COMPARATIVE ANALYSIS OF LINEAR AND NON-LINEAR REGRESSION MODELS ON IMPACT OF NIGERIAN POPULATION AND DEBT ON GROSS DOMESTIC PRODUCT (GDP)

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Abstract

Country's population is a 2way adaptor, either beneficiary or detrimental depending on how the government channels it. But national debt is short term beneficiary and become economic trap if persistent in the long run on economic growth of the nation. There had been numerous studies that aimed at examining the effects of public debt and population growth on economic growth carried out over time across countries of the world, models of both linear and non-linear had been applied to solve problems of public debt and population on Gross Domestic product most of which are bereft of strategic empirical evidences suitability in developing countries. We consider Linear and Exponential growth model estimators for estimating the significant of population and National debt on the economic growth of Nigeria measured by GDP and realized that population growth rate has exerted significant positive impact on economic growth (GDP) While, National debt does not have any significant impact on economic growth. Moreover, AIC still remain the best model selection criterial and that linear regression model is recommended in fitting Nigeria economic growth on the population and public Debt.

Key Words: Akaike Information Criterial (AIC), Exponential growth rate, Gross Domestic Product (GDP), Linear growth rate, Population, Public Debt.

INTRODUCTION

The currently estimated population in Nigeria is 224,036,297 as of Monday, July 17, 2023, based on (Worldometer). Bremner, López-Carr, Suter and Davis (2010) observed that continuous population growth militates against economic growth through the inducement of poverty, falling medical care/services, as well as environmental degradation. Vitalis and Oruonye (2021) in their study argued that population increase is not a problem in itself to any nation, and that there are some impeding factors associated with population growth such as corruption, inadequate planning, inappropriate implementation of development plans, poor budget/implementation a complacency in developing human capital. Many analysts believe that economic growth in high-income countries is likely to be relatively slow in coming years in part because population growth in these countries is predicted to slow considerably (Baker, Delong & Krugman2005). Adewole (2012) examined effect of population on economic development in Nigeria from 1981 to 2007 using ordinary least square method of analysis. The result reveals that population growth exert positive and significant effect on economic growth measured as Per Capita Income (PCI) and Real Gross Domestic Product (RGDP) in Nigeria between 1981 and 2007. Previous studies frequently made the incorrect assumption that the relationship between public debt and growth is linear and symmetric, leading to empirical results that is frequently disputed and imprecise. Yussuf and Muhammed (2023) realized that external debt has a significant positive and symmetric impact on economic growth in the long and short run, while debt service payment supporting the debt overhang hypothesis activated a symmetric effect that stifle growth. Domestic debt retarded growth asymmetrically in the short term and linearly over the long term. Reinhart and Kenneth (2010), claimed that there is nonlinear relationship between National Debt and Economic Growth.

While Birungi, et.al (2023) shows that linear regression model fits in modelling the effect of the increase in population on economic growth in Uganda.

Statistical modeling quantifies uncertainty within a system of ideas. Statisticians use statistical models that generate intuitive visualizations to estimate reality and establish correlations between random and non-random variables within a set. It focuses on judging the quality of data analysis practices in an organization, which utilizes quantitative evidence to help researchers in understanding which data is qualitatively reliable for their analysis. Statistical models are becoming the most widely acceptable tools for finding reliable and reasonable conclusions from extensive data sets. Bates and Watts (2007) claimed that the main advantages of nonlinear models are parsimony, interpretability, and prediction also, another advantage of using nonlinear regression models is that their predictions tend to be more robust that competing polynomials, especially outside the range of observed data (i.e., extrapolation). Model selection plays a fundamental role in choosing a best model from a series of candidate models for data driven modelling and system identification problems. In general, system identification and data-driven modelling consists of several important steps, including data collection, data processing, selection of representation functions, model structure selection, model validation and model refinement (Preacher & Merkle, 2012; Solares, Wei, & Billings, 2017; Söderström & Stoica, 1989). Upon the numerous studies that had aimed at examining the effects of public debt and population growth on economic growth that had been carried out over time across countries of the world. Noticeably, a significant number of these studies and other related researches are bereft of strategic empirical evidences in developing countries, examines the effect of public debt on economic growth like in Nigeria. The objective of this research work Research work is to compare the statistical modeling of Linear and non - linear Regression of Nigeria GDP on population and debt, fitting non-linear

regression model of Nigeria GDP on population and debt and to select the best models using the most commonly applied model selection criteria.

2. Materials and Methods

2.1 Data Collection

Secondary source of data on the hypothesized variables from 1990 to 2021 were from Central Bank of Nigeria (CBN) Statistical Bulletin (2023) based on the objective of the study which was to examine the significant impact of population growth rate and National debt on economic growth of Nigeria. The variables employed are gross domestic product, population growth rate and National debt

2.2 Models of the Study

2.2.1 Linear Growth Model

Regression model specifies the relation of dependent variable (Y) to a function combination of independent variables (X) and unknown parameters β

$$Y \approx f(X, \beta) \quad (1)$$

The formulae for the regression equation can be written as;

$$y = a + bx \quad (2)$$

Where

$$a = \frac{\sum y - b\sum x}{n} \quad (3)$$

The structural form of the model described in the study can be written as:

$$y = a + b_1 x_1 + b_2 x_2 + \varepsilon \quad (4)$$

where y = GDP, x_1 = Population and x_2 = National Debt, while a, b_1, and b_2, are the coefficients.

