DEVELOPMENT OF LOGIT SKEWED EXPONENTIAL POWER (LSEP) DISTRIBUTION FOR MODELLING INFLATION RATE

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ABSTRACT

This study focuses on the development and application of the Logit Skewed Exponential Power (LSEP) distribution to model rates and proportion. The data considered were the Nigeria's inflation rate from 2008 to 2024. The study employs a systematic methodology that involves parameter estimation, graphical analysis using histograms, P-P plots, Q-Q plots, and model adequacy evaluation through information criteria such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The estimated parameters for the LSEP model—Alpha (0.3001), Beta (1.2915), Miu (-0.7066), and Sigma (0.0197)—demonstrated its ability to address the skewness and scale of the inflation rate distribution. The Beta distribution's parameters—Alpha (1.367) and Beta (6.678)—revealed a simpler structure that was less capable of capturing the data's complexity. The evaluation of model adequacy through information criteria showed that the LSEP model had a lower log-likelihood value (66.26637) and lower AIC (-124.5327) and BIC (-115.9602) compared to the Beta distribution log-likelihood value (52.8371) and higher AIC (-101.6742) and BIC (-97.38794). The LSEP model's parameter estimates indicate its robustness in addressing the complexities of the inflation rate, with lower AIC and BIC values compared to the Beta distribution.

1.0 Introduction

Traditional models for inflation rate forecasting, such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), have been widely used in economic analysis (Box & Jenkins, 1970). While these models provide useful insights, they often fall short in capturing the complex, nonlinear, and asymmetric nature of inflation data. Inflation rates can exhibit skewness and kurtosis that these traditional models may not adequately address. Traditional inflation models, such as those based on the Consumer Price Index (CPI) and Producer Price Index (PPI), are widely used but may not align well with empirical inflation data due to their underlying assumptions (Fuhrer, 2010). Recent studies have highlighted the inadequacy of standard models in capturing the skewness and kurtosis observed in inflation rates, suggesting a shift towards more flexible and robust statistical distributions (Giacomini, 2015).
The SEP distribution is known for its flexibility in modelling data with varying degrees of skewness and kurtosis (Clark & Baccar, 2018). By incorporating the logit transformation, the Logit SEP (LSEP) distribution can effectively handle bounded data, which is particularly useful for inflation rates that are typically confined within a specific range. The logit transformation maps the bounded inflation rates onto an unbounded scale, allowing for more efficient modelling and analysis. This transformation is essential because it addresses the constraints of traditional models that assume normally distributed errors and linear relationships (Clark & Baccar, 2018).
The Skewed Exponential Power distribution itself is a generalization of the Exponential Power distribution, allowing for asymmetry in the data (Bernardi et al., 2018). This characteristic

makes it suitable for modelling financial and economic data, which often exhibit skewed distributions. The flexibility of the SEP distribution lies in its ability to adjust for different shapes and tails of the distribution, providing a more accurate representation of real-world data compared to symmetric distributions like the normal distribution (Weldensea, 2019).

Consequently, there is a growing interest in applying advanced statistical distributions and methods that can more accurately represent the underlying dynamics of inflation rates (Engle et al., 2022). The limitations of traditional models underscore the need for more advanced statistical approaches that can handle the complexities of inflation data. The normal distribution, for example, assumes symmetry and may not adequately represent the skewness observed in inflation data (Martínez-Flórez et al. 2021). Similarly, the log-normal distribution, commonly used to model positive-valued variables, may not capture the extreme values or heavy tails sometimes observed in inflation rates (Wilson et al., 2022). These limitations highlight the need for statistical distributions that can better capture the characteristics of inflation data.

