DELAY PARAMETER OF TRANSITION VARIABLE IN THE SMOOTH TRANSITION AUTOREGRESSIVE (STAR) MODEL: IMPORTANT DETERMINANT

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ABSTRACT

This study investigates some determinant of appropriate delay parameter of the transition variable in the Smooth Transition Autoregressive (STAR) models, with emphasize on the model type and data characteristics through a refine method. Daily share price data were sourced from the Nigerian Exchange Limited, covering 10 years (January 2, 2014 to December 29, 2023), comprising 2,472 observations for each of the selected stocks: GTCO and STANBIC from the financial sector, and DANGCEM, BETAGLASS, and WAPCO from the industrial sector. Linearity tests demonstrated that DANGCEM, GTCO, and STANBIC share returns exhibit nonlinear characteristic of financial time series (FTS), whereas WAPCO returns remain linear and the most suitable delay parameter for each stock returns was determined. The Escribano-Jorda procedure was employed to select appropriate transition function. An asymmetric transition function was specified for DANGCEM and GTCO stock returns, while a symmetric transition function was identified for STANBIC and BETAGLASS returns. Consequently, asymmetric STAR models were fitted to DANGCEM and GTCO, and symmetric STAR models were fitted to BETAGLASS and STANBIC, using delay lengths within the range $(1 \leq d \leq p)$. *The results indicated that the APLSTAR, LSTAR, and SPLSTAR models with the initially chosen delay lengths were optimal for DANGCEM, GTCO, and BETAGLASS stock returns, respectively. However, for STANBIC returns, the optimal SPLSTAR model was associated with a delay length different from the initially selected delay parameter. This study concludes that while the delay parameter of the transition variable is typically determined from the characteristics of the FTS, STAR model type also significantly influences the selection of an appropriate delay parameter.*

Keywords: Delay parameter, correlation matrix, financial time series, transition variable and stock returns

I.0 Introduction

Determining the most suitable delay parameter of the transition variable is important in modeling a financial time series (FTS) with a smooth transition autoregressive (STAR) model. The standard procedure for selecting the value of the delay parameter, d, is to estimate a threshold autoregressive (TAR) model for each potential value of d and choose d with the smallest value of the residual sum of squares (RSS). Also, one can select a delay parameter from the range of values $1 \leq d \leq p$ which leads to the smallest information selection criteria (Enders, 2015). Terasvirta (1994) opined that the lag length of a linear model is determined first, followed by the determination of d by varying it and choosing the value minimizing the p-value of the linearity test. The delay parameter is the delay length between regimes and a key component of the transition function. According to Effiong *et al.* (2024), the efficiency of STAR models varies from series to series and depends largely on the transition function. Consequently, researchers modified the transition functions of various STAR models either from symmetric to asymmetric and vice versa ensuring that the salient dynamics of financial time series (FTS) necessary to achieve optimal forecasts by a particular STAR model are modeled (Terasvirta, 1994; Anderson, 1997; Liew *et al.*, 2003; Lundberg *et al.*, 2003; Siliverstovs, 2005; Dueker *et al.*, 2007 Shangodoyin *et al.*, 2009; Ajmi and El-Montasser, 2012 and Yaya and Shittu, 2016). Each used a specified delay parameter (*d*) for a specific FTS to compare different STAR models. While some of the analysts who studied the dynamics of time series using STAR models also used specific delay parameters of the transition variable, $y_{t-d}, d \leq p$, where p is the lag length of a linear model (Terasvirta and Anderson, 1992; Sarantis, 1999; Boero and Marrocu, 2002; Terasvirta *et al.*, 2005; Baharumshah and Liew, 2006; Yoon, 2010 and Hsu and Chiang, 2011). Only Dijk *et al.* (2002) considered a twelve-month difference with three different delay parameters ($\Delta_{12} y_{t-d}, d = 1,2,3$) as transition variable for

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modeling the US unemployment rate with logistic STAR (LSTAR) models and the LSTAR models with different delay parameters ($d = 1, 2, 3$) provide a comparable in-sample fit. Contrary to the known method of determining the delay parameter of the transition variable, and by evaluating the dynamics of stock returns and various STAR model specifications, this study investigates the role of the delay parameter in improving the precision of Smooth Transition Autoregressive (STAR) models applied to Nigerian stock indices, emphasizing the impact of model type and data characteristics on determining optimal delay lengths through a refine method.

2.0 Methodology

For the specification of STAR models, we employ the Escribano-Jorda procedure proposed by Escribano and Jorda (2001), while parameter estimation is carried out using the nonlinear least squares (NLS) method. Model evaluation metrics are applied to evaluate the adequacy and performance of the fitted STAR models.

