Unit Root Testing with Neural Network Nonlinearity: An Application to GDP Per Capita

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Abstract

This study investigates the existence of the unit root hypothesis in the GDP per capita of African countries. We apply the nonlinear unit root testing framework of the autoregressive neural network ADF (ANN-ADF) to the panel SUR Dickey-Fuller method. This method combines the distinct properties of individual time series with the wider panel structure, producing a more robust tool for empirical economic analysis. Various empirical applications with GDP per capita across several countries is carried out. Using GDP per capita data from several countries, we discover that most of them do not have unit roots. This implies mean-reverting behaviour and suggesting economic stability in the region. Consequently, result of the panel SUR Dickey-Fuller techniques outperform the traditional unit root testing methods.

This result does not only contribute to the understanding of economic growth in Africa but also highlight the importance of sophisticated econometric methods in capturing the complexities of real-world data.

Keyword: Unit root, GDP per capita, African countries, Neural network, SUR-ADF

1.0 Introduction

Unit root testing, a fundamental step in time series analysis and econometric modelling, is conducted to determine the stationarity level of a time series before further model estimation for data (Box and Jenkins, 1976). Unit root testing procedures are commonly employed in economics and finance (Choi, 2015). Unit roots are detected in nonstationary autoregressive (AR) or autoregressive moving average (ARMA) time series processes that can have no intercept term, intercept and/or trend. Thus, the testing procedure, founded on the Dickey-Fuller regression relies on an AR(1) process assumed to contain a unit root (nonstationarity).

In Time series analysis and econometric Modelling, a unit root testing is essential step performed to check the level of stationarity of a time series before estimating any model on it (Box and Jenkins, 1976). The unit root test checks if a time series has a unit root (stochastic trend) or has trend-stationary, meaning it revert around a deterministic trend. The traditional unit root tests, such as the Augmented Dickey–Fuller (ADF) test, the Philips-Perron (PP) test, and the Kwiatkowski–Phillips Schmidt–Shin (KPSS) test, have been widely used for this purpose due to their theoretical basis (Dickey & Fuller, 1979; Phillips & Perron, 1988; Kwiatkowski et al, 1992). However, these tests are based on linearity assumption which may not be true in practice.

In the case of developing countries, Economic and financial data may exhibit nonlinear characteristics and therefore could not efficiently be explained using the traditional linear tests. As an example, gross domestic product (GDP) per capita– an important economic performance indicator– could be affected by a variety of nonlinear factors such as abrupt policy changes, technological innovations or external disturbances (Kapetanios, Shin, & Snell, 2003). Indeed, in the presence of both types of nonlinearity, uncritical use of the linear unit root tests would give rise to misleading inferences about the persistence of shocks or the impact of permanent economic policies.

The recent advancements in the computational method helped the artificial neural networks (ANNs) to be used in the analysis of time series because of their ability to fit nonlinear relationships with flexibility without the imposition of strict parametric structures (Hornik, Stinchcombe, & White, 1989). While ANNs have been utilized for forecasting and volatility modelling (Zhang, Patuwo, & Hu, 1998; Tkacz, 2001), applications to formal unit root testing have been rather scarce. Indeed, the majority of the applications have been on the univariate series and there has been little work on the case of data panel, where comparisons across countries are to be made and a pooled inference is sought (Pesaran, 2007).

This study fills this gap by adapting new unit root tests which account for nonlinearity using neural network for both univariate and panel testing framework. In the first place, we explored a single-equation ANN-ADF test to account for nonlinear stationarity dynamics. Secondly, we extend the framework to a panel structure using Seemingly Unrelated Regression (SUR) to accommodate cross-sectional dependence (Zellner, 1962). In this paper, the we applied these methods to data on GDP per capita in 20 African countries which has not been properly explored in order to examine for mean-reversion in long-term economic growth patterns or if they are a driven by persistence trend changes alone.

This paper aimed to introduce neural network architecture into unit root testing. In this way, it provides a more flexible and better framework for evaluating the dynamic properties of macroeconomic indicators. The findings have practical implications for understanding economic resilience, policy sustainability, and the long-run effects of economic shocks in developing economies.

