TRANSMUTED SINE GENERALIZED EXPONENTIAL DISTRIBUTION: DISTRIBUTIONAL PROPERTIES AND APPLICATIONS

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Abstract:

This research paper introduces a novel probability distribution called the Transmuted Sine Generalized-Exponential (T-SGEx) distribution, which is developed by combining the transmutation technique with the sine function and the Generalized-Exponential distribution. The distribution is designed to offer greater flexibility in modeling real-life data, particularly for datasets exhibiting complex behaviors such as skewness, heavy tails, or multimodal patterns. To understand the properties of the T-SGEx distribution, several distributional characteristics, including probability density function (PDF), cumulative distribution function (CDF), hazard rate function, moments, and quantile function, were examined. The study also explored the reliability properties of the distribution, which are critical for applications in survival analysis and engineering.

The estimation of the T-SGEx distribution parameters was performed using Maximum Likelihood Estimation (MLE). Real-life datasets from two previous studies were used to evaluate the performance of the T-SGEx distribution. The goodness-of-fit of the T-SGEx distribution was compared with two competing distributions: the New Weighted Exponential Distribution (NWED) and the standard Exponential Distribution (ED). Model comparison was conducted using the Akaike Information Criterion (AIC) and the negative log-likelihood (NLL). The results demonstrated that the T-SGEx distribution provides a superior fit to the datasets, as evidenced by lower AIC and NLL values compared to the NWE and Exponential distributions.

The findings of this study highlight the practical application of the T-SGEx distribution in modeling real-life data, particularly in fields such as reliability engineering, survival analysis, and environmental studies.

Keywords: Distribution, Quadratic rank transmutation map, Maximum likelihood estimation, Order Statistics, Transmuted Sine-G distribution

1 Introduction

For the past two decades, developing compound distributions has received wide attention from both the mathematics and statistics research communities with the aim of developing flexible probabilistic models that fit in unique stochastic phenomena. This phenomenon occurs in several sectors of human endeavor such as engineering, survival analysis, marine sciences and many more. The unique nature of different behaviors of datasets from several random processes as a result of technological development has been the key motivational phrase for developing new probability distributions. Varieties of techniques have been developed in the literature used for defining new compound distributions such as link function, odd function, generalization, transmutation and many more. Several families for generating new distributions and their respective sub-families have been developed in the literature. The distribution has been used for modelling different datasets studied by so many researchers. Azzalini (1985) proposed a new method for inducing a skewness parameter to the normal distribution based on a weighted function and obtained the skew-normal distribution. Azzalini's idea has been applied to other symmetric distributions and many skew-symmetric distributions have been developed; for example, the skew-logistic distribution due to Nadarajah (2009). The study showed that the weighted exponential distribution can be used to analyze positively skewed data, like the distributions mentioned above, and data coming from hidden truncated models. Gupta and Kundu (2009) proposed the weighted exponential distribution but Bashir and Naqvi (2016) used Azzalini's Method to develop a new weighted exponential distribution, which is simpler and mathematically easier to handle.

In reliability and lifespan data analysis, several modified distributions with extra parameters have recently been developed and investigated as lifetime distributions. This includes the Odd Generalized Exponential-New Weighted Exponential Distribution by Abdullahi and Abba (2017), Transmuted Odd Generalized Exponential- Exponential Distribution by Abdullahi et al. (2018), Transmuted Exponential Lomax Distribution by Abdullahi and Ieren (2018), Odd Generalized Exponential-Inverse Lomax Distribution by Falgore et al. (2018), and Generalized transmuted-Inverted exponential Distribution by Usman et al. (2019).