The application of the LSEP distribution to inflation rate modelling involves several steps. First, the mathematical formulation of the LSEP distribution needs to be developed, incorporating the logit transformation and the skewness parameters. This formulation should be theoretically sound and capable of capturing the key characteristics of inflation data. Next, the LSEP model needs to be applied to actual inflation rate data, requiring careful data collection and preprocessing. The data preprocessing steps may include handling missing values, normalizing the data, and ensuring that the inflation rates are appropriately bounded for the logit transformation (Khashimova & Buranova, 2018). Parameter estimation is a critical aspect of developing the LSEP model. Techniques such as Maximum Likelihood Estimation (MLE) or Bayesian methods can be employed to estimate the parameters of the LSEP distribution. These methods provide robust estimates that are essential for accurate modelling and forecasting (Pérez-Dattari & Kober, 2023). Interpreting the estimated parameters in the context of inflation rates is also important, as it provides insights into the underlying dynamics and potential policy implications (Khashimova & Buranova, 2018). Evaluating the adequacy of the LSEP model involves comparing it with traditional models using information criteria such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Deviance Information Criterion (DIC) (Thomas et al., 2022, Khashimova & Buranova, 2018). These criteria provide a quantitative measure of model fit and complexity, allowing for an objective assessment of the LSEP model's performance. A lower value of these criteria indicates a better fit, balancing model complexity and goodness of fit. Diagnostic tests and validation techniques are also employed to ensure the model's robustness and reliability (Thomas et al., 2022).

The development and application of the LSEP model for inflation rate modelling have significant implications for economic policy and decision-making. Accurate inflation forecasting can help central banks and policymakers implement timely and effective measures to control inflation, stabilize the economy, and achieve sustainable economic growth. For instance, if the LSEP model indicates an upcoming increase in inflation, policymakers can pre-emptively adjust interest rates or implement other monetary policies to mitigate the impact (Khashimova & Buranova, 2018). Furthermore, the LSEP model can provide valuable insights into the factors driving inflation and their relative importance. By analysing the estimated parameters, policymakers can identify the key determinants of inflation and design targeted interventions to address them.

2.0 Methodology

2.1 Source of data

The data is a secondary datasets. The datasets is on Nigeria inflation rates from 2008-2024, particularly for Nigeria, is sourced from the world bank, This data is crucial for understanding economic trends and making informed decisions in areas such as monetary policy, investment, and international trade. The in action rate dataset is gotten at https://www.macrotrends.nets/countries/NGA/nigeria/in action-rate-cpi.

2.2 Development of Model

The aim is to introduce Logit-SEP(θ) distribution, let x be an independent and identically observations from the logit - SEP(θ) distribution, where $\theta = (\sigma, \alpha, \beta, \infty)$.
The motivation for our parameterization of the skewed exponential power distribution follows from a version of the definition of the a quantile for the random variable X given by the value of θ satisfying

$$f(x) = \frac{k}{\sigma} \exp\left(-\frac{1}{2}(|w| + (2\alpha - 1)w)^{2l(1+\beta)}\right) \quad (1)$$

Where:

$$k^{-1} = \Gamma\left(1 + \tfrac{1+\beta}{2}\right) 2^{1+\frac{1}{2}(1+\beta)} l(4\alpha(1-\alpha)),\ 0 < \alpha < 1 -1 < \beta \leq 1, -\infty < \theta < \infty,$$

$w = [logit(x) - \theta)]/\sigma$ and $\Gamma(.)$ is the Euler gamma function. In terms of this parameterization θ serves as the location parameter and σ serves as the scale parameter. As we will see below α and β are shape parameters.
The distribution function is then given as

$$F(x) = \begin{cases} \frac{\alpha\Gamma\left(\frac{\beta+1}{2}\right)}{2^{\frac{\beta+1}{2}}} w^{\frac{2}{\beta+1}} - \frac{1}{\alpha-1} w^{\frac{2}{\beta+1}} \Gamma\left(\frac{\beta+1}{2}\right), & \text{if } w \leq 0, \\ 1 - \frac{1-\alpha\Gamma\left(\frac{\beta+1}{2}\right)}{2^{\frac{\beta+1}{2}}} \frac{1}{\alpha z^{\frac{2}{\beta+1}} - 2^{\frac{\beta+1}{2}} \Gamma\left(\frac{\beta+1}{2}\right)}, & \text{if } w > 0, \end{cases} \quad (2)$$

where $-1 < \beta \leq 1, -\infty < \theta < \infty, \sigma > 0$, $w = [logit(x) - \theta]/\sigma$, $\Gamma(,)$ is the incomplete gamma function and $\Gamma()$ is the complete gamma function. We can write the cumulative distribution function (c.d.f.) more compactly as

