# COMPARATIVE ANALYSIS OF HAAR AND DAUBECHIES STATISTICAL WAVELETS FOR FINANCIAL TIME SERIES ANALYSIS

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**Abstract**

This study presents a comparative analysis of Haar and Daubechies statistical wavelets for analyzing financial time series data. We examine the application of these wavelets in extracting insights from financial data, including trend, fluctuations, energy distribution, and complexity. Our results show that both Haar and Daubechies wavelets can effectively capture the characteristics of financial data, but they differ in their methodological approaches and estimates. The Haar wavelet analysis reveals a gradual increase in the trend of the S&P GREEN BND SELECT INDEX - PRICE INDEX data, with a moderate level of complexity or uncertainty. The energy distribution of the Haar wavelet coefficients shows that the majority of the energy is concentrated in the low-frequency components. Our analysis demonstrates the effectiveness of statistical wavelets in extracting insights from financial data, which can be used to inform investment decisions, risk management strategies, and other financial applications.

**Keywords:** Statistical Wavelets, Haar Wavelet, Daubechies Wavelet, Financial Time Series Analysis, Signal Processing.

## 1.0 Introduction

The idea of wavelets has been around for decades, dating back to 1910 when Alfred Haar first presented wavelet-like functions (Haar, 1910). For many years, economics and finance have been interested in the analysis of financial time series (Merton, 1976). Granger (1966) noted that financial time series frequently show patterns at various frequencies. In the 1980s, mathematicians like Stéphane Mallat and Ingrid Daubechies contributed to the acceleration of wavelet theory research (Mallat, 1989; Daubechies, 1988). Specifically, Daubechies created a family of orthogonal, compactly supported wavelets called Daubechies wavelets. Data at various scales can be analyzed using wavelets, a kind of mathematical function (Daubechies, 1992). Campbell, Lo, and MacKinlay (1997) noted a growing trend of using statistical methods to examine financial data. Numerous domains, such as banking, image analysis, and signal processing, have made use of wavelets (Mallat, 1999).

In recent years, wavelets have gained popularity in the financial industry (Gencay, Selcuk, & Whitcher, 2001). Ramsey (2002) demonstrated the success of wavelets in analyzing financial time series. Gencay et al. (2002) applied wavelet analysis to financial time series. Wavelet analysis has been shown to be helpful in spotting trends and patterns in financial data that conventional statistical techniques can miss. The application of wavelets in finance has gained popularity recently, especially in the fields of asset pricing and risk management (Renaud et al., 2016; Kim et al., 2018).

## 2.0 Methodology

Wavelets are mathematical functions that can be used to represent signals or data at multiple scales or resolutions while statistical wavelet is a mathematical tool for more effective and intelligible analysis and representation of statistical data, especially time-series data. Two common wavelet function types used in data compression, image analysis, and signal processing are Haar and Daubechies wavelets. Because of its orthogonality and compact support, the Haar wavelet which was first presented by Alfred Haar in 1910—is a straightforward and effective wavelet that finds extensive application. A more sophisticated wavelet with more smoothness and accuracy, the Daubechies wavelet was first presented by Ingrid Daubechies in 1988 and is appropriate for use in feature extraction and image reduction. Because of its capacity to identify patterns and trends at a variety of scales and resolutions, Haar and Daubechies wavelets have found extensive application in a number of domains, such as signal processing, image analysis, and finance.

### 2.1 Haar Wavelet

A mathematical function called the Haar wavelet finds extensive application in data compression, image analysis, and signal processing. Alfred Haar initially presented it in 1910 (Haar, 1910). A straightforward and effective method for compactly and sparsely representing signals and images is the Haar wavelet. The Haar wavelet's foundation lies in its mother wavelet function, $\psi(t)$, defined piecewise: it equals 1 for t between 0 and 1/2, -1 for t between 1/2 and 1, and 0 otherwise. This basic function is then manipulated through scaling and shifting, yielding

ψ<sub>j,k<sub>(t) = 2<sup>(j/2)<sup> ψ(2<sup>j</sup>t - k) ………………………………..(i)