2.1 The Model

2.1.1 Smooth transition autoregressive (STAR) models

A two-regime STAR model for a univariate time series z_t, which is observed at $t = 1 - p, 1 - (1 - p), \dots, -1, 0, 1, \dots, T - 1, T$ is given by

$$z_t = \mathbf{\Pi}_1' \boldsymbol{w_t}(1 - \mathrm{H}(z_{t-d}; \gamma, \mathrm{c})) + \mathbf{\Pi}_2' \boldsymbol{w_t} \mathrm{H}(z_{t-d}; \gamma, \mathrm{c}) + \boldsymbol{\varepsilon}_t, \quad (1)$$

where $\mathbf{\Pi}_\mathrm{i} = (\pi_{\mathrm{i},0}, \pi_{\mathrm{i},1}, \cdots, \pi_{\mathrm{i},\mathrm{p}})'$ for $i = 1,2$, $\boldsymbol{w}_t = (1, z_{t-1}, \dots, z_{t-p})'$, γ is the scale parameter, c is the location parameter, $\boldsymbol{\varepsilon}_t$ is a white noise process and $\mathrm{H}(z_{t-d}; \gamma, \mathrm{c})$ is a transition function that is at least twice differentiable (Lundbergh *et al.*, 2003). The most commonly used transition functions that give rise to different types of regime-switching behaviour are the following:

The first-order logistic function (LSTR1) proposed by Terasvirta (1994) given by

$$\mathrm{H}(z_{t-d}; \gamma, \mathrm{c}) = (1 + \exp[-\gamma(z_{t-d} - \mathrm{c})])^{-1}, \ \gamma > 0. \quad (2)$$

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and the STAR model (1) with (2) is called the logistic STAR (LSTAR) model.

The second-order logistic (LSTR2) function is given by

$$H(z_{t-d};\gamma,c) = (1+\exp[-\gamma(z_{t-d}-c_1)(z_{t-d}-c_2)])^{-1}, c_1 \le c_{2,}\ \gamma > 0, \quad (3)$$

where $c = (c_1, c_2)'$, as proposed by Jansen and Terasvirta (1996).

The exponential function proposed by Terasvirta (1994) given by

$$H(z_{t-d};\gamma,c) = 1-\exp[-\gamma(z_{t-d}-c)^2], \ \ \gamma > 0. \quad (4)$$

Model (1) with (4) is called the exponential STAR (ESTAR) model.

The error logistic function proposed by Shangodoyin *et al.* (2009) given by

$$H(z_{t-d};\gamma,c) = \{1+\exp\,[-\gamma(e_t(AR(p))-c]\}^{-1}\ , \quad (5)$$

Model (1) with (5) is called the error logistic smooth transition regression (ELSTR) model

Yaya and Shittu (2016) proposed an absolute error logistic function given by

$$H(z_{t-d};\ \gamma,c) = \{1+\exp\,[-\gamma/\varepsilon_{t-d}/-c]\}^{-1}, c > 0 \quad (6)$$

Model (1) with (6) is called the absolute error logistic STAR (AELSTAR) model

The Power Logistic (PL) Function proposed by Effiong *et al*. (2023) is given by

$$H(z_{t-d};\gamma,c) = \left\{1+0.5\exp[-\gamma\left(z_{t-d}^{i}-c\right)]\right\}^{-2}, \gamma > 0,\ \ i = 1,2, \quad (7)$$

Model (1) with (7) is called the Power Logistic STAR (PLSTAR) model.

(7) is called the asymmetric power logistic (APL) function when $i = 1$ and the corresponding STAR is APLSTAR, while (7) is a symmetric power logistic (SPL) function when $i = 2$ and the corresponding STAR model is the SPLSTAR model.

2.1.2 Specification of STAR model

According to Terasvirta (1994), the Lag length (p) of the linear model is determined either by Akaike information criterion (AIC) or Bayesian information criterion (BIC) provided the estimated residuals of the selected linear model are free from serial correlation, followed by testing for the

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linearity of the conditional mean model, If linearity is rejected, d is determined from the range of values $1 \leq d \leq P$ considered appropriate.

Escribano and Jorda procedure (EJP) is applied to specify the appropriate transition function based on the following two hypotheses within the auxiliary regression equation:

$$z_t = K_0' w_t + K_1' w_t z_{t-d} + K_2' w_t z_{t-d}^2 + K_3' w_t z_{t-d}^3 + K_4' w_t z_{t-d}^4 + \varepsilon_t \quad (8)$$

$H_{0L}: K_2 = K_4 = 0$ with an F-test (F_L)

$H_{0E}: K_1 = K_3 = 0$ with an F-test (F_E)

If the minimum p-value corresponds to F_E, select LSTAR model. Otherwise, select the ESTAR model.