$$F(x) = \begin{cases} \alpha P\left(\frac{\beta+1}{2}\right) \cdot 2^{\frac{2}{\beta+1}} w^{\frac{2}{\beta+1}} - \frac{1}{\alpha-1} w^{\frac{2}{\beta+1}}, & \text{if } w \leq 0, \\ 1 - \frac{1-\alpha P\left(\frac{\beta+1}{2}\right) \cdot 2^{\frac{2}{\beta+1}}}{2^{\frac{2}{\beta+1}}} \frac{1}{\alpha w^{\frac{2}{\beta+1}} - 2^{\frac{\beta+1}{2}}}, & \text{if } w > 0, \end{cases} \quad (3)$$

where $P(a,w) = \Gamma(a,w)/\Gamma(a)$ is the regularized gamma function or the survival function of a standard gamma distribution. Again we have $-1 < \beta \leq 1, -\infty < \theta < \infty, \sigma > 0$ and $w = [logit(x) - \theta]/\sigma$. Inverting the c.d.f. at yields the corresponding quantile function. The plot of the pdf as shown in figure 1, 2, 3, shows that the shape of the pdf could be left skewed, platykurtic, leptokurtic also right skewed, platykurtic and leptokurtic. These properties makes it possible to model non-normal data sets.

$$F(x)=\begin{cases}\dfrac{\alpha\Gamma\left(\frac{\beta+1}{2},2^{\frac{2}{\beta+1}-1}((\alpha-1)w)^{\frac{2}{\beta+1}}\right)}{\Gamma\left(\frac{\beta+1}{2}\right)}, & \text{if } w\le 0\\[2ex] 1-\dfrac{(1-\alpha)\Gamma\left(\frac{\beta+1}{2},2^{\frac{2}{\beta+1}-1}\left(\frac{1}{\alpha w}\right)^{-\frac{2}{\beta+1}}\right)}{\Gamma\left(\frac{\beta+1}{2}\right)} & \text{if } w>0\end{cases} \tag{4}$$

$$F(x)=\begin{cases}\alpha P\left(\frac{\beta+1}{2},2^{\frac{2}{\beta+1}-1}((\alpha-1)w)^{\frac{2}{\beta+1}}\right), & \text{if } w\le 0\\[2ex] 1-(1-\alpha)P\left(\frac{\beta+1}{2},2^{\frac{2}{\beta+1}-1}\left(\frac{1}{\alpha w}\right)^{-\frac{2}{\beta+1}}\right) & \text{if } w>0\end{cases} \tag{5}$$

$$Q(u)=\begin{cases}\theta-\sigma\dfrac{\left(2^{\left(\frac{1}{2}-\frac{1}{\beta+1}\right)}\sqrt{P^{-1}\left(\frac{\beta+1}{2},\frac{u}{\alpha}\right)}\right)^{\beta+1}}{1-\alpha}, & \text{if } u\le\alpha\\[3ex] \theta+\sigma\dfrac{\left(\dfrac{2^{\left(\frac{1}{\beta+1}-\frac{1}{2}\right)}}{\sqrt{P^{-1}\left(\frac{\beta+1}{2},\frac{1-u}{1-\alpha}\right)}}\right)^{-(\beta+1)}}{\alpha}, & \text{if } u>\alpha\end{cases} \tag{6}$$