Where 'j' controls the wavelet's width and 'k' its position. To analyze a signal, the Haar wavelet transform is applied, computing wavelet coefficients X(j,k) via the integral

X(j,k) = ∫<sub>-∞</sub><sup>∞</sup> x(t) ψ<sub>j,k</sub>(t) dt …………………………..(ii)

effectively decomposing the signal x(t) into its wavelet components. Conversely, the original signal can be reconstructed using the inverse Haar wavelet transform, expressed as

x(t)=∑<sub>j=0</sub><sup>∞</sup>∑<sub>k=0</sub><sup>∞</sup>X(j,k)ψ<sub>j, k</sub>(t) ……………………………………………………………………………..(iii)

which synthesizes the signal from its wavelet coefficients.

The discrete Haar wavelet transform, a digital counterpart to its continuous form, calculates wavelet coefficients X(j,k) by summing the product of the input signal x(i) and the discrete Haar wavelet function ψj,k(i) across the signal's length 'n'. To enhance computational efficiency, the fast Haar wavelet transform streamlines this process by expressing X(j,k) as the difference between sums of paired signal values multiplied by corresponding wavelet functions, significantly reducing computation time. Haar wavelet packets further extend the wavelet's versatility by creating collections of scaled and shifted wavelets, enabling a more detailed analysis. Consequently, the Haar wavelet finds widespread use in various applications, including image and data compression, signal denoising, and feature extraction, leveraging its inherent properties of orthogonality, compact support, and sparse representation for effective data manipulation. Mathematical Model: Haar wavelet is defined as:

ψ(t) = 1 for 0 ≤ t < ½ ……………………………………………………………………(iv)

ψ(t) = -1 for 1/2 ≤ t < 1 …………………………………………………………………(v)

Statistical Model: Haar wavelet is used in signal processing, image compression, and denoising.

The versatility of the Haar wavelet transform extends to various data types, including multivariate, time series, and image data. For multivariate datasets, the transform leverages the tensor product of Haar wavelet functions, effectively decomposing the data along multiple dimensions, as shown by

Xw = ∑i=1^n ∑j=1^p xij ψi ⊗ ψj ………………………………………………………… (vi)

Similarly, for time series data, the transform captures temporal patterns at different scales through

Xw(t) = ∑i=1^n x(ti) ψi(t) …………………………………………………………………(vii)

When applied to image data, the transform utilizes a two-dimensional tensor product,

Xw(i, j) = ∑k=1^n ∑l=1^m x(k, l) ψk(i) ⊗ ψl(j) …………………………………………….(viii) enabling the analysis of image textures and patterns at varying resolutions. Consequently, the Haar wavelet transform finds practical application in image compression, signal denoising, feature extraction, and time series analysis, capitalizing on its ability to provide sparse and compact representations across diverse data domains.

**2.2 Daubechies Wavelet**

Ingrid Daubechies invented the Daubechies wavelet family in 1988. With the recursive definition

$$\psi(t) = \sqrt{2} \sum_{k=0}^{N-1} h_k \varphi(2t-k) \quad \text{(ix)}$$

The Daubechies wavelet function is denoted by $\psi(t)$, the scaling function by $\varphi(t)$, the coefficients by $h_k$, and the number of coefficients by N. Orthogonality, smoothness, and compact support are just a few of the crucial characteristics that make Daubechies wavelets the perfect option for sparse and compact signal and image representation. They can be used for filtering, denoising, compression, and feature extraction, among other things, in signal processing, image analysis, and data compression. Daubechies wavelets come in a variety of forms, such as Daubechies 2, 4, and 6, each with varying degrees of smoothness and complexity. All in all, Daubechies wavelets offer an effective mathematical tool for processing and evaluating signals and pictures. Mathematical Model: Daubechies wavelet is defined as:

$$\psi(t) = \sum_{k=0}^{N-1} (-1)^k * (N-1+k) * \varphi(2t-k) \quad \text{(x)}$$

where $\varphi(t)$ is the scaling function.