2.0 Material and Methods

This study examines annual time series data on GDP per capita for 20 African economies in current US dollars. The dataset is obtained from World Development Indicators (WDI) website (https://www.wdi.worldbank.org). The following Africa countries are considered; Algeria, Benin, Botswana, Cameroon, Congo, Egypt, Equatorial Guinea, Ethiopia, Gabon, Ghana, Kenya, Lesotho, Mali, Morocco, Namibia, Nigeria, South Africa, Tunisia, Uganda, and Zambia with each series spanning 1991 to 2023.

2.1 ADF test regression from Autoregressive model (AR(p))

The augmented Dickey-Fuller unit root test for an autoregressive model is determined by considering the autoregressive model of order p (AR(p))

$$X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \phi_3 X_{t-3} + \ldots + \phi_p X_{t-p} + \varepsilon_t \tag{1}$$

where X_t is the time series, μ is the constant term, $\phi_1, \phi_2, \ldots, \phi_p$ are the parameters of the autoregressive model and ε_t is the error term which is assumed to be a white noise process.

Then the ADF test Regression obtained from (1) is given as

$$\Delta X_t = \mu + \beta X_{t-1} + \sum_{j=1}^{p} \alpha_J \Delta X_{t-j} + \varepsilon_t \tag{2}$$

2.2 The ANN-form Nonlinearity

ANN model for a single hidden-layer model is defined as: `

$$x_t = \phi' w_t + \sum_{j=1}^{q} \left[\theta_j F\left(\gamma_j' w_t\right)\right] + \varepsilon_t \tag{3}$$

where x_t is the time series, $\phi' = \left(\phi_0, \phi_1, \ldots, \phi_j\right)$ are the parameters of the linear AR part of the model, $w_t = \left(1, x_{t-1}, x_{t-2}, \ldots, x_{t-j}\right)$, $\theta = \left(\theta_1, \theta_2, \ldots, \theta_q\right)'$ as the 'connector strength' parameters, $\gamma_j = \left(-c, \gamma_{11}, \ldots, \gamma_{1j}\right)'$ which is a $\left(j+1\right)\times 1$ vector of parameters of weights of the j^{th} hidden unit, ε_t is the error term. It has been theoretically shown that one hidden layer is enough to approximate any non-linear function (see Hornok et al, 1989, Udomboso and Saliu, 2016).

The Logistic Transfer Function used in (2) is given as

$$F(\gamma_j' w_t) = \{1 + \exp(-\gamma_j' w_t)\}^{-1} - \frac{1}{2} \tag{4}$$

Then, (3) can be expressed as an ADF regression

$$(1-L)x_t = \alpha + \rho x_{t-1} + \sum_{k=1}^{p} \delta_k (1-L)x_{t-k} + \sum_{j=1}^{q} \theta_j F(\gamma_j' w_t) + \varepsilon_t \tag{5}$$

where x_t is the time series, $\phi' = (\phi_0, \phi_{1,}, \dots \phi_p)$ are the parameters of the linear AR part of the model, $w_t = (1, x_{t-1}, x_{t-2}, \dots, x_{t-p})$, $\theta_j = (\theta_1, \theta_2, \dots \theta_q)'$ as the 'connector strength' parameters, $\gamma_j = (-c, \gamma_{11}, \dots \gamma_{1j})'$ which is a $(j+1) \times 1$ vector of parameters of weights of the j^{th} hidden unit, ε_t is the error term. By using the third-order Taylor series expansion on the logistic function, (5) results into:

$$\Delta x_t = \alpha + \rho x_{t-1} + \sum_{i=1}^{q}\sum_{j=i}^{q} m_{ij} w_{ti} w_{tj} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{ijl} w_{ti} w_{tj} w_{tl} + \sum_{k=1}^{p} \delta_k (1-L)x_{t-k} + \tilde{\varepsilon}_t \tag{6}$$

The null and alternative hypotheses for nonlinearity of the time series is

$$H_0: m_{ij} = m_{ijl} = 0$$

$$H_1: m_{ij} \neq m_{ijl} \neq 0$$

The rejection of the joint null hypothesis implies non-linearity of the time series otherwise there is linearity in the time series, that is the case of ADF.