2.1.3 Estimation of STAR model

The parameters of the STAR model (1) can be estimated by nonlinear least squares (NLS) method. Let $f(w_t; \Pi_1', \Pi_2', \gamma, c) = \Pi_1' w_t \big(1 - F(z_{t-d}: \gamma, c)\big) + \Pi_1' w_t F(z_{t-d}: \gamma, c)$, then (1) becomes

$$z_t = f(w_t; \Xi) + \varepsilon_t\,, \quad (9)$$

where $\Xi = (\Pi_1', \Pi_2', \gamma, c)$

The parameters $\Xi = (\Pi_1', \Pi_2', \gamma, c)$ can be estimated using NLS method as follows:

$$\hat{\Xi} = \underset{\Xi}{\operatorname{argmin}}\, Q_T(\Xi) = \underset{\Xi}{\operatorname{argmin}} \sum_{t=1}^{T} [z_t - f(w_t; \Xi)]^2 = \underset{\Xi}{\operatorname{argmin}} \sum_{t=1}^{T} {\varepsilon_t}^2 \quad (10)$$

The nonlinear least squares estimate is the value of $\hat{\Xi}$ that minimizes (10).

If ε_t are normally distributed, NLS is equivalent to maximum likelihood. Otherwise, the NLS estimates can be interpreted as quasi-maximum likelihood estimates. Under certain regularity conditions, White and Domowitz, (1984), Gallant (1987) and P¨otscher and Prucha (1997) showed that NLS estimates are consistent and asymptotically normal. Mathematically,

$$\sqrt{T}\big(\hat{\Xi} - \Xi_0\big) \to N(0, C) \quad (11)$$

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where Ξ_0 denotes the true parameter values. The asymptotic covariance-matrix C of $\hat{\Xi}$ can be estimated consistently as $\widehat{\mathrm{A}}_{\mathrm{T}}^{-1}\widehat{\mathrm{B}}_{\mathrm{T}}^{-1}\widehat{\mathrm{A}}_{\mathrm{T}}^{-1}$, where $\hat{A}_T$ is the Hessian evaluated at $\hat{\Xi}$.

$$\hat{A}_T = \frac{1}{T}\sum_{t=1}^{T}\nabla^2 \mathrm{q_t}(\hat{\Xi}) = \frac{1}{T}\sum_{t=1}^{T}(\nabla\mathrm{F}(w_t;\hat{\Xi})\nabla\mathrm{F}(w_t;\hat{\Xi})' - \nabla^2\mathrm{F}(w_t;\hat{\Xi})\hat{\varepsilon}_t) \quad (12)$$

Where $\mathrm{q_t}(\hat{\Xi}) = [\mathrm{z_t} - \mathrm{f}(\mathrm{w_t};\hat{\Xi})]^2$,

$\nabla\mathrm{F}(w_t;\hat{\Xi}) = \frac{\partial\mathrm{F}(w_t;\hat{\Xi})}{\partial\Xi}$ and

$$\widehat{\mathrm{B}}_{\mathrm{T}} == \frac{1}{T}\sum_{t=1}^{T}\nabla\mathrm{q_t}(\hat{\Xi})\nabla\mathrm{q_t}(\hat{\Xi})' = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t^2(\nabla\mathrm{F}(w_t;\hat{\Xi})\nabla\mathrm{F}(w_t;\hat{\Xi})' \quad (13)$$

Where $\widehat{\mathrm{B}}_{\mathrm{T}}$ is the outer product of the gradient.

2.1.4 Estimation of Linear Model by Least Squares Method

Estimating the parameters of the AR(p) model

$$Z_t = \mu + \pi_1 Z_{t-1} + \cdots + \pi_p Z_{t-p} + e_t \quad (14)$$

Ordinary least squares (OLS) estimates are obtained by rewriting (14) as

$$Z_t - \mu = \pi_1(Z_{t-1} - \mu) + \cdots + \pi_p(Z_{t-p} - \mu) + e_t \quad (15)$$

and treating it as a regression model to estimate $\pi_1, \cdots, \pi_p$ and μ. Letting $t = p+1, p+2, \cdots, n$ in (15), we have

$$\begin{aligned} Z_{p+1} - \mu &= \pi_1(Z_p - \mu) + \cdots + \pi_p(Z_1 - \mu) + e_{p+1} \\ Z_{p+2} - \mu &= \pi_1(Z_{p+1} - \mu) + \cdots + \pi_p(Z_2 - \mu) + e_{p+2} \\ \vdots \quad & \quad\quad \vdots \quad\quad\quad \vdots \\ Z_n - \mu &= \pi_1(Z_{n-1} - \mu) + \cdots + \pi_p(Z_{n-p} - \mu) + e_n \end{aligned} \quad (16)$$