A random variable *X* is said to have a transmuted sine-G family as developed by Dewu *et al.* (2025) if its *pdf* and *cdf* are respectively given by:

$$f(x) = \pi \cos\left\{\pi \frac{G(x;\eta)}{1+G(x;\eta)}\right\}\left(\frac{g(x;\eta)}{\left(1+G(x;\eta)\right)^2}\right)\left[1+\lambda-2\lambda \sin\left\{\pi \frac{G(x;\eta)}{1+G(x;\eta)}\right\}\right]; x \in \mathbb{R} \tag{1}$$

And

$$F(x;\eta) = (1+\lambda) sin\left\{\pi \frac{G(x;\eta)}{1+G(x;\eta)}\right\} - \lambda\left(sin\left\{\pi \frac{G(x;\eta)}{1+G(x;\eta)}\right\}\right)^2 ; x \in \mathbb{R} \tag{2}$$

Where; $x > 0$, and $-1 \leq \lambda \leq 1$ is the transmuted parameter, *G(x)* is the *cdf* of any continuous distribution while *f(x)* and *g(x)* are the associated *pdf* of *F(x)* and *G(x)*, respectively.

The *pdf* and *cdf* of exponential distribution with parameter β is defined as:

$$f(x;\beta) = \beta e^{-\beta x} \tag{3}$$

$$F(x;\beta) = 1 - e^{-\beta x} \tag{4}$$

2. The proposed T-SGEx Distribution

Using equations (3) and (4) in (1) and (2) and simplifying, we obtain the *pdf* and *cdf* of the transmuted Sine G Exponential (T-SGEx) distribution as follows:

$$f(x;\lambda,\beta) = \pi \cos\left\{\pi \frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\left(\frac{\beta e^{-\beta x}}{(1+1-e^{-\beta x})^2}\right)\left[1+\lambda-2\lambda \sin\left\{\pi \frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right]; x \in \mathbb{R} \qquad (5)$$

$$F(x;\lambda,\beta) = (1+\lambda)\sin\left\{\pi \frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\} - \lambda\left(\sin\left\{\pi \frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right)^2; x \in \mathbb{R} \qquad (6)$$

Where; $x > 0, \beta > 0, -1 \leq \lambda \leq 1$, β *and* λ are the scale and transmuted parameters respectively. The *pdf* and *cdf* of the *T-SGEx distribution* using some parameter values are displayed in **Figures 2.1** and **2.2** as follows:

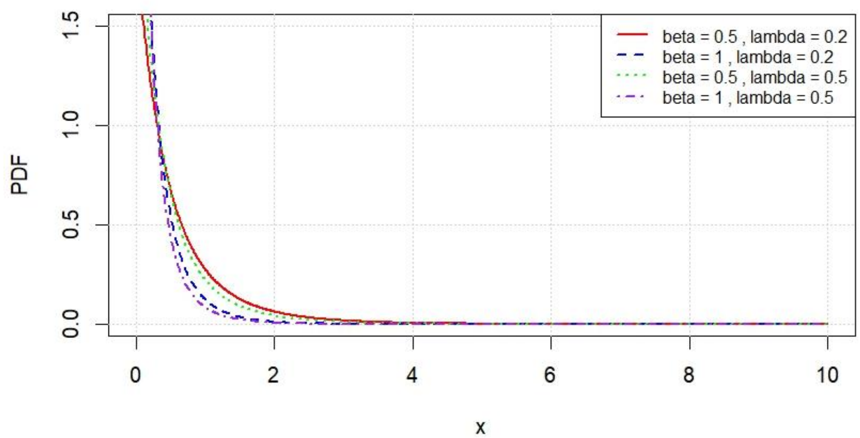

Figure 2.1: The graph of *pdf* of the *T-SGEx* at some chosen parameter values of λ and β.

