse Gaussian distribution. Their result indicates that IGARCH (1,1) with Student's-t distribution among other models is selected as the best model prior to and during the financial crisis. Furthermore, the IGARCH (1,1) with NIG distributed error provided a better out-of-sample forecasts of volatility before and during the financial crisis.

Recently, Adenomon & Idowu (2023) studied the impact of the COVID-19 on some selected Nigeria sectorial stocks using GARCH models with structural breaks. In their analysis, two error distributions, Student's-t and skewed Student's-t distributions, were used. Their findings indicated that GARCH models with Student's-t innovations outperformed models utilizing skewed Student's-t innovations across most of the sectors. Despite these efforts, gaps remain in capturing the complex stochastic behaviour of financial returns. This paper introduces the Odd Generalized Exponential Laplace Distribution error distribution aimed at addressing these limitations by accurately modelling the unpredictability of asset returns.

2. METHODOLOGY

2.1 Proposed Error Distribution

David and Obalowu (2023) defined the Odd Generalized Exponential Laplace Distribution as:

$$g(y)=\frac{m\theta\exp\left(-\frac{|y-\lambda|}{\tau}\right)}{2\tau\left(1-\left(\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]\right)\right)^{2}}\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]\right)}\right)\right\}$$
$$\times\left[1-\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|y-\lambda|}{(y-\lambda)}\left[1-\exp\left(-\frac{|y-\lambda|}{\tau}\right)\right]\right)}\right)\right\}\right]^{\theta-1} \quad (1)$$

where $m, \theta, \tau > 0, -\infty < \lambda, y > \infty$. In (1), m and θ are shape parameters that control kurtosis, τ is a scale parameter that controls skewness and λ is a location parameter.

Following the procedure of Ghalanos (2022), let, $a_t = y-\lambda$, where λ is a zero-mean process so that:

$$f(a_t)=\frac{m\theta\exp\left(-\frac{|a_t|}{\tau}\right)}{2\tau\left(1-\left(\frac{1}{2}+\frac{1}{2}\frac{|a_t|}{(a_t)}\left[1-\exp\left(-\frac{|a_t|}{\tau}\right)\right]\right)\right)^{2}}\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|a_t|}{(a_t)}\left[1-\exp\left(-\frac{|a_t|}{\tau}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|a_t|}{(a_t)}\left[1-\exp\left(-\frac{|a_t|}{\tau}\right)\right]\right)}\right)\right\}$$
$$\times\left[1-\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|a_t|}{(a_t)}\left[1-\exp\left(-\frac{|a_t|}{\tau}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|a_t|}{(a_t)}\left[1-\exp\left(-\frac{|a_t|}{\tau}\right)\right]\right)}\right)\right\}\right]^{\theta-1} \quad (2)$$

The innovation (error term) follows a GARCH model if it is specified in the form,

$$\varepsilon_t = a_t\tau_t \quad (3)$$

where a_t in its standard form, represents an independent and identically distributed process with a mean of zero and a variance of one, τ_t is the conditional standard deviation. From (3), $a_t = \frac{\varepsilon_t}{\tau_t}$ is standardized residual series and the density is given as:

$$g(\varepsilon_t) = \frac{1}{\tau_t} f(a_t) \tag{4}$$

Thus,

$$f(\varepsilon_t) = \frac{m\theta \exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)}{2\tau_t^2\left(1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)\right)^2} \exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)}\right)\right\}$$
$$\times\left[1-\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)}\right)\right\}\right]^{\theta-1} \tag{5}$$

is the error distribution of the OGELAD.

2.2 Model Specification

The proposed error distribution is integrated into a GARCH model framework with the following equations:

$$r_t = \lambda + \varepsilon_t \tag{6}$$

$$\varepsilon_t = a_t \tau_t \tag{7}$$

GARCH (1,1): $\tau_t^2 = \kappa + a_1\varepsilon_{t-1}^2 + b_1\tau_{t-1}^2$ (8)

EGARCH (1,1): $\log_e\left(\tau_t^2\right) = \kappa + a_1\varepsilon_{t-1} + \gamma_1\left(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|\right) + b_1\log_e\left(\tau_{t-1}^2\right)$ (9)