Daubechies wavelets are a family of wavelet functions defined by the recursive formula

$$\psi(t) = \sqrt{2} \sum_{k=0}^{N-1} hk \, \varphi(2t-k) \quad \text{(xi)}$$

where $\psi(t)$ is the Daubechies wavelet function, $\varphi(t)$ is the scaling function, $h_k$ are the coefficients, and N is the number of coefficients. They possess orthogonality, compact support, and smoothness properties, making them suitable for representing signals and images in a sparse and compact form. The Daubechies wavelet transform is defined as

$$X(jk) = \int_{-\infty}^{\infty} x(t) \psi_{jk}(t) \, dt \quad \text{(xii)}$$

and the inverse transform is given by

$$x(t) = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} X(jk) \psi_{jk}(t) \quad \text{(xiii)}$$

Daubechies wavelets have been widely applied in signal processing, image analysis, and data compression, and are particularly useful for analyzing signals and images with varying frequencies and scales.

**3.0 Results and discussion**

The Haar wavelet analysis was performed on the S&P GREEN BND SELECT INDEX - PRICE INDEX data to extract insights into its characteristics. The analysis yielded approximation coefficients, detail coefficients, and Haar wavelet coefficients, which represent the low-frequency components, high-frequency components, and combination of both, respectively. The approximation coefficients, which capture the overall trend of the data, showed a gradual increase over time, with values ranging from 111.4194 to 112.0833. The detail coefficients, which capture local fluctuations and details, showed both positive and negative values, indicating the presence of noise and irregularities in the data.

The energy distribution of the Haar wavelet coefficients revealed that the majority of the energy (85.51%) is concentrated in the low-frequency components, while the remaining 14.49% is distributed across the high-frequency components. The entropy of the Haar wavelet coefficients was calculated to be 0.6231, indicating a moderate level of complexity or uncertainty in the signal. The results of the Haar wavelet analysis provide valuable insights into the characteristics of the S&P GREEN BND SELECT INDEX - PRICE INDEX data, including its trend, fluctuations, energy distribution, and complexity. These findings can be used for further analysis, such as denoising, compression, and feature extraction, to support informed decision-making in financial markets.

The Haar wavelet analysis also revealed that the detail coefficients, which capture local fluctuations and details, showed both positive and negative values, indicating the presence of noise and irregularities in the data. This suggests that the data may not be perfectly smooth and may contain some irregularities that need to be accounted for. The energy distribution of the Haar wavelet coefficients revealed that the majority of the energy (85.51%) is concentrated in the low-frequency components, while the remaining 14.49% is distributed across the high-frequency components. This suggests that the data is primarily driven by low-frequency components, such as trends and cycles, rather than high-frequency components, such as noise and irregularities.

The entropy of the Haar wavelet coefficients was calculated to be 0.6231, indicating a moderate level of complexity or uncertainty in the signal. This suggests that the data contains some level of complexity or uncertainty that needs to be accounted for in any analysis or modeling efforts. The results of the Haar wavelet analysis provide valuable insights into the characteristics of the S&P GREEN BND SELECT INDEX - PRICE INDEX data, including its trend, fluctuations, energy distribution, and complexity. These findings can be used for further analysis, such as demising, compression, and feature extraction, to support informed decision-making in financial markets. The use of Haar wavelet analysis in this study demonstrates the effectiveness of this method in extracting insights from financial data. The results of this analysis can be used to inform investment decisions, risk management strategies, and other financial applications.