OLS estimates are the values of $\hat{\mu}, \hat{\pi}_1, \cdots, \hat{\pi}_p$ that minimize the conditional sum of squares given by

$$S_c = \sum_{t=p+1}^{n}\hat{e}_t^2 = \sum_{t=p+1}^{n}\{Z_t - \hat{\mu} - \hat{\pi}_1(Z_{t-1} - \hat{\mu}) - \cdots - \hat{\pi}_p(Z_{t-p} - \hat{\mu})\}^2 \quad (17)$$

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2.2 Evaluation measures

Relative forecast performance is used as a model selection criterion or as an alternative or complement to an in-sample comparison of different models (Dijk *et al.,* 2002). The ratio of the root mean square error (RMSE) of the nonlinear model to that of the corresponding benchmark AR model will provide the relative performance of the two models. The RMSE of the in-sample forecast is the square root of the Mean square error (MSE). The standard error of the residuals of the competing models will be used to choose the best among them. .

3.0 Results and discussion

3.1 Data

Five Nigerian stock indices are considered. Share prices of GTCO and STANBIC from the financial sector and share prices of DANGCEM, BETAGLASS, and WAPCO from the industrial sector each comprising 2,472 observations spanning from January 2, 2014 to December 29, 2023, were obtained from Nigerian Exchange Limited.

3.2 Preliminary Analysis

STANBIC stock index is positively correlated with BETAGLASS, DANDCEM, and GTCO stock indices, but negatively correlated with WAPCO stock index based on the correlation matrix. Hence, the STANBIC stock index has interaction with BETAGLASS, WAPCO, DANDCEM, and GTCO stock indices. Correlation matric also reveals a negative relationship between the BETAGLASS stock index and the WAPCO stock index (Figure 1).

DANGCEM	GTCO	STANBIC	WAPCO	BETAGLAS	
1.0000	0.3790	0.5765	−0.1289	−0.0055	DANGCEM
	1.0000	0.7100	−0.1687	0.4228	GTCO
		1.0000	−0.5235	0.5192	STANBIC
			1.0000	−0.6827	WAPCO
				1.0000	BETAGLAS

Figure 1. Correlation Matrix of selected Nigerian stock indices

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Time series plots reveal decelerated growth in the share prices of DANDCEM, GTCO, STANBIC and WAPCO between 2014 and 2016, except BETAGLASS stock index which experienced accelerated growth between 2014 and 2015, but sudden fall in 2016 (Figures 2, 3, 4 and 5).

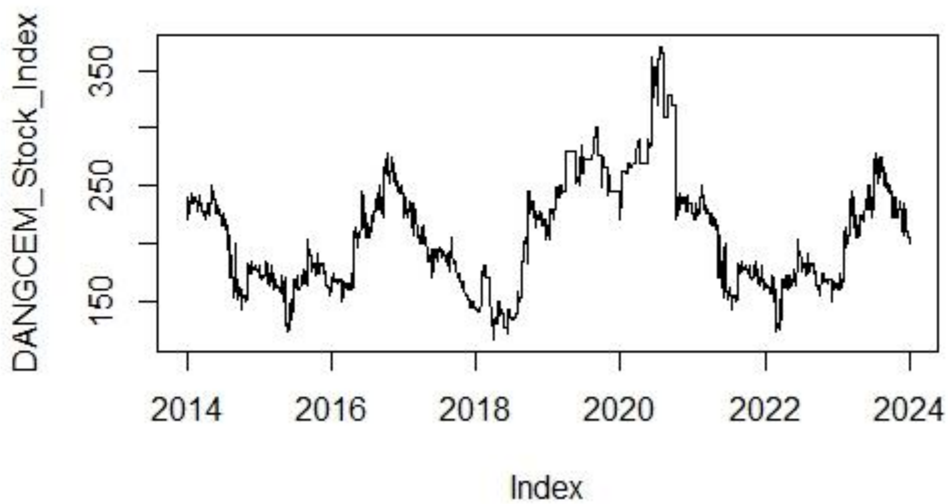

Figure 2: Time series plot of DANGCEM stock index

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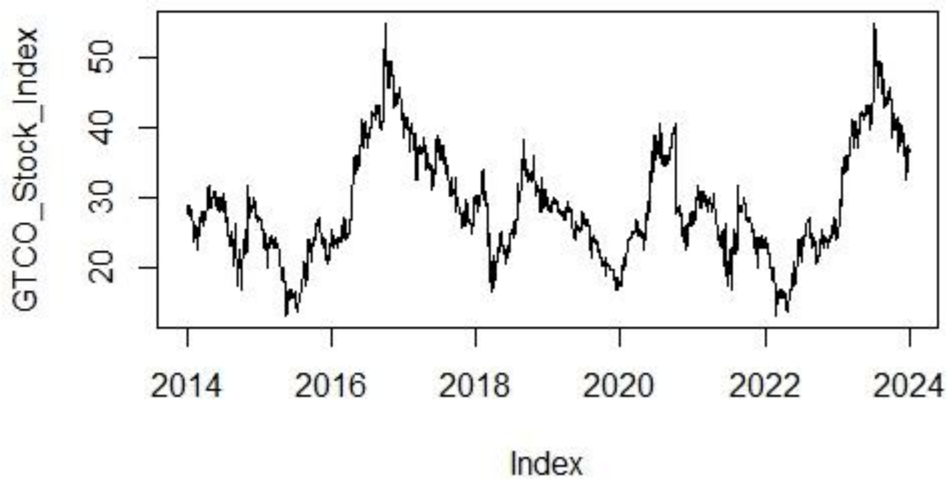