TGARCH (1,1): $\tau_t = \kappa + a_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right) + b_1\tau_{t-1}$ (10)

GJR-GARCH (1,1): $\tau_t^2 = \kappa + a_1\left(\left|\varepsilon_{t-1}\right| - \gamma_1\varepsilon_{t-1}\right)^2 + b_1\tau_{t-1}^2$ (11)

where a_t in Equation (7) follows the proposed error distribution. Equation (6) is the mean equation of the variance equations given in Equations (8) – (11). In Equations (8), (10), and (11), the parameters κ, a_1 and b_1 are non-negative. Furthermore, γ_1 represents the coefficient explaining the leverage or asymmetric effect.

3.2 Parameter Estimation

Parameters are estimated using the Maximum Likelihood Estimation (MLE) method. The likelihood function for observed realizations from a given distribution is

$$L(x;\theta) = f_X(x_1;\theta) \times f_X(x_2;\theta) \times f_X(x_3;\theta) \times \ldots \times f_X(x_n;\theta) = \prod_{i=1}^{n} f\left(x_i;\boldsymbol{\theta}\right) \tag{12}$$

where θ represents the set of distribution parameters.

The likelihood function of the error distribution of OGELAD is:

$$L(\varepsilon_t;\theta) = \prod_{t=1}^{n} \frac{m\theta \exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)}{2\tau_t^2\left(1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)\right)^2} \exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)}\right)\right\}$$
$$\times\left[1-\exp\left\{-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]}{1-\left(\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left[1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right]\right)}\right)\right\}\right]^{\theta-1} \tag{13}$$

and the log likelihood function is given as:

$$
\begin{aligned}
\ln(L) = {} & n\left[\ln m + \ln\theta - \ln\left(\tau_t^2\right) - \ln 2\right] - \sum_{t=1}^{n}\frac{|\varepsilon_t|}{\tau_t^2} - 2\sum_{t=1}^{n}\ln\left[\frac{1}{2}-\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left(1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right)\right] \\
& - m\sum_{t=1}^{n}\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left(1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right)}{\frac{1}{2}-\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left(1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right)}\right) + (\theta-1)\sum_{t=1}^{n}\left(\ln\left[\left[\left(1-\exp\left(-m\left(\frac{\frac{1}{2}+\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left(1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right)}{\frac{1}{2}-\frac{1}{2}\frac{|\varepsilon_t|}{\varepsilon_t}\left(1-\exp\left(-\frac{|\varepsilon_t|}{\tau_t^2}\right)\right)}\right)\right)\right)\right]\right]\right)
\end{aligned} \tag{14}
$$

In Equation (14), τ_t^2 is the variance of the volatility equations given in (8) – (11). The log-likelihood function for GARCH processes is maximized with respect to the parameters of the model. The parameter values obtained from this optimization represent the Quasi Maximum-Likelihood Estimator (QMLE) for the GARCH process parameters (Zivot, 2009). Several optimization methods have been proposed in the literature to address the complexity of the conditional variance which exhibit non-linear structure (Hill & McCullough, 2019). Key examples include the Nelder-Mead method (Nelder & Mead, 1965), the Berndt-Hall-Hall-Hausman (BHHH) algorithm (Berndt, Hall, Hall, & Hausman, 1974), and the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm, introduced independently by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970). These algorithms, widely implemented in statistical software like *R*, are designed to maximize the likelihood function, which quantifies how well the model fits the observed data.

3. RESULTS AND DISCUSSION

3.1 Simulation Study Setup

A simulation is conducted to compare the proposed distribution against normal, skew normal and skew GED. The robustness of the estimated GARCH models with specified error distributions have been evaluated using data simulated from a known error distribution. In this study, the skewed Student's-t error distribution has been used. The simulated data procedures are:

i. assume good parameters values for the student's-t error distribution;

ii. simulate the data using the maximum likelihood estimation;

iii. fit the different GARCH models under selected error distributions;

iv. compare the parameter estimates for the fitted models;

vi. examine the adequacy of the fitted models using LM test for heteroscedasticity;

vii. make forecast using fitted models.