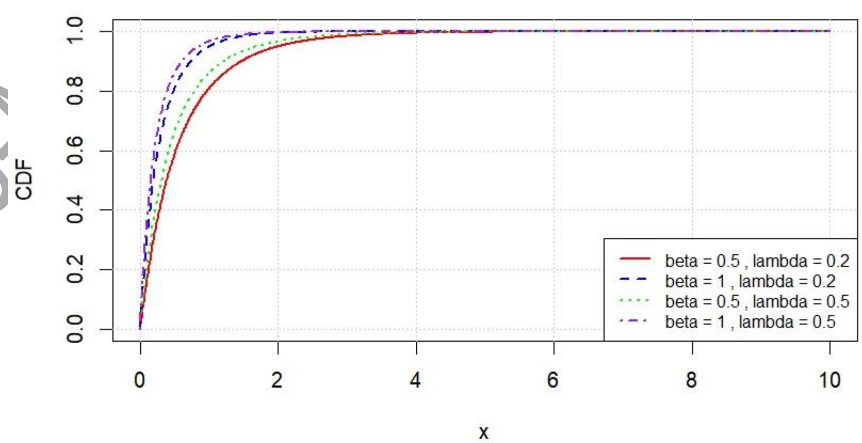

Figure 2.2: The graph of *cdf* of the *T-SGEx* at some chosen parameter values of β and λ.

The plot for the *pdf* reveals that the *T-SGEx* is positively skewed and, therefore, will be a good model for positively skewed datasets, while that of the *cdf* shows that it is also a valid distribution since limits of 0 and 1 are proven as *X* approaches zero and positive infinity, respectively.

2.2 The Reliability Analysis (Survival and Hazard Functions) of the T-SGEx Distribution

Simplifying equations (3), (4), (5) and (6), we obtain the survival and hazard functions of the T-SGEx as follows:

$$S(x;\lambda,\beta) = 1 - \left((1+\lambda)sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\} - \lambda\left(sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right)^2\right); x \in \mathbb{R} \quad (7)$$

The corresponding hazard function is:

$$H(x;\lambda,\beta) = \frac{\pi\, cos\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\left(\frac{\beta e^{-\beta x}}{\left(1+1-e^{-\beta x}\right)^2}\right)\left[1+\lambda-2\lambda\, sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right]}{1-\left((1+\lambda)sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}-\lambda\left(sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right)^2; x \in \Re\right)}; x \in \mathbb{R} \quad (8)$$

Respectively. Where; $x > 0, \beta > 0, -1 \le \lambda \le 1, \beta$ is the scale parameter while λ is the transmuted parameter. The *survival* and *hazard functions* of the *T-SGEx distribution* are displayed in Figures 2.3 and 2.4 as follows:

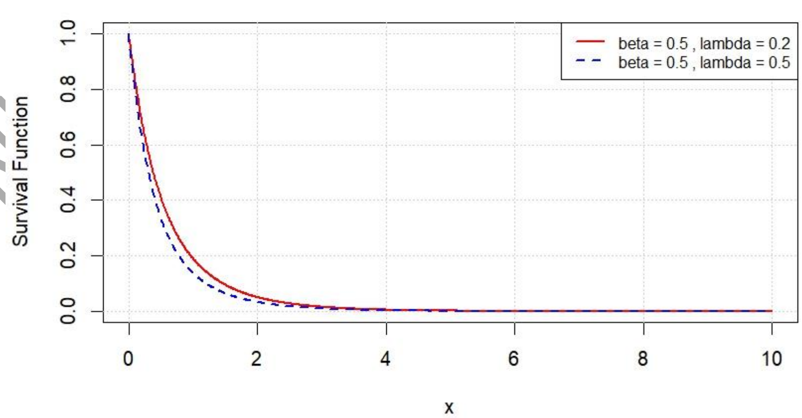

Figure 2.3: The graph of the survival function of the *T-SGEx*

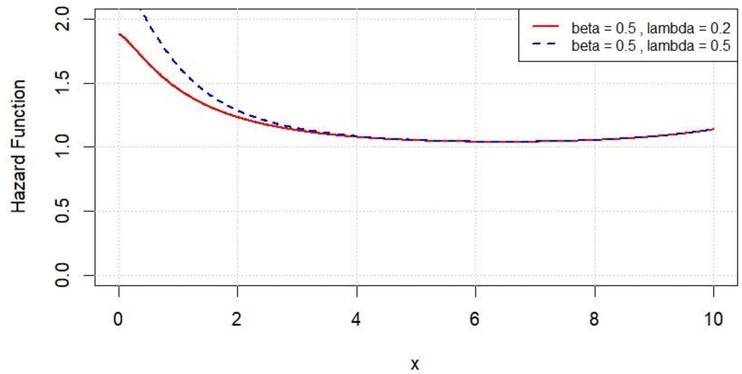

Figure 2.4: The graph of hazard function of the *T-SGEx*

3. Properties

In this section, we define and discuss some properties of the *T-SGEx* distribution.