2.3 The SUR-ANN-ADF test model

The Yaya et al (2021) is extended into the panel settings. Thus, the panel unit root test based on SUR system for ANN-ADF regression is then given as,

$$\Delta x_{i,t} = \alpha_i + \rho_i x_{i,t-1} + \sum_{i=1}^{q}\sum_{j=i}^{q} m_{i,ij} w_{ti} w_{tj} + \sum_{i=0}^{q}\sum_{j=i}^{q}\sum_{l=j}^{q} m_{i,ijl} w_{ti} w_{tj} w_{tl} \sum_{j=1}^{k_2} \delta_{i,j} \Delta x_{i,t-j} + \varepsilon_{i,t} \qquad t=1,..,T \quad (7)$$

One of the advantages of this unit root testing technique over the existing ones is the ability to identify the series that are stationary among the panel members with the inclusion of a neural network model. The null and alternative hypotheses can be tested separately for each of the members of the panel within the SUR framework. The hypothesis to be tested for each member of the panel is written as

$$
\begin{array}{lll}
H^1{}_0: \rho_1 = 0 & \text{vs} & H^1{}_1: \rho_1 < 0 \\
H^2{}_0: \rho_2 = 0 & \text{vs} & H^2{}_1: \rho_2 < 0 \\
\vdots & & \vdots \\
H^N{}_0: \rho_N = 0 & \text{vs} & H^N{}_1: \rho_N < 0
\end{array}
$$

3.0 Result and Discussion

Table 1: Data Summary for GDP per Capita

Country	Code	1991	2023	Min.	Max.
Algeria	DZA	1749.29	5260.21	1466.54	6164.64
Benin	BEN	375.29	1434.66	269.79	1434.66
Botswana	BWA	2855.95	7249.80	2832.81	7726.11
Cameroon	CMR	1005.30	1673.65	692.86	1673.65
Congo	COD	1110.97	2508.82	662.88	3753.86
Egypt	EGY	637.90	3512.58	637.90	4295.41
Gabon	GAB	229.55	7066.62	186.68	19849.72
Equatorial Guinea	GNQ	5349.45	8420.10	3705.82	10273.80
Ghana	GHA	416.78	2238.16	253.38	2422.09
Kenya	KEN	340.81	1949.89	226.52	2099.30
Lesotho	LSO	384.98	878.01	384.98	1265.86
Mali	MLI	298.57	897.45	214.36	897.45
Morocco	MAR	1282.58	3672.11	1217.43	3767.52
Namibia	NAM	2117.01	4742.78	1773.43	6017.18
Nigeria	NGA	609.37	1621.12	494.13	3200.95
Rwanda	SOM	255.37	1000.22	111.94	1000.22
South Africa	ZAF	3304.83	6253.16	2708.42	8737.04
Tunisia	TUN	1516.29	3895.39	1516.29	4398.64
Uganda	UGA	182.79	1014.21	151.98	1014.21
Zambia	ZMB	428.50	1369.123	353.83	1840.32

Currency is given in US dollar

The dataset is summarised in presented Table 1 which shows the GDP per capita in 1991 and 2023, with the minimum and maximum values in the sampled period across those countries. The least and highest GDP per capita in 1991 are Uganda and Equatorial Guinea, while the least and highest GDP per capita in 2023 are Lesotho and Equatorial Guinea respectively. Also, we find all countries have higher rates of GDP per capita in 2023 than in 1991.