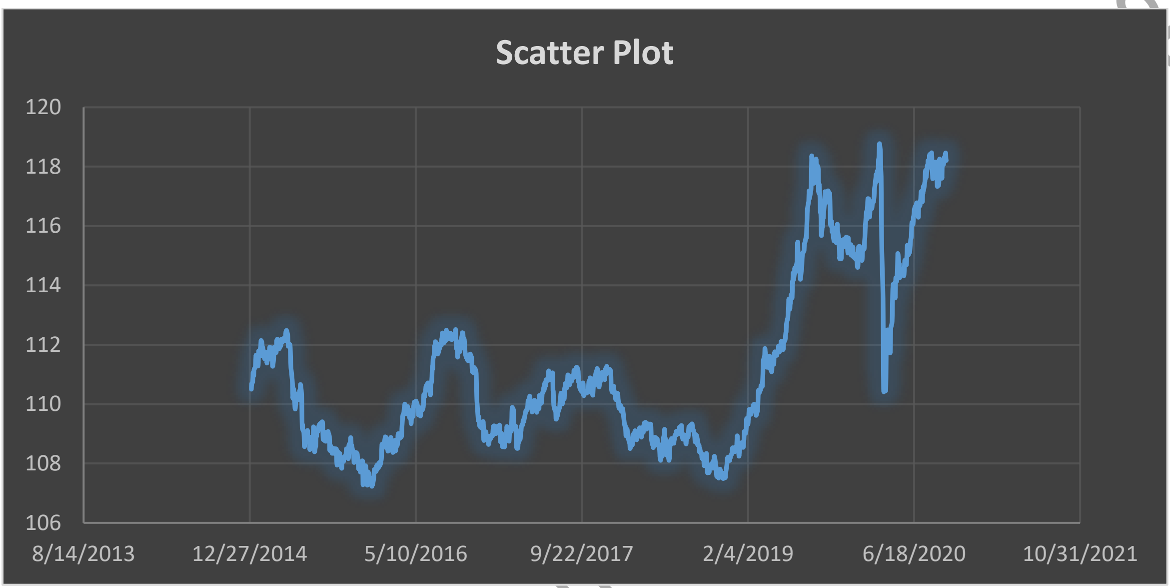


Figure 1: scatter plot

This scatter plot illustrates the price fluctuations of the Invesco QQQ Trust over time, specifically from August 2013 to October 2021. The horizontal axis represents this time period, while the vertical axis tracks the trust's price, ranging from approximately 106 to 120. Despite noticeable volatility, the plot reveals a general upward trend in the trust's value throughout this period. Significant price swings, particularly around 2020, likely reflect market reactions to major events like the COVID-19 pandemic. Overall, the chart suggests the Invesco QQQ Trust experienced growth, but also highlights its sensitivity to market volatility, emphasizing the importance of considering long-term investment strategies.

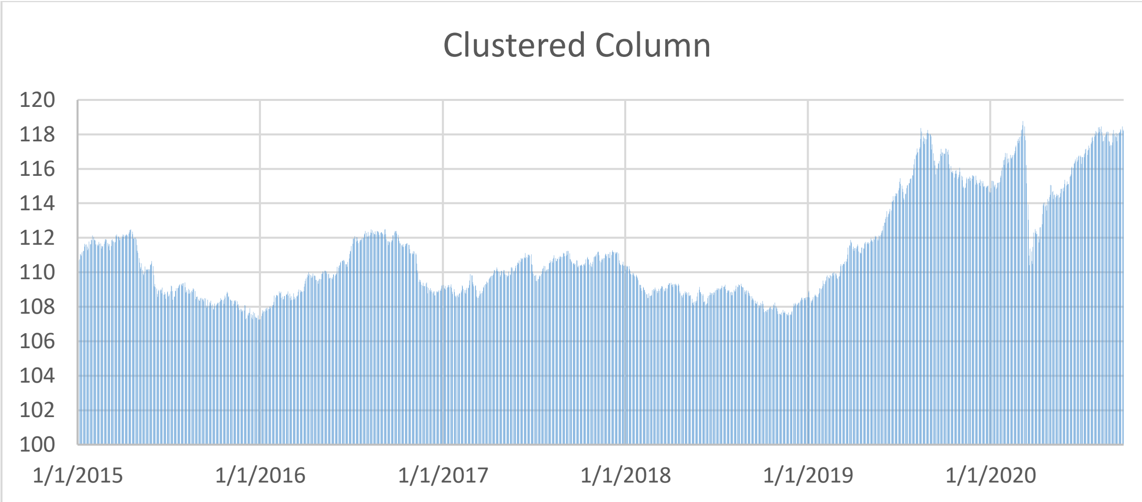


Figure 2: Clustered column chart

This clustered column chart displays a single data series over time, beginning January 1, 2015, and extending beyond January 1, 2020. The vertical axis, ranging from 100 to 120, suggests the data represents an index or normalized value. The chart shows a period of relative stability from 2015 to early 2019, with values fluctuating within a narrow range. However, starting in early 2019, a clear upward trend emerges, accompanied by increased volatility, as indicated by more frequent and significant fluctuations in the column heights. Without further context regarding the specific metric being measured, the chart indicates a notable shift in the data's behavior, showing a period of steady growth and increased instability in the later years.