3.1 Moments

Let *X* denote a continuous random variable, the n^{th} moment of *X* is given by;

$$\mu_n' = E(X^n) = \int_{-\infty}^{\infty} x^n f(x)dx = \int_0^{\infty} x^n f(x)dx \qquad (9)$$

Taking *f(x)* as the *pdf* of the *T-SGEx* as given in equation (18), the n^{th} moment of *X* is obtained using integration by substitution and is given as:

$$\mu_n = \int_0^{\infty} x^n 2\sin\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right) cos\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right)\left(\frac{2\pi\beta e^{-\beta x}}{(1+1-e^{-\beta x})^2}\right)dx \qquad (10)$$

The Mean

The mean of the T-SGEx can be obtained from the nth moment of the distribution when n=1 as follows:

$$\mu_1 = \int_0^{\infty} x 2\sin\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right) cos\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right)\left(\frac{2\pi\beta e^{-\beta x}}{(1+1-e^{-\beta x})^2}\right)dx \qquad (11)$$

Also, the second moment of the T-SGEx is obtained from the nth moment of the distribution when n=2 as:

$$\mu_2 = \int_0^{\infty} x^2 2\sin\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right) cos\left(\frac{\pi(1-e^{-\beta x})}{1+1-e^{-\beta x}}\right)\left(\frac{2\pi\beta e^{-\beta x}}{(1+1-e^{-\beta x})^2}\right)dx \qquad (12)$$

The Variance

The n[th] central moment or moment about the mean of X, say μ_n, can be obtained as:

$$\mu_n = E(X - \mu_1')^n = \sum_{i=0}^{n} (-1)^i \binom{n}{i} \mu_1'^i \mu_{n-i}' \quad (13)$$

The variance of *X* for *T-SGEx* distribution is obtained from the central moment when *n=2*, that is;

$$Var(X) = E(X^2) - \{E(X)\}^2 \quad (14)$$

$$Var(X) = \mu_2' - \{\mu_1'\}^2 \quad (15)$$

Where $\mu_1'$and $\mu_2'$are the mean and second moment of the T-SGEx distribution respectively.

The variation, skewness and kurtosis measures can also be calculated from the non-central moments using some well-known relationships.

3.2 Moment Generating Function

The moment-generating function (*mgf*) is an important shape characteristic of a distribution and is always in one function that represents all the moments.

Thus, the *mgf* of the *T-SGEx* distribution is given as:

$$M_x(t) = \int_0^\infty e^{tx} \left[(1+\lambda)\cos\big(z(x)\big)\frac{d}{dx}z(x) - 2\lambda \sin\,(z(x))\cos\,(z(x))\frac{d}{dx}z(x)\right]dx \quad (16)$$

4. Estimation of Parameters of T-SGEx distribution using the Maximum Likelihood Method

Let $X_1 \ldots X_n$ be a sample of size '*n*' of independently and identically distributed random variables from the *T-SGExD* with unknown parameters α, β, λ defined previously.