2.2.2 Exponential Growth Model

Exponential regression is used when the relationship between variables can be represented by an exponential function rather than a linear model. The general form of an exponential regression model is:

$$y = ae^{bx} + \varepsilon \quad (5)$$

2.2.3 Model Selection Criterion

When we are dealing with multiple models, the question is how to find the best model among competing models. Depending on the structure of the models, different statistical criteria can be used to find the best model: *F* test, Akaike information criterion (AIC), Bayesian information criterion (BIC), or the likelihood ratio test (Zucchini, 2000; Burnham and Anderson, 2002; Hoffmann, 2005; Ritz and Streibig, 2008; Lewis et al., 2011).

In order to select the best model that best fit the data of the study, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were adopted in this study to statistically measure the best model for the study.

The formula for AIC is given by

$$AIC = 2k - 2\ln\left(\hat{L}\right) \quad (7)$$

Where:

k is the number of model parameters (degrees of freedom)

$\hat{L}$ is the maximum value of the likelihood function for the model

The formula for BIC can be written as;

$$BIC = k\ln(n) - 2\ln(\hat{L}) \quad (8)$$

Where n is the number of observations in the dataset.

The BIC differs from the AIC only in the second term, which depends on *n*. Clearly as *n* increases, the BIC favors the simpler models (fewer parameters). This explains why sometimes the AIC and BIC indices disagree.

3. Result and Discussion

The result in table 3.1 present the results of different models considered this study with their respective metrics: RSE (Residual Standard Error), AIC (Akaike Information Criterion), and BIC (Bayesian Information Criterion). A lower RSE indicates a better fit of the model to the data. In this case, the Linear Growth Model has a slightly lower RSE (3.172) compared to the Exponential Growth Model (3.259), suggesting that the Linear Growth Model might have a slightly better overall fit to the data.

Lower AIC values indicate a better trade-off between model fit and simplicity. The Linear Growth Model has a lower AIC (169.5403) compared to the Exponential Growth Model (171.2655), indicating that the Linear Growth Model might be a better choice according to the AIC criterion

(see Table 3.1). Similar to AIC, BIC is used for model selection, but it penalizes complex models more heavily.

Like AIC, lower BIC values indicate a better trade-off between goodness of fit and model complexity. In this study, the Linear Growth Model has a lower BIC (175.4033) compared to the Exponential Growth Model (177.1285), suggesting that according to BIC, the Linear Growth Model might be more preferable. We therefor concluded that the Linear Growth Model performs slightly better in terms of RSE, AIC, and BIC compared to the Exponential Growth Model based on the provided metrics.

The population and National debt variables VIF values are close to 1 (1.023013 and 1.023 respectively), suggesting that there is minimal multicollinearity present between these variables (Table 3.2). Breusch-Pagan test checks whether the variance of the residuals is dependent on the independent variables. The test result (see Table 3.2) is presented with a test value of 0.28394 and a p-value of 0.8676 which suggests that there is no strong indication that the variance of residuals is not constant across the values of the independent variables. These findings indicate that, based on these specific diagnostics, there are no significant issues with multicollinearity or heteroscedasticity detected in the model used for the study.

The correlation coefficient in Table 3.3 between GDP and Population is 0.6455. This positive value suggests a moderately strong positive linear relationship between GDP and Population. When one variable (GDP) increases, the other variable (Population) tends to increase as well, and vice versa, but the relationship is not perfect. The correlation coefficient between GDP and Debt is -0.0856 (Table 3.3). This value is close to zero, indicating a very weak negative linear

relationship between GDP and Debt. There's almost no apparent linear association between changes in GDP and changes in Debt based on this correlation.

The coefficient for the population variable is 25.7627 (see Table 3.4). This means that, holding other variables constant, a one-unit increase in population is associated with an estimated increase of approximately 25.7627 units in the dependent variable (GDP). The coefficient for the National debt variable is 0.0028. This suggests that, with all other variables held constant, a one-unit increase in National debt is associated with an estimated increase of 0.0028 units in the dependent variable (GDP).

The R-squared measures the proportion of variance in the dependent variable that is explained by the independent variables. In this case, the R-squared is 0.4168, indicating that approximately 41.68% of the variability in the dependent variable (GDP) is explained by the regression model. The model as a whole is statistically significant according to the F-test, and approximately 41.68% of the variability in the gross domestic product of Nigeria is explained by the population growth rate included in the model.

3.2 Result Presentation

Table 3.1: Model Selection Criterion

Model	**RSE**	**AIC**	**BIC**
Linear Growth Model	3.172	169.5403	175.4033
Exponential Growth	3.259	171.2655	177.1285

Table 3.2: Diagnostic Check

Collinearity	**Population**	**Debt**
Vif	1.023013	1.023
Breusch-Pagan test		
	test value	**p-value**
BP	0.28394	0.8676

Table 3.3: Pearson Correlation

	GDP	**Population**	**Debt**
GDP	1	0.6455	-0.0856
Population	0.6455	1	-0.1500
Debt	-0.0856	-0.1500	1

Table 3.4: Linear Growth Model

Variable	**Estimate**	**Std. Error**	**t – value**	**Pr (>|z|)**
Intercept	-62.7829	14.9056	-4.212	0.000224***
Population	25.7627	5.7100	4.512	9.83e-05***
Debt	0.0028	0.0358	0.079	0.9375
R-squared	0.4168			
Adjusted R-squared	0.3765			
F-statistic	10.36			
p – value	0.0004025			

4. Conclusion

We consider Linear and Exponential growth model estimators for estimating the significant of population and National debt on the economic growth of Nigeria measured by GDP. Findings from the comparative study revealed that linear growth model estimator is a better when compared exponential growth model. The study found that population growth rate has exerted significant positive impact on economic growth (GDP). However, National debt does not have any significant impact on economic growth. Our study provide evidence that 41.68% of the variability in the gross domestic product of Nigeria is explained by the population growth rate.

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