Figure 3: Time series plot of GTCO stock index

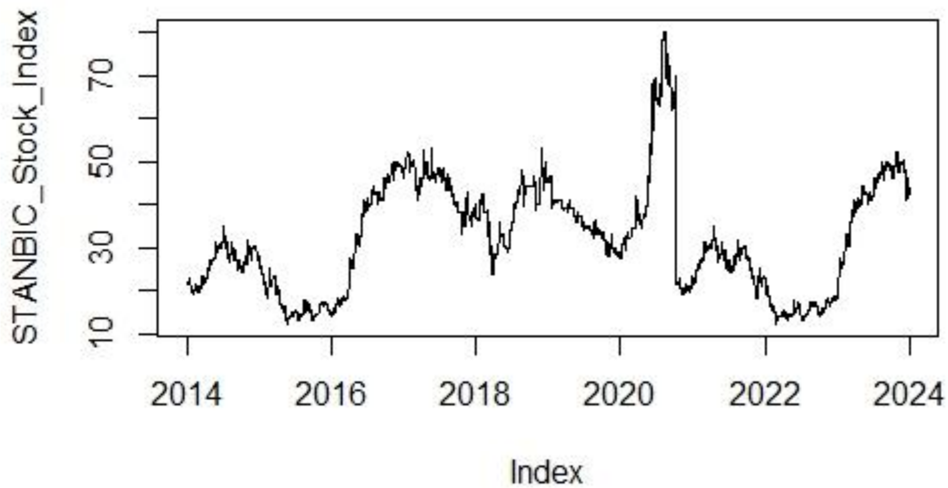

Figure 4: Time series plot of STANBIC stock index

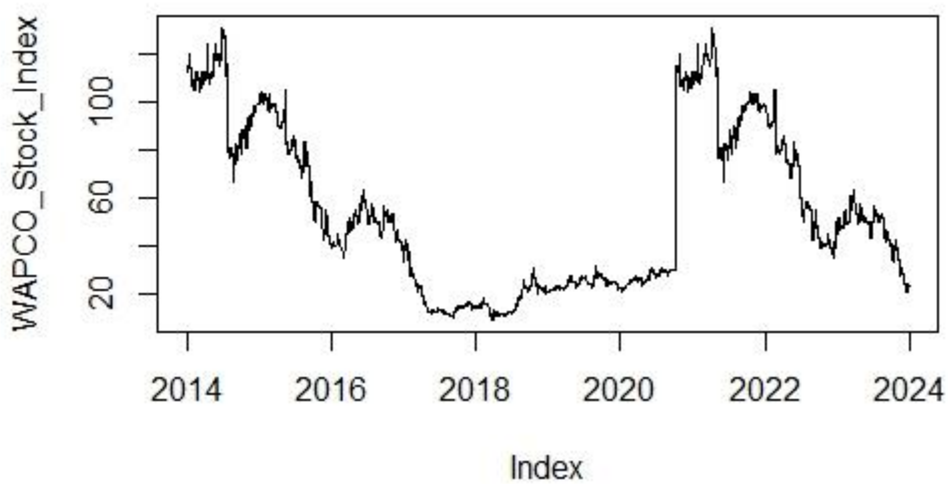

Figure 5: Time series plot of WAPCO stock index

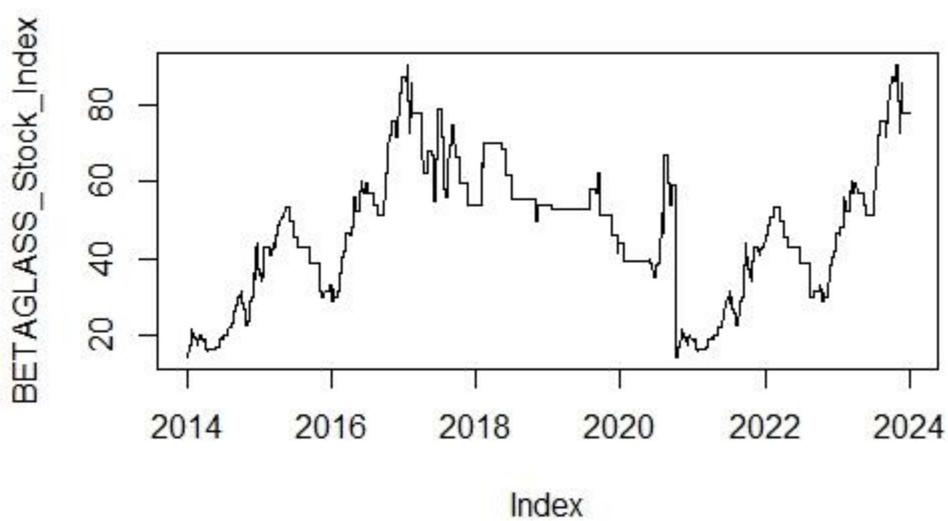

Figure 6: Time series plot of BETAGLASS stock index

Based on Table 1, the Augmented Dickey Fuller tests reveal the presence of unit root in the five selected stock indices, but their return series are stationary.

Table 1: Augmented Dickey-Fuller tests

Nigerian stock index	Dickey Fuller Statistic (Original series)	Lag order	Dickey Fuller Statistic	
			Logarithmic series	**Return series**
GTCO	-2.4515 (0.3872)	15	-2.5662 (0.3386)	-15.9 (<0.01)
DANGCEM	-2.4594 (0.3838)	15	-2.5726, (0.3359)	-15.683 (<0.01)
STANBIC	-2.477 (0.3764)	15	-2.0878 (0.5411)	-15.466 (<0.01)
WAPCO	-2.252 (0.4716)	15	-1.7269, (0.694)	-14.76, (<0.01)
BETAGLASS	-2.486 (0.373)	15	-2.595 (0.326)	-14.783 (<0.01)

Where the values in the parentheses are p-values of Ljung-Box statistic

Akaike information criterion (AIC) was applied to identify ARIMA(4,1,0), ARIMA(3,1,0), ARIMA(2,1,0) and ARIMA(2,1,0) models for modeling DANGCEM stock returns (DSR), GTCO stock returns (GSR), WAPCO stock returns (WSR) and STANBIC stock returns, respectively, while Schwarz Bayesian information criterion (BIC) was used to identify ARIMA(2,1,0) model for modeling BETAGLASS stock returns (BSR) as shown in Table 2.

Table 2: Selection of lag length of linear models

Lags	AIC				BIC
	DANGCEM	**GTCO**	**STANBIC**	**WAPCO**	**BETAGLASS**
1	5.769350	1.938049	-4.626324	3.225303	-5.130259
2	5.769325	1.935509	**-4.627136***	**3.213153***	**-5.130612***
3	5.769209	**1.935249***	-4.627074	3.213692	-5.128225
4	**5.768368***	1.935262	-4.626355	3.213314	-5.125956
5	5.768871	1.935781	-4.626148	3.214117	-5.124151