The likelihood function is given by:

$$L(X_1, X_2, \ldots\ldots, X_n/\beta, \lambda) = \prod^{\pi\, cos\left\{\pi \frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\left(\frac{\beta e^{-\beta x}}{\left(1+1-e^{-\beta x}\right)^2}\right)\left[1+\lambda-2\lambda\, sin\left\{\pi\frac{1-e^{-\beta x}}{1+1-e^{-\beta x}}\right\}\right]} \quad (17)$$

Let the log-likelihood function, $l = log\, L\,(X_1, X_2, \ldots\ldots, X_n/\alpha, \beta, \lambda)$, therefore;

$$LL = n\,log(\pi) + \sum_{i=1}^{n} log\left(cos\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}\right) + n\,log\,\beta - \beta\sum_{i=1}^{n} x_i - 2\sum_{i=1}^{n} log\left(1+1-e^{-\beta x_i}\right) + \sum_{i=1}^{n} log\left[1+\lambda - 2\lambda\,sin\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}\right] \quad (18)$$

Differentiating l partially with respect to $\beta\ and\ \lambda$ respectively gives;

$$\frac{\partial l}{\partial\beta} = \sum_{i=1}^{n}\left(-tan\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}\left\{\pi\frac{\left(1-e^{-\beta x_i}\right)x_i e^{-\beta x_i} + x_i e^{-\beta x_i}\left(1+1-e^{-\beta x_i}\right)}{\left(1+1-e^{-\beta x_i}\right)^2}\right\}\right) + \frac{n}{\beta} - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\frac{x_i e^{-\beta x_i}}{2-e^{-\beta x_i}} + \sum_{i=1}^{n}\left[\frac{-2\lambda\,cos\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}}{\left(1+\lambda-2\lambda\,sin\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}\right)} \times \left\{\pi\frac{\left(1+1-e^{-\beta x_i}\right)x_i e^{-\beta x_i} + x_i e^{-\beta x_i}\left(1-e^{-\beta x_i}\right)}{\left(1+1-e^{-\beta x_i}\right)^2}\right\}\right] \quad (19)$$

$$\frac{\partial l}{\lambda} = \sum_{i=1}^{n}\left[\frac{1-2\lambda\,sin\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}}{1+\lambda-2\lambda\,sin\left\{\pi\frac{1-e^{-\beta x_i}}{1+1-e^{-\beta x_i}}\right\}}\right] \quad (20)$$

Equating equations (30) and (31) to zero and solving for the solution of the non-linear system of equations will give us the maximum likelihood estimates of parameters $\beta\ and\ \lambda$, respectively. However, the solution cannot be obtained analytically except numerically with the aid of suitable statistical software like Python, R, SAS, etc., when data sets are given.

5. Application to real-life datasets

This section presents two datasets, their summary of descriptive statistics and applications to some selected extensions of the Exponential distribution. We have compared the performance of the Transmuted Sine Exponential distribution (*T-SGExD*) to those of the New Weighted Exponential distribution (*NWED*) and the classical Exponential distribution (*ED*).

We have evaluated the performance of the above-listed models using the *AIC* (Akaike Information Criterion). It is considered that the model with the smallest value of AIC will be chosen as the best model to fit the data.

Data set I: The first data set from (Chen et al., 2010) and (Abdullahi et al., 2023) corresponds to fifty-two ordered annual maximum antecedent rainfall measurements in mm from Maple Ridge in

British Columbia, Canada. The data are: 264.9, 314.1, 364.6, 379.8, 419.3, 457.4, 459.4, 460, 490.3, 490.6, 502.2, 525.2, 526.8, 528.6, 528.6, 537.7, 539.6, 540.8, 551.0, 573.5, 579.2, 588.2, 588.7, 589.7, 592.1, 592.8, 600.8, 604.4, 608.4, 609.8, 619.2, 626.4, 629.4, 636.4, 645.2, 657.6, 663.5, 664.9, 671.7 ,673.0, 682.6, 689.8, 698, 698.6, 698.8, 703.2, 755.9, 786, 787.2, 798.6, 850.4, 895.1. It's summarized as follows:

Table 1: Summary Statistics for Dataset I

Parameters	n	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	52	264.9	528.1	596.8	672.0	595	895.1	16158	-0.1895	0.3454