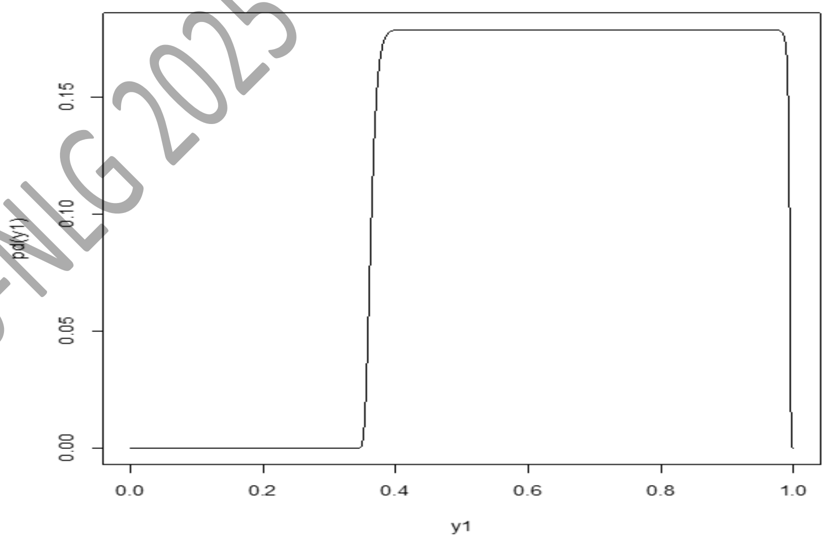

Figure 1: The shape of the pdf left tailed and platykurtic

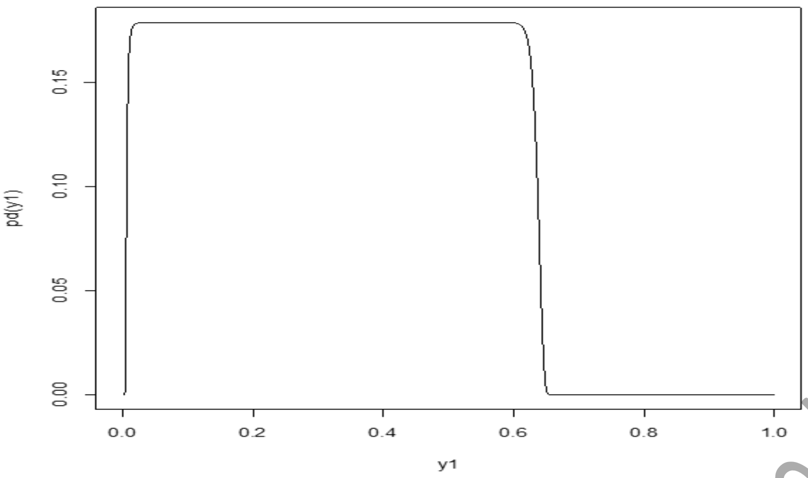

Figure 2: The shape of the pdf right tailed and platykurtic

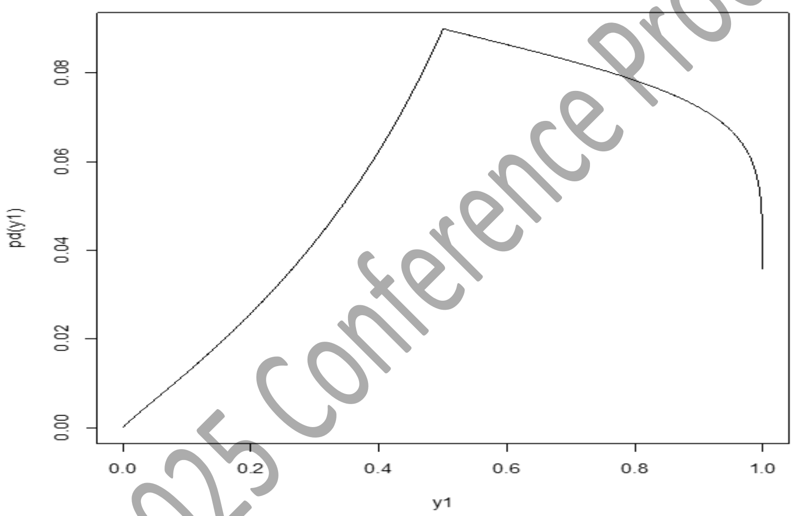

Figure 3: The shape of the pdf left tailed and leptokurtic

2.3 Maximum Likelihood

Maximum likelihood estimation (MLE) is a statistical estimation that allows us to use a sample to estimate the parameters of the probability distribution that generated the sample. MLE nds the parameter values that makes the observed data the most probable under the assumed model.