A total of 8,000 returns are generated following the outlined procedures. To address potential starting errors, the most recent 4,000 observations are selected for further analysis. Of these, 3,800 observations are used for model fitting, while the remaining 200 observations are set aside to evaluate the model's performance.

3.2 Results and Discussion

Table 1 presents the parameter estimates of specified GARCH models under various error distributions, including the normal, skewed normal, skewed generalized error, and odd generalized exponential Laplace distributions. In the GARCH (1,1) model, the ARCH term (a_1) is estimated to be zero across all error distributions, indicating no significant short-term volatility effects. Furthermore, the skew parameter of the OGELAD error distribution is also estimated to be zero, suggesting symmetry.

For the EGARCH (1,1) model, no parameter values could produce a convergent result for any of the error distributions. This suggests potential convergence issues or a lack of suitable parameter estimates for the EGARCH (1,1) model, highlighting the importance of carefully considering the model selection and potential limitations when analyzing the data.

In contrast, the TGARCH (1,1) model provides statistically significant estimates for both the ARCH term (a_1) and the GARCH term (b_1) across all error distributions, suggesting robust modelling of short- and long-term volatility components. Additionally, the asymmetric term (γ_1) in the OGELAD error distribution is statistically significant at all levels. This suggests the presence of leverage effects and that the TGARCH (1,1) model with the OGELAD error distribution successfully captures the asymmetric effects in the simulated returns.

Similarly, the GJR-GARCH (1,1) model shows statistical significance for most parameters across all error distributions. However, the ARCH term (a_1) is consistently estimated as zero, suggesting that the lagged squared residuals do not contribute significantly to the volatility dynamics in the GJR-GARCH (1,1) model. The asymmetric parameter (γ_1) is negative and statistically significant for all error distributions except the OGELAD error distribution, where it is not significant. This indicates that while the GJR-GARCH model effectively captures asymmetry in most cases – where negative shocks have a larger impact on volatility compared to positive shocks of the same magnitude, it may not align well with the OGELAD distribution for modelling leverage effects.

Table 1: Estimation of GARCH models for simulated returns

Model	Error Distribution	Estimates					
		κ	a_1	b_1	γ_1	Skew	Shape
GARCH (1,1)	NORM	0.112630^{*}	0.000000	0.995550^{***}			
	SNORM	0.111180^{*}	0.000000	0.995600^{***}		0.981700^{***}	
	SGED	0.109800^{*}	0.000000	0.995660^{***}		0.982190^{***}	1.975460^{***}
	NIG	0.123279^{*}	0.000000	0.995135^{***}		-0.068376	24.999810^{*}
	OGELAD	0.120000^{***}	0.000000	0.980000^{***}		0.000000	0.000020^{***}
EGARCH (1,1)	NORM	NO CONVERGENCE					
	SNORM						
	SGED						
	NIG						
	OGELAD						
TGARCH (1,1)	NORM	0.460734^{*}	0.013629^{*}	0.897723^{***}	0.999999		
	SNORM	0.460931^{*}	0.013715^{*}	0.897614^{***}	1.000000	0.981084^{***}	
	SGED	0.461176^{*}	0.013711^{*}	0.897569^{***}	1.000000	0.981294^{***}	1.988835^{***}

	NIG	0.466553*	0.013847*	0.896488***	1.000000	-0.069850	24.999999*
	OGELAD	0.480000***	0.01100***	0.900000***	0.980000***	0.0000000	0.000030***
GJR-GARCH (1,1)	NORM	0.114699**	0.000000	0.998126***	-0.005258***		
	SNORM	0.113802**	0.000000	0.998129***	-0.005193***	0.983822***	
	SGED	0.114726**	0.000000	0.998113***	-0.005235***	0.984320***	1.975334***
	NIG	0.058776***	0.000000	1.000000***	-0.004601***	-0.059078***	24.999935***
	OGELAD	0.110000***	0.000000	0.990000***	-0.005200	0.000000	0.000020***

Note: Estimated parameters are significant at: 5% level ‘*’, 1% level ‘**’, and 0.1% level ‘***’

Following the fitting of various models, the standardized residuals and squared standardized residuals are examined for the presence of ARCH effects. Result shows that there is no ARCH effects left in the standardized residuals and squared standardized residuals of the fitted models for simulated returns. This suggests that the models adequately capture and account for the volatility dynamics present in the data.