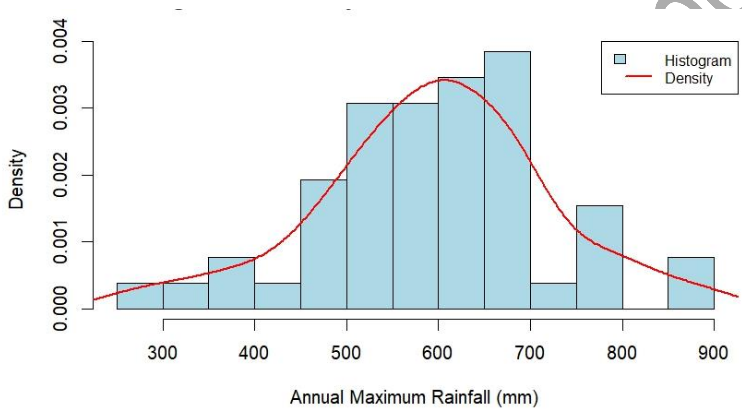

Figure 5.5: A Histogram and density plot for Dataset I

From the descriptive statistics in Table 1 as well as the histogram and density plot shown above in Figure 5.1, we observed that the data set is approximately normal and therefore not suitable for skewed distributions such as the proposed model.

Table 2: The performance of the selected models using the *AIC* value of the models evaluated at the *MLEs* based on Dataset I.

Distributions	Parameter estimates (standard errors)	-ll=(-log-likelihood value)	*AIC*	Ranks
T-SGEx	$\lambda = -1$ (0.38309) $\beta = 0.001$ (0.00000839)	-373.4173	750.8347	**1**
NWED	$\lambda = 0.06685$ (0.27784) $\beta = 0.06516$ (0.27751)	-384.2055	772.4109	**3**

ED	$\beta = 0.00168\ (0.00023)$	-384.2053	770.4106	**2**

Using table 2 and comparing the value of the *AIC* for each model show that the *T-SGEx* distribution has better performance compared to the *NWED* and *ED*. This is due to the decision rule, which says that the distribution or model with a smaller value of the test statistics (AIC) will be taken as the most adequate or efficient model.

Data set II: This second data represents the survival times of 121 patients with breast cancer obtained from a large hospital in a period from 1929-1938 from Lee (1992), Ramos *et al*. (2013) and Oguntunde *et al.* (2017). The observations are as follows: 0.3, 0.3, 4.0, 5.0, 5.6, 6.2, 6.3, 6.6, 6.8, 7.4, 7.5, 8.4, 8.4, 10.3, 11.0, 11.8, 12.2, 12.3, 13.5, 14.4, 14.4, 14.8, 15.5, 15.7, 16.2, 16.3, 16.5, 16.8, 17.2, 17.3, 17.5, 17.9, 19.8, 20.4, 20.9, 21.0, 21.0, 21.1, 23.0, 23.4, 23.6, 24.0, 24.0, 27.9, 28.2, 29.1, 30.0, 31.0, 31.0, 32.0, 35.0, 35.0, 37.0, 37.0, 37.0, 38.0, 38.0, 38.0, 39.0, 39.0, 40.0, 40.0, 40.0, 41.0, 41.0, 41.0, 42.0, 43.0, 43.0, 43.0, 44.0, 45.0, 45.0, 46.0, 46.0,47.0, 48.0, 49.0, 51.0, 51.0, 51.0, 52.0, 54.0, 55.0, 56.0, 57.0, 58.0, 59.0, 60.0, 60.0, 60.0, 61.0, 62.0, 65.0, 65.0, 67.0, 67.0, 68.0, 69.0, 78.0, 80.0, 83.0, 88.0, 89.0, 90.0, 93.0, 96.0, 103.0, 105.0, 109.0, 109.0, 111.0, 115.0, 117.0, 125.0, 126.0, 127.0, 129.0, 129.0, 139.0, 154.0. Its summary is given as follows:

Table 3: Summary Statistics for the dataset II

Parameter	N	Minimum	Q_1	Median	Q_3	Mean	Maximum	Variance	Skewness	Kurtosis
Values	121	0.30	17.5	40.00	60.0	46.33	154.00	1244.464	1.04318	0.40214