6	5.768943	1.936136	-4.625352	3.214903	-5.120760

The asterisks above indicate the best (minimized) values of the respective information criteria, AIC = Akaike criterion and BIC = Schwarz Bayesian criterion.

The five specified linear models were estimated and based on Table 3, the p-values of Ljung-Box statistic reveal that the residuals of the estimated models are free from serial correlation. Tests of

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linearity against the alternative STAR type nonlinearity based on the five selected stock returns were carried out.

Table 3: Estimation of parameters of linear models

Nigerian stock index	Model	Parameter	Estimate	Ljung-Box Statistic	σ_{ε_t}
DANGCEM	ARIMA (4,1,0)	π_1	0.029535 (0.07423)	0.00074302 (0.9783)	0.02135
		π_2	-0.011327 (0.49374)		
		π_3	-0.022315 (0.17779)		
		π_4	-0.033440 (0.04341)		
GTCO	ARIMA (3,1,0)	π_1	0.0970506 (4.363e-09)	0.0038825 (0.9503)	0.02391
		π_2	-0.0058603 (0.724261)		
		π_3	-0.0556463 (0.0007631)		
STANBIC	ARIMA (2,1,0)	π_1	0.016408 (0.321047)	0.00086177 (0.9766)	3.06369
		π_2	-0.048281 (0.003499)		
WAPCO	ARIMA (2,1,0)	π_1	-0.5676 (9.63e-196)	265.434 (0.1902)	0.03313
		π_2	-0.3261 (5.86e-066)		
BETAGLASS	ARIMA (2,1,0)	π_1	0.027236 (0.000)	0.0014947 (0.9692	3.27
		π_2	0.025474 (0.003)		

where the values in the parentheses are p-values of estimated parameters and σ_{ε_t} is the standard error of the residuals.

According to Table 4, there is evidence of nonlinearity in the BSR, DSR, GSR and STANBIC stock returns at 5% level of significance, while for WSR, linear model is not rejected at 5% level of significance. Hence, WSR is a linear FTS and is modeled with ARIMA (2, 1, 0) model; BSR, DSR, GSR and STANBIC stock returns are classified as nonlinear FTS and are modeled with STAR models.