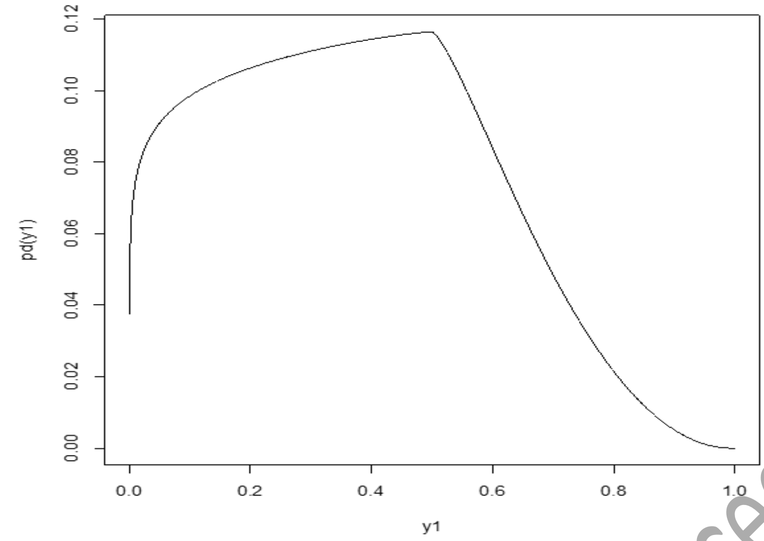

Figure 4: The shape of the pdf right tailed and leptokurtic

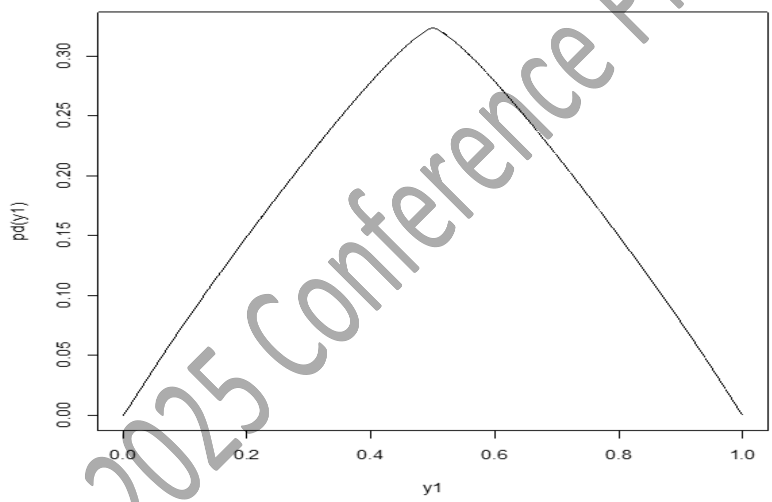

Figure 5: The shape of the pdf can be leptokurtic

2.4 Quantile function

A Quantile function is known as the Cumulative Distribution Function, its is the mathematical function that map a probability to the value in the such that the probabiliy of obseving a value less than or equal to that value is equal to that given probability, it gives you the value corresponding to a given probability in a distribution. The quantile function of Logit-SEP distribution is given in equation (6).

3.0 Result and Discussion

This chapter presents the modeling and analysis of Nigeria's inflation rate from 2008 to 2024, using the Logit Skewed Exponential Power (LSEP) and Beta distributions. The results compare these models in terms of fit and effectiveness in capturing inflation dynamics.

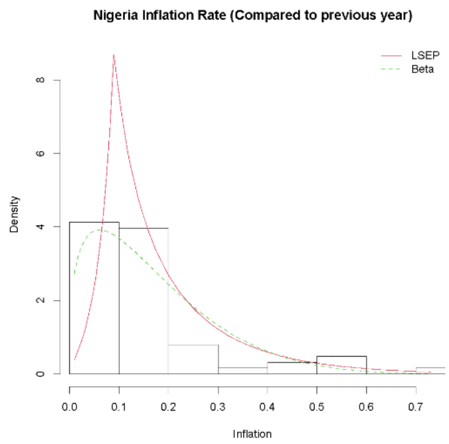

Figure 6: Nigeria Inflation Rate from 2008-2024 (compared to previous year)

The chart in Figure 6 illustrates the distribution of Nigeria's inflation rate compared to the previous year. It combines a histogram with density curves representing the LSEP and Beta distributions. The histogram shows a concentration of inflation rates between 0 and 0.2, with a few higher values. The LSEP curve, in red, closely follows the histogram's shape, suggesting it might be a suitable fit for the data. The Beta curve, in green, is a poorer fit, deviating from the histogram's pattern, especially in the tails. In essence, the chart provides a visual representation of inflation rate distribution in Nigeria, comparing it to theoretical distributions (LSEP and Beta) to assess potential fitting models.