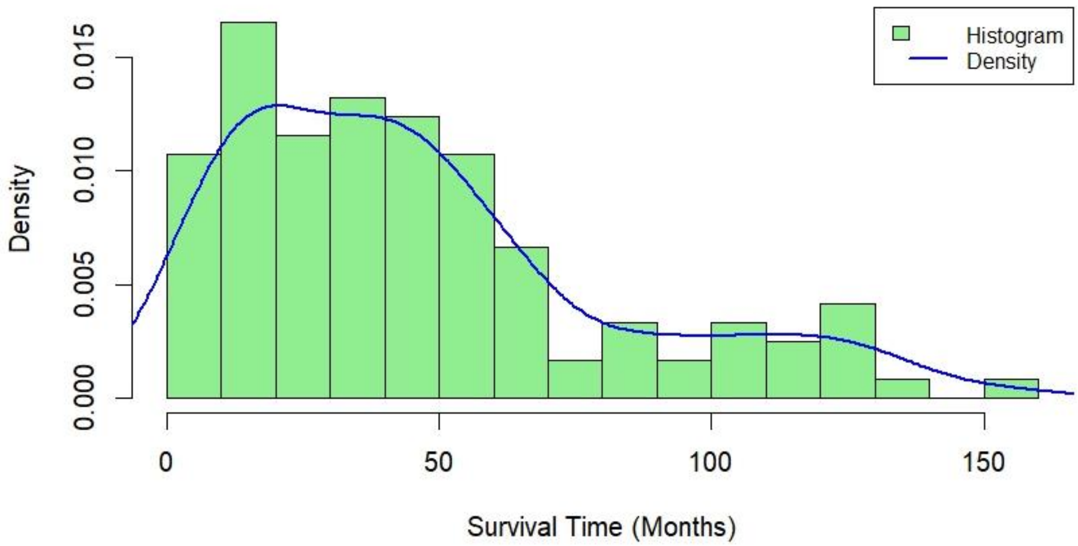

Figure 5.8: A Histogram and density plot for Data set II

From the descriptive statistics in table 3 and the histogram as well as the density plot shown in Figure 5.8 above, we observed that dataset II is positively skewed with greater peakness which is not normally distributed as clearly demonstrated and therefore suitable for distributions that are skewed to the right and have a variety of shapes just like the T-SGExD.

Table 4: The performance of the selected models using the *AIC* value of the models

Distributions	Parameter estimates (standard errors)	-ll=(-log-likelihood value)	*AIC*	Ranks
T-SGEx	α =-0.8414 (0.1161) β =0.0106 (0.00094)	-580.0007	1164.001	**1**
NWED	α =0.4031 (NA) β =0.3816 (NA)	-585.1278	1174.256	**3**
ED	λ =0.02159 (0.00196)	-585.1277	1172.255	**2**

Now, comparing the value of the *AIC* for all the distributions, it is very clear that the proposed *T-SGExD* performs better than the *NWED* and the classical exponential distribution (*ED).* This conclusion is again due to the decision rule, which says that the distribution or model with a smaller value of the test statistics (AIC) should be considered the most fitted or efficient model.

6 Conclusion

This article shows the distributional properties of a new distribution called a Transmuted Sine Exponential distribution (*T-SGEx)*. The derivations of some expressions for its moments, moment generating function, survival function, and hazard function have also been done. The model parameters have been estimated using the method of maximum likelihood estimation. The implications of the plots for the survival function indicate that the transmuted new weighted Exponential distribution could be used to model age-dependent events or variables whose survival decreases as time grows or whose survival rate decreases with time. The performance of the *T-SGEx* distribution has been evaluated using two real-life datasets from previous research and the results show that the proposed distribution fits the datasets better compared to the fits of the other two distributions (New Weighted Exponential distribution and Exponential distribution) considered in this study.

Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest

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