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To determine whether the class or type of STAR model is a determining factor of appropriate delay parameter of the transition variable. We first determined a delay parameter within the range of values $1 \leq d \leq p$ with the smallest value of the residual sum of squares for each stock return and based on Table 4, the delay parameters for BSR, DSR, GSR and STANBIC stock returns are 2, 2, 1 and 2, respectively. Escribano-Jorda procedure was adopted to specify appropriate transition function and its corresponding STAR model.

Table 4: Linearity Test

Nigerian stock index	F- Statistic for Null hypothesis of linearity	Residual sum of squares (RSS) of the Delay parameter (d)			
		1	2	3	4
DANGCEM	7.8179 (<0.001)	4557.7828	**45359.5409**	45386.4523	45361.1632
GTCO	3.3731 (0.0092)	**985.3175**	986.3200	988.8851	-
STANBIC	24.0271 (0.0072)	2100.0888	**2094.4873**	**-**	-
BETAGLASS	8.657488 (<0.001)	2345.3747	**2342.5337**	**-**	-
WAPCO	0.430583 (0.7311)	-	-	-	-

The values in the parentheses are p-values of F- Statistics.

From Table 5, First-order logistic function (asymmetric transition function) was specified for DSR and GSR since p-value $= 0.1025 >$ p-value$= (< 0.001)$ and p-value$= 0.0003 >$ p-value$= 0.0002$, respectively, while exponential function (symmetric transition function) was identified for STANBIC stock returns and BSR since p-value$= 0.1427 <$ p-value$= 0.5571$ and p-value$= 0.0022 <$ p-value$= 0.0652$. Consequently, asymmetric STAR models were fitted to DANGCEM and GTCO stock returns, while symmetric STAR models were fitted to BSR and STANBIC stock returns with different delay lengths to determine the best delay parameter at the evaluation stage based on the efficiency of the model.

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Table 5: Escribano-Jorda Tests

Nigerian stock index	F- Statistic		Transition function
	H_{0L}	H_{0E}	
DANGCEM	2.668256 (0.1025)	11.64640 (<0.0001)	First-order logistic function
GTCO	6.2076 (0.0003)	8.7702 (0.0002)	First-order logistic function
STANBIC	1.9483 (0.1427)	0.3448 (0.5571)	Exponential function
BETAGLASS	4.890444 (0.0022)	2.409339 (0.0652)	Exponential function

The values in the parentheses are p-values of F- Statistics, while H_{0L} and H_{0E} are null hypotheses.

In accordance with Table 6, two asymmetric STAR (ELSTR and APLSTAR) models fitted to DSR have the smallest standard error of residual and AIC when $d = 2$ similar to the specified delay parameter, while LSTAR model has the lowest standard error of residual and AIC when $d = 4$ completely different from the initial choice of delay parameter ($d = 2$) for DSR. Yet the best model for modeling DSR is APLSTAR model with $= 2$.

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Table 6: Evaluation of asymmetric STAR models fitted to DANGCEM stock returns for $1 \leq d \leq 4$

Model	Delay Parameter	Ljung-Box Statistic	σ_{ε_t}	AIC
LSTAR	1	0.038819 (0.8438)	4.495	55070
	2	0.021161 (0.8843)	4.50	55110
	3	-	-	-
	4	0.046528 (0.8292)	**4.48**	**54950**
ELSTR	1	6.3122 (0.01199)	2.169	10853.3
	2	5.1402 (0.12338)	**2.167**	**10849.31**
	3	2.0053 (0.1567)	2.174	10865.19
	4	-	-	-
APLSTAR	1	4.4057 (0.03582)	2.167	10847.96
	2	1.4376 (0.2305)	**2.163**	**10839.96**
	3	0.018308 (0.8924)	2.174	10864.4
	4	-	-	-

The values in the parentheses are p-values of Ljung-Box statistics

Based on Table 7, LSTAR and ELSTR models are very efficient when $d = 1$, the same delay length specified for modeling GTCO stock returns. APLSTAR model is efficient when d=3 completely different from the specified $d = 1$. The best model for modeling GSR is LSTAR model with $d = 1$.