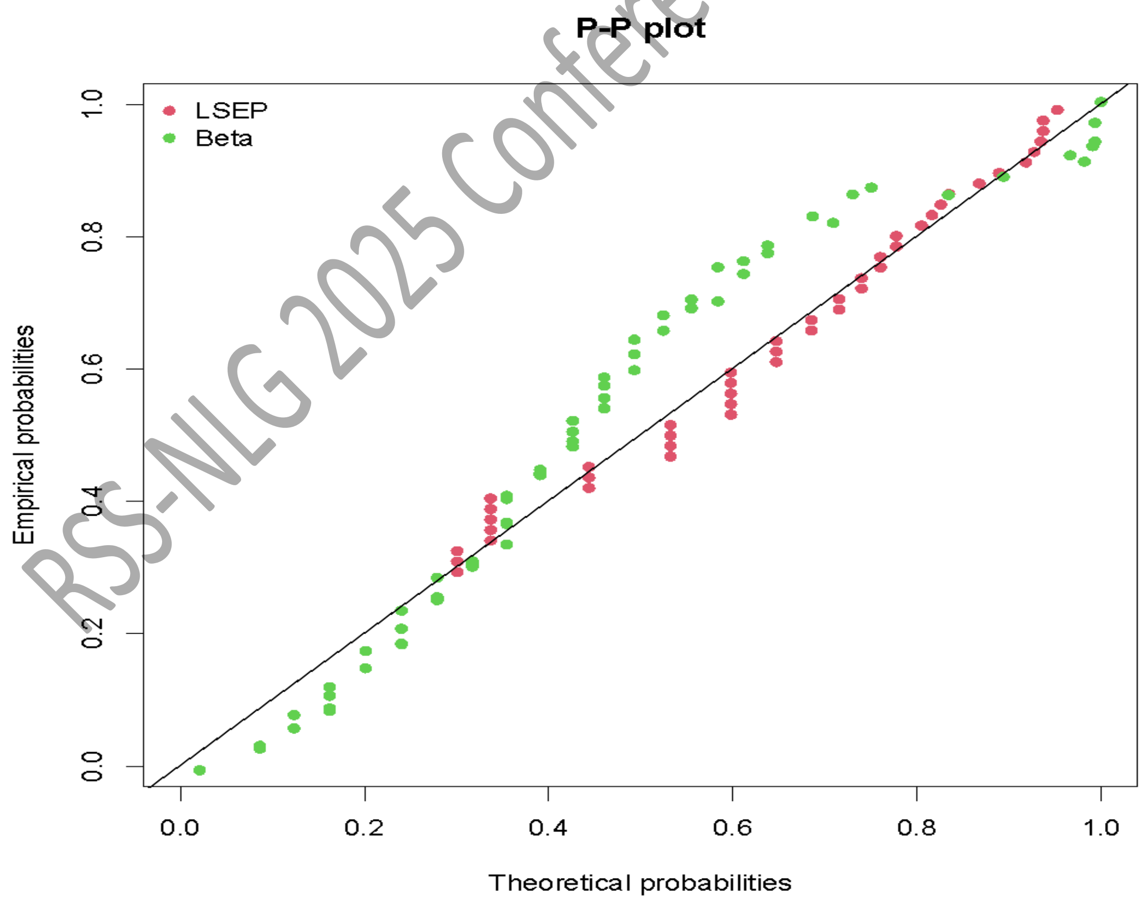

Figure 7: P-P Plot

The P-P plot in figure 7 above compares the cumulative distribution function (CDF) of the observed data (empirical probabilities) to the CDF of a theoretical distribution (in this case, LSEP and Beta).
For LSEP: The points largely follow the diagonal line, indicating a reasonable fit between the observed data and the LSEP distribution. However, there are some deviations, particularly in the tails, suggesting that the LSEP might not perfectly capture the extreme values in the data.
For Beta: The points deviate significantly from the diagonal line, especially in the central part of the plot. This indicates a poor fit between the observed data and the Beta distribution. The Beta distribution is not suitable for modeling the data.
Overall, the LSEP distribution appears to be a better fit for the data compared to the Beta distribution.