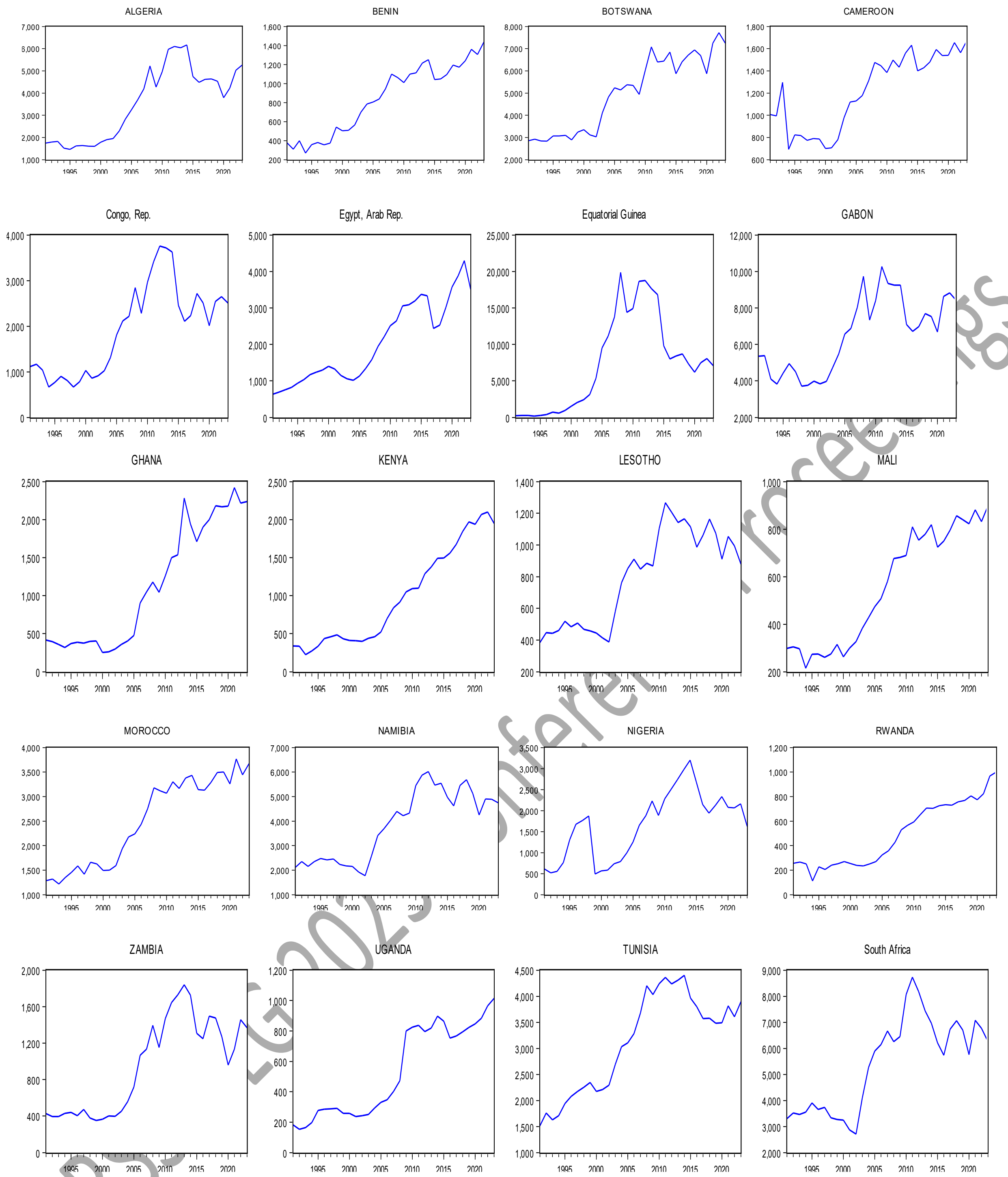

Figure 1: Time Plot of the GDP per capita data across the twenty Africa Countries

Plots of the twenty GDP per capita are given in Figure 1, where we observed episodic trends/cycles over the historic period. However, all the selected Africa countries' GDP per capita appear to be characterized by non-linearity. Thus, mean and variance structure of the series are time variant, and non-stationarity is suspected.