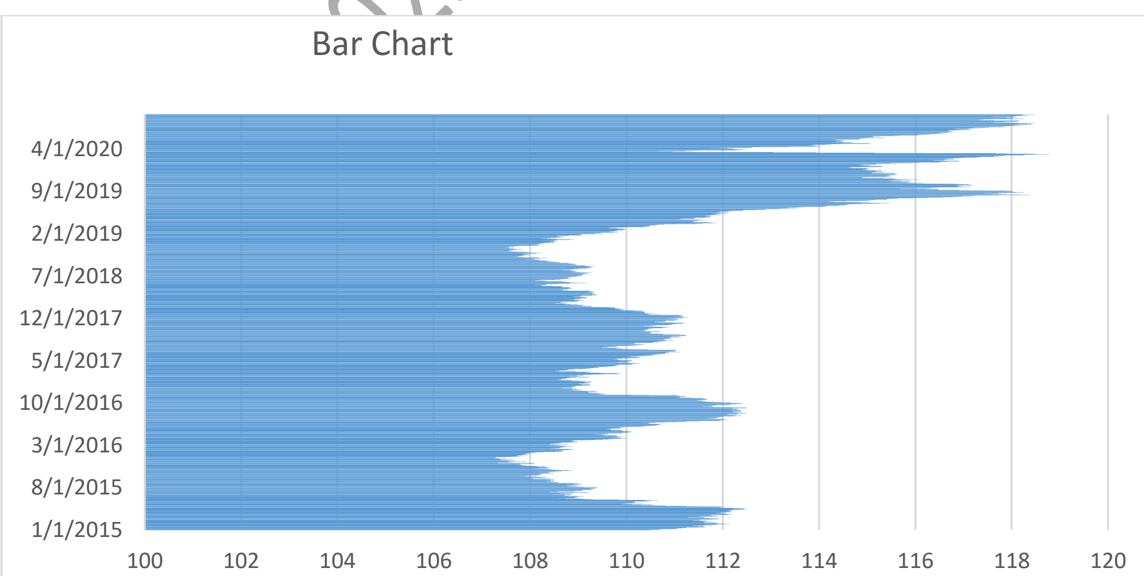


Figure 3: Horizontal bar chart

This horizontal bar chart illustrates the progression of a metric over time, from January 1, 2015, to April 1, 2020. The horizontal axis, ranging from 100 to 120, indicates the magnitude of the measured value, suggesting it's likely an index or normalized figure. From 2015 to early 2019, the chart shows a relatively stable period, with the bars indicating fluctuations within a narrow range between 106 and 112. However, beginning in early 2019, a clear upward trend emerges, with the bars progressively lengthening, signifying a substantial increase in the measured value. This upward trend is also accompanied by increased volatility, as the bars show more significant and frequent fluctuations, particularly towards the end of the time period. Without additional context regarding the specific metric being represented, the chart demonstrates a notable shift from a period of stability to one of growth and instability.

## 4.0 Conclusion

This horizontal bar chart visually represents the change of a specific metric over time, from January 1, 2015, to April 1, 2020. The chart uses horizontal bars, with the length of each bar corresponding to the magnitude of the measured value at each point in time. The horizontal axis, ranging from 100 to 120, suggests the data likely represents an index or normalized value, as the values are constrained within a relatively small range. From the beginning of the time period to early 2019, the bars are consistently short, indicating a period of relative stability. The measured value fluctuates within a narrow range, roughly between 106 and 112, with minimal variation. However, starting in February 2019, a noticeable upward trend begins. The bars progressively lengthen, signifying a substantial increase in the measured value, which continues until April 2020.

Furthermore, the latter part of the time period, particularly from late 2019 to April 2020, displays increased volatility. The bars show more frequent and significant changes in length, indicating more rapid fluctuations in the measured value. Overall, the chart illustrates a clear shift from a stable period to one of significant growth and increased volatility, suggesting a notable change in the underlying dynamics of the measured metric.



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