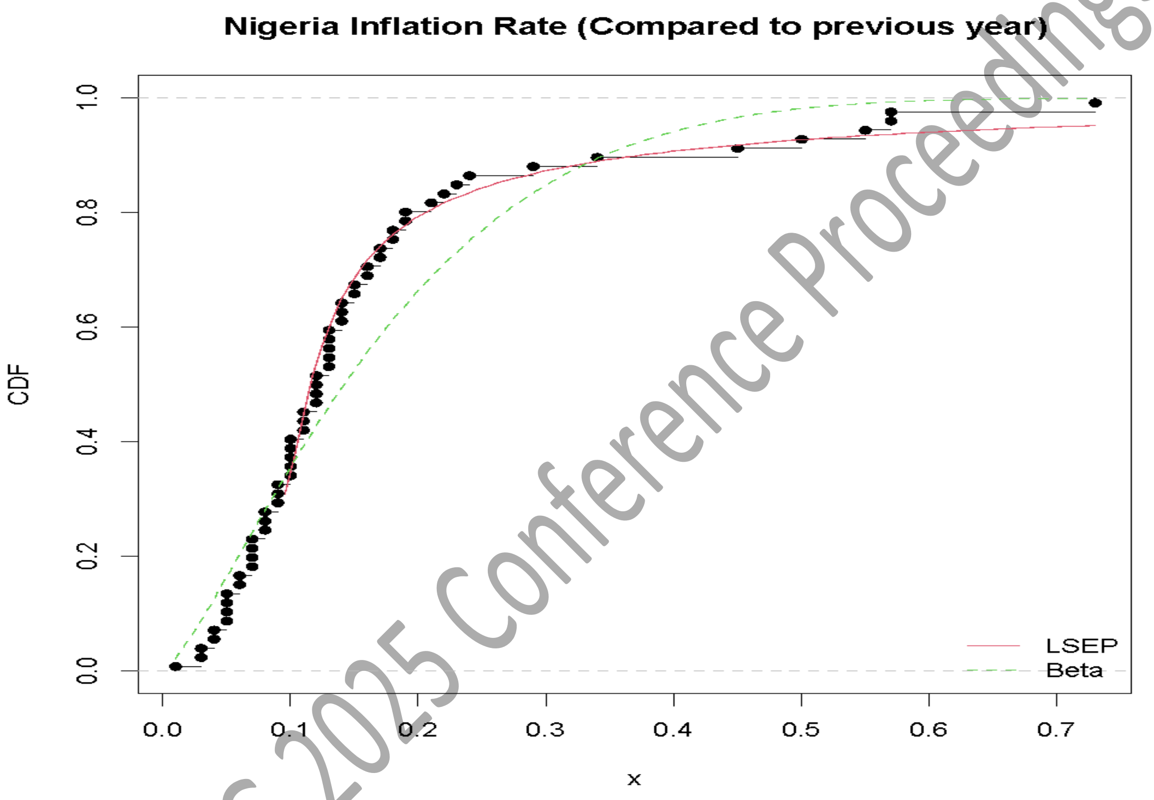

Figure 8: Nigeria Inflation Rate from 2008-2024 (compared to previous year)

The Q-Q plot in Figure 8 compares the empirical cumulative distribution function (ECDF) of the inflation data to the theoretical cumulative distribution functions of the LSEP and Beta distributions.
For LSEP: The points largely follow the diagonal line, indicating a reasonable fit between the observed data and the LSEP distribution. However, there are some deviations, particularly in the tails, suggesting that the LSEP might not perfectly capture the extreme values in the data.
For Beta: The points deviate significantly from the diagonal line, especially in the central part of the plot. This indicates a poor fit between the observed data and the Beta distribution. The Beta distribution is not suitable for modeling the data.
Overall, the LSEP distribution appears to be a better fit for the data compared to the Beta distribution.

3.2 Parameter Estimation

The Logit Skewed Exponential Power (LSEP) model parameters were estimated (Table 1), yielding the following values: Alpha = 0.3001 with a standard error of 0.0452, Beta = 1.2915 with a standard error of 0.481, Miu = -0.7066 with a standard error of 0.0105, and Sigma = 0.0197 with a standard error of 0.0035. These parameter estimates highlight the model's ability to capture the distribution's skewness and scale effectively. The log-likelihood value of 66.26637 suggests a good fit of the model to the data. Additionally, the model's adequacy was evaluated using information criteria: the Akaike Information Criterion (AIC) was -124.5327, and the Bayesian Information Criterion (BIC) was -115.9602. Both criteria indicate that the model provides a strong fit to the inflation rate data, favoring the LSEP model over more traditional approaches. These results underscore the LSEP model's effectiveness in capturing the characteristics of inflation data, particularly its ability to handle asymmetry and heavy tails.