Table 2: Results of ADF and PP unit root test

	ADF			PP		
	None	Intercept only	Intercept and trend	None	Intercept only	Intercept and trend
Algeria	-0.9065[0]	-3.6502[8]	-3.5613[8]	-0.8647[3]	-0.8791[3]	-2.1208[4]
Angola	-1.1373[2]	-3.3256[1]	-3.5281[1]	-0.8887 [3]	-0. 8301 [3]	-1.3507 [2]
Benin	-0.7213[0]	-1.8644[0]	-2.1949[0]	-0.3335[9]	-2.3884[9]	-2.4569[9]
Botswana	0.3861[3]	-3.5953[1]	-3.6405[1]	0.6776[4]	-2.6213[1]	-2.5904[1]
Cameroon	-1.9035[1]	-1.9493[1]	-1.8192[1]	-2.3352[3]	-1.4333[3]	-0.8549[2]
Congo	1.4367[2]	-0.7598[3]	-3.1501[8]	-0.7237[0]	-0.7237[0]	-1.9477[1]
Egypt	-0.5110[1]	-2.7523[3]	-2.7877[3]	-0.5854[2]	-1.8317[2]	-1.7329[2]
Equatorial Guinea	2.1131[2]	-0.4586[2]	-1.4208[2]	2.2581[15]	-1.9258[5]	-2.5855[5]
Ghana	-0.5978[2]	-1.0392[2]	-2.8056[2]	-0.5725[3]	-1.4885[1]	-2.7169[8]
Kenya	0.0969[1]	-1.7557[1]	-2.5037[1]	1.3825[3]	0.3736[3]	-0.6322[2]
Lesotho	1.5532[8]	-0.5519[8]	-1.3397[8]	0.7548[11]	-2.00913]	-2.4220[5]
Mali	1.1756[1]	2.1563[5]	0.9899[5]	0.8928[1]	-0.6988[3]	-2.0046[3]
Morocco	-1.4109[0]	-1.0508[0]	-1.5945[0]	-1.5214[2]	-0.9979[2]	-1.6038[2]
Namibia	-0.1409[0]	-3.6410[1]	-3.8617[1]	-0.0508[9]	-2.2217[6]	-2.1012[10]
Nigeria	0.5389[7]	-3.1197[3]	-3.2255[3]	-0.5214[1]	-1.3979[0]	-1.6279[1]
Rwanda	0.0842[3]	-1.9473[4]	-1.5032[4]	0.7775[11]	-1.4033[3]	-1.5206[5]
South Africa	-0.8932[7]	-1.4444[7]	-1.9346[7]	1.8581[3]	0.6954[2]	-1.5881[1]
Tunisia	-1.8133[0]	2.1104[2]	-1.8139[0]	-0.3643[17]	-2.3938[4]	-2.3131[5]
Uganda	-0.3467[0]	-2.3784[0]	-2.5637[1]	-0.5830[4]	-2.9585[3]	-2.9620[3]
Zambia	-2.3253[2]	-4.1771[1]	-1.4898[2]	-4.4360[31]	-1.1019[18]	-1.8825[10]

Note, t statistics for both ADF and PP test are reported with corresponding optimal lag length based on Akaike information criterion (as in the case of the ADF test), and the optimal Bartlett kernel bandwidth number are reported in parentheses (as in the case of PP test). Level of significance is 5% while none of the t statistics is significant, thus non-rejection of the null of unit root. As part of the baseline check, both the ADF and PP unit root tests are performed, and the findings from these tests are provided in Table 2. These unit root tests have the null hypothesis that the GDP per capital is nonstationary I(1) against the alternative that the rate is stationary I(0), and it is found that the unit root null is unrejected.