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Table 7: Evaluation of symmetric STAR models fitted to GTCO stock returns for $1 \leq d \leq 3$

Model	Delay Parameter	Ljung-Box Statistic	σ_{ε_t}	AIC
LSTAR	1	0.0020976 (0.9635)	**2.3770**	**6337**
	2	0.0034241 (0.9533)	2.3787	6342
	3	-	-	-
ELSTR	1	0.021508 (0.8834)	**2.396**	**11344.68**
	2	0.8746 (0.3497)	2.406	11365.91
	3	0.12993 (0.7185)	2.398	11349.01
APLSTAR	1	2.9195 (0.08751)	2.404	11361.16
	2	3.9695 (0.04633)	2.409	11371.41
	3	0.37672 (0.5394)	**2.401**	**11356.51**

The values in the parentheses are p-values of Ljung-Box statistics

According to Table 8, ESTAR and AELSTAR (symmetric STAR) models fitted to STANBIC stock returns have the smallest standard error of residuals and AIC value when $d = 2$, similar to the initial delay length specified for modeling STANBIC stock returns, whereas SPLSTAR has the lowest standard error of residuals and AIC value when $d = 1$ completely different from the delay parameter specified ($d = 2$) for modeling STANBIC stock returns. SPLSTAR model with $d = 1$ is the best model for modeling STANBIC stock returns.

From Table 9, the three symmetric STAR model fitted to BSR have the smallest standard error of residuals and AIC value when $d = 2$, the value of the delay length specified for modeling BSR. But the most efficient STAR model is SPLSTAR model with $d = 2$. In view of the foregoing results, the type of STAR model is very important in the determination of delay parameter of the transition variable in a STAR model.

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Table 8: Evaluation of symmetric STAR models fitted to STANBIC stock returns for $1 \leq d \leq 2$

Model	Delay Parameter	Ljung-Box Statistic	σ_{ε_t}	AIC
ESTAR	1	0.012211 (0.912)	3.373	13035.47
	2	0.73869 (0.3901)	**3.369**	**13030.58**
AELSTAR	1	0.69662 (0.4039)	3.346	13001.41
	2	0.56555 (0.452)	**3.336**	**12986.76**
SPLSTAR	1	0.86039 (0.3536)	**3.317**	**12953.56**
	2	0.86039 (0.3536)	3.368	13029.49

The values in the parentheses are p-values of Ljung-Box statistics

Table 9: Evaluation of symmetric STAR models fitted to BETAGLASS stock returns for $1 \leq d \leq 2$

Model	Delay Parameter	Ljung-Box Statistic	σ_{ε_t}	AIC
ESTAR	1	-	-	-
	2	0.010244 (0.9194)	**3.4**	**13075.73**
AELSTAR	1	-	-	-
	2	0.011699 (0.9139)	**3.399**	**13078.76**
SPLSTAR	1	0.000565 (0.981)	3.404	13081.81
	2	0.71487 (0.3978)	**3.398**	**13073.8**

The values in the parentheses are p-values of Ljung-Box statistics

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3.4 Discussion of findings

BETAGLASS, GTCO, STANBIC, DANGCEM, and WAPCO are the five Nigerian stock indices obtained from Nigerian Exchange Limited for analyses. STANBIC stock index is associated with the BETAGLASS, DANDCEM, GTCO, and WAPCO stock indices, while the BETAGLASS index is related to the WAPCO stock index. Time series plots reveal decelerated growth among the series between 2014 and 2016, except BETAGLASS stock index which experienced a sudden rise between 2014 and 2015. ADF tests indicate the presence of unit roots in the five selected stock indices, while their return series are stationary.

Linearity tests revealed that BSR, DSR, GSR, and STANBIC share returns are nonlinear FTS, while WSR is a linear FTS. The suitable delay parameters were initially determined for BSR, DSR, GSR, and STANBIC share returns using known approach commonly applied by time series analysts (Terasvirta, 1994; Liew *et al.*, 2003; Siliverstovs, 2005; Dueker *et al.*, 2007; Shangodoyin *et al.*, 2009; Yaya and Shittu, 2016; Ajmi and El-Montasser, 2012, Effiong et al., 2023 and Effiong et al.. 2024).

Escribano-Jorda procedure specified first-order logistic function (asymmetric transition function) for DSR and GSR, while exponential function (symmetric transition function) was identified for BSR and STANBIC stock returns. Consequently, asymmetric STAR models were fitted to DSR and GSR, and symmetric STAR models were fitted to BSR and STANBIC stock returns with the delay lengths within the range $1 \leq d \leq p$ to investigate the role of STAR model type in the determination of appropriate delay parameter of the transition variable. Indeed, APLSTAR, LSTAR, and SPLSTAR models with initially chosen delay lengths are optimal for modeling DSR, GSR, and BSR, respectively. However, SPLSTAR model with a delay length completely different from the initially chosen delay length is optimal for modeling STANBIC stock returns. Hence, although the delay parameter of the transition variable is usually based on the characteristics of the FTS, the type of STAR model also influences the choice of the appropriate delay parameter.

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4.0 Conclusion

This study concludes that, although the delay parameter of the transition variable is generally determined based on the characteristics of the financial time series (FTS), the type of STAR model chosen also plays a critical role in selecting an appropriate delay parameter. These findings enhance the accuracy and reliability of STAR model applications in financial time series analysis.

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