The model selection for the simulated returns have been assessed using the log likelihood and some information criteria. Result from Table 2 shows the GARCH models with OGELAD error distribution have superior performance to other error distributions. Specifically, the GJR-GARCH (1,1) model with the OGELAD error distribution is selected as the best fitting model based on the maximum log likelihood and the lowest values for AIC, BIC, and HQIC. This suggests that the GJR-GARCH (1,1) model with the OGELAD error distribution provides the best fit to the volatility patterns in the simulated returns among the considered models and error distributions.

Table 2: Selecting a volatility model for simulated returns

Model	Error Distribution	Log likelihood	AIC	BIC	HQIC
GARCH (1,1)	NORM	-11540.170	7.2145	7.2202	7.2165
	SNORM	-11539.870	7.2149	7.2225	7.2176
	SGED	-11539.820	7.2155	7.2250	7.2189
	NIG	-11540.290	7.2159	7.2254	7.2193
	OGELAD	**52062.450**	**-32.5340**	**-32.5189**	**-32.5286**
TGARCH (1,1)	NORM	-11535.780	7.2117	7.2174	7.2138
	SNORM	-11535.460	7.2122	7.2198	7.2149
	SGED	-11535.450	7.2128	7.2223	7.2162

	NIG	-11536.210	7.2133	7.2227	7.2167
	OGELAD	**43856.440**	**-27.4053**	**-27.3901**	**-27.3998**
GJR-GARCH (1,1)	NORM	-11537.330	7.2127	7.2184	7.2147
	SNORM	-11537.100	7.2132	7.2208	7.2159
	SGED	-11537.040	7.2138	7.2233	7.2172
	NIG	-11537.360	7.2142	7.2236	7.2176
	OGELAD	**52867.200**	**-33.0370**	**-33.0218**	**-33.0316**

Table 3 provides an evaluation of the predictive accuracy of the specified models. Notably, the OGELAD error innovation demonstrates superior forecast accuracy compared to the other error distributions for both the GARCH (1,1) and GJR-GARCH (1,1) models. Overall, the adequacy measures consistently indicate that the GARCH (1,1) model with the OGELAD error innovation outperforms other error innovations in actual out-of-sample forecast. This suggests that the GARCH (1,1) model with the OGELAD error distribution is well-suited for predicting the future volatility dynamics of the simulated returns.

Table 3: Forecasting accuracy of volatility models for simulated returns

Model	Error Distribution	MAE	RMSE
GARCH (1,1)	NORM	2.4142	2.9427
	SNORM	2.4137	2.9423
	SGED	2.4237	2.9423
	NIG	2.4161	2.9443
	OGELAD	**2.0595**	**2.7161**
TGARCH (1,1)	NORM	2.4211	2.9478
	SNORM	2.4211	2.9478
	SGED	2.4207	2.9475
	NIG	2.4221	2.9486
	OGELAD	2.3129	3.1309
GJR-GARCH (1,1)	NORM	2.4009	2.9322
	SNORM	2.4020	2.9332
	SGED	2.4022	2.9333
	NIG	2.3723	2.9094
	OGELAD	2.0742	2.7590

The proposed distribution bridges the gap between traditional and modern approaches to modeling asset return unpredictability. Its flexibility in capturing higher moments enhances risk assessment and derivative pricing applications.

6. Conclusion

This study considers the error distribution for the odd generalized exponential Laplace distribution tailored for asset return modelling, addressing limitations of conventional distributions. The proposed distribution bridges the gap between traditional and modern approaches to modelling asset return unpredictability. All error distributions effectively eliminated any traces of ARCH effects in the residuals of the specified volatility models, as confirmed on the simulated data. Furthermore, the positive and significant asymmetry parameter in the selected model indicated that positive shocks have lower influence on volatility as opposed to negative shocks of same size. The symmetric and asymmetric models proved successful in capturing the volatility patterns of the simulated data. The proposed error distribution displayed clear advantages over existing distributions in the context of modelling and forecasting volatility. Thus, it can be extended to multivariate models for portfolio risk management. Likewise, the distribution can be applied to high-frequency financial data to assess robustness under different market conditions.

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