Table 1: Logit-SEP Parameter Estimation

Parameter	Estimates	Standard error
Alpha	0.3001	0.0452
Beta	1.2915	0.481
Miu	-0.7066	0.0105
sigma	0.0197	0.0035

Log Likelihood: 66.26637 AIC: -124.5327 BIC: -115.9602

3.3 Beta distribution

The Beta distribution model parameters were estimated (Table 2) as follows: **Alpha = 1.367** with a standard error of 0.2206 and **Beta = 6.678** with a standard error of 1.2433. The log-likelihood value for this model was **52.8371**, indicating a reasonable fit to the data. The Akaike Information Criterion (AIC) was **-101.6742**, and the Bayesian Information Criterion (BIC) was **-97.38794**, both lower than those obtained for the Logit Skewed Exponential Power (LSEP) model. These results suggest that while the Beta distribution offers a decent fit, it may not capture the complexity of the inflation rate data as effectively as the LSEP model. The Beta distribution’s simpler structure, compared to the LSEP model, limits its ability to account for the asymmetry and heavy tails observed in the data.

Table 2: Beta distribution

Parameter	Estimates	Standard error
Alpha	1.367	0.2206
Beta	6.678	1.2433

Log Likelihood: 52.8371 AIC: -101.6742 BIC: -97.38794

3.4 Discussion

The study analysed Nigeria's inflation rate from 2008 to 2024, focusing on modelling it using the Logit Skewed Exponential Power (LSEP) and Beta distributions. The analysis aimed to identify the most suitable distribution for capturing the characteristics of the inflation data, particularly its skewness and heavy tails.

Inflation Rate Distribution: A histogram of the inflation rate compared to the previous year revealed a concentration of inflation rates between 0 and 0.2, with a few higher values. The LSEP distribution, represented by a density curve, closely followed the histogram, indicating a strong fit. In contrast, the Beta distribution deviated from the histogram, especially in the tails, suggesting a poorer fit for the data.

Goodness-of-Fit Analysis: The P-P and Q-Q plots further confirmed the LSEP model's superior fit compared to the Beta model. The P-P plot showed that the LSEP distribution's points closely followed the diagonal line, indicating a reasonable fit, although some deviations occurred in the tails. The Beta distribution, however, exhibited significant deviations, particularly in the central region of the plot, confirming its inadequacy for modeling the inflation data.

Parameter Estimation: The LSEP model parameters were estimated as Alpha = 0.3001, Beta = 1.2915, Miu = -0.7066, and Sigma = 0.0197. These estimates highlight the model's ability to capture the inflation data's skewness and scale effectively. The model's performance was also assessed using information criteria, with a log-likelihood of **66.26637**, AIC of **-124.5327**, and BIC of **-115.9602**, indicating a strong fit.

Comparison with Beta Distribution: The Beta distribution, with parameters Alpha = 1.367 and Beta = 6.678, had a log-likelihood of **52.8371**, AIC of **-101.6742**, and BIC of **-97.38794**. These values were lower than those for the LSEP model, confirming that the Beta distribution provides a less effective fit for the inflation rate data.

4.0 Conclusion

This study explored the modeling of Nigeria's inflation rate from 2008 to 2024 using the Logit Skewed Exponential Power (LSEP) and Beta distributions. The findings indicate that the LSEP distribution is a superior model for capturing the skewness and heavy tails inherent in the inflation data. The LSEP model closely followed the actual data distribution, as demonstrated by the density curves, P-P plots, and Q-Q plots, outperforming the Beta distribution. The LSEP model's parameter estimates, along with its favorable log-likelihood, AIC, and BIC values, further reinforce its suitability for this analysis. In contrast, the Beta distribution, with its simpler structure, failed to adequately model the complexities of the inflation rate data. Overall, the LSEP distribution provides a more accurate and robust tool for modeling inflation in Nigeria, making it a valuable approach for future economic analysis and policy development.

Given the superior performance of the LSEP model, it is recommended that this model be adopted for future analyses of inflation data in Nigeria and similar economic contexts. Policymakers and researchers should leverage the insights gained from the LSEP model to better understand inflation dynamics and develop strategies to mitigate economic volatility. Additionally, further research should explore the application of the LSEP model to other economic indicators, broadening its utility in economic forecasting and policy formulation.

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