Table 8: Results of Unconditional Correlations for GDP per capita

	Algeria	Benin	Botswana	Cameroon	Congo	Egypt	Guinea	Gabon	Ghana	Kenya	Lesotho	Mali	Morocco	Namibia	Nigeria	Rwanda	SAfrica	Tunisia	Uganda	Zambia
Benin	**0.915****																			
Botswana	**0.922****	0.964**																		
Cameroon	0.896**	0.925**	0.912**																	
Congo	0.981**	0.868**	0.880**	0.872**																
Egypt	0.814**	0.899**	0.889**	0.822**	0.770**															
Guinea	0.908**	0.747**	0.729**	0.722**	0.918**	0.565**														
Gabon	0.955**	0.868**	0.884**	0.880**	0.961**	0.745**	0.902**													
Ghana	0.841**	0.919**	0.923**	0.904**	0.807**	0.938**	0.582**	0.789**												
Kenya	0.780**	0.911	0.909**	0.863**	0.730**	0.950**	0.497**	0.724**	0.977											
Lesotho	0.947**	0.893	0.940**	0.877**	0.936**	0.804**	0.841**	0.901**	0.851	0.805										
Mali	0.924**	0.978**	0.971**	0.952**	0.884**	0.914**	0.729**	0.880**	0.959	0.943	0.923									
Morocco	0.931**	0.983**	0.969**	0.927**	0.886**	0.901**	0.768**	0.889**	0.937	0.923	0.931	0.988								
Namibia	0.958**	0.898**	0.945**	0.894**	0.945**	0.812**	0.836**	0.916**	0.871	0.825	0.989	0.936	0.940							
Nigeria	0.854**	0.746**	0.772**	0.739**	0.851**	0.764**	0.778**	0.814	0.776	0.719	0.868	0.791	0.819	0.863						

Rwanda	0.860**	0.938**	0.936**	0.910**	0.810**	0.957**	0.596**	0.803	0.969	0.975	0.842	0.967	0.942	0.869	0.744				
SAfrica	0.940**	0.866**	0.912**	0.864**	0.940**	0.739**	0.882**	0.939	0.789	0.737	0.971	0.888	0.903	0.975	0.827	0.794			
Tunisia	0.959**	0.920**	0.899**	0.844**	0.925**	0.797**	0.914**	0.909	0.791	0.751	0.934	0.902	0.937	0.927	0.848	0.813	0.927		
Uganda	0.882**	0.924**	0.922**	0.878**	0.841**	0.954**	0.660**	0.813	0.937	0.937	0.881	0.955	0.946	0.900	0.818	0.965	0.840	0.865	
Zambia	0.985**	0.886**	0.911**	0.880**	0.973**	0.807**	0.883**	0.937	0.859	0.791	0.950	0.915	0.919	0.970	0.877	0.859	0.939	0.931	0.885

To determine panel variable dependencies for SUR modelling, a Pearson moment correlation analysis was performed (Table 3). Most pairing have substantial correlations at the 1% and 5% levels.

Table 4 Results of GDP per Capita in Africa for Unit root testing

Country	ADF	ANN-ADF	SUR-ADF	SUR-ANN-ADF
Algeria	-0.976	**-5.099****	0.101	**0.860****
Benin	-0.377	**-6.483****	0.193	**-1.571****
Botswana	-0.748	**-5.678****	-0.320	**0.914****
Cameroon	-0.523	**-3.530****	-0.198	**2.252****
Congo	-1.249	**-8.965****	0.330	**5.649****
Egypt	-1.191	**-3.010****	-2.474	**-11.577****
Equatorial Guinea	**-22.812****	**-23.401****	0.503	**5.986****
Gabon	-1.157	**-5.711****	-0.975	**3.248****
Ghana	-0.135	**-4.370****	-0.458	**0.722****
Kenya	-0.169	**-2.050****	0.077	**12.154****
Lesotho	-1.409	**-5.804****	-0.462	**2.758****
Mali	-0.017	**3.248****	-0.043	**6.283****
Morocco	-0.642	**-4.448****	-0.606	**-3.349****
Namibia	-1.238	**-5.300****	-0.825	0.072
Nigeria	-1.998	**-2.376****	-0.401	**3.176****
Rwanda	1.027	-0.351	0.422	**-4.062****
South Africa	-3.618	**-2.476****	-1.398	**-8.045****
Tunisia	-1.279	**-4.166****	-0.643	**-2.210****
Uganda	-0.563	**-5.402****	-0.714	**6.675****
Zambia	-0.695	**-5.509****	1.552	**4.350****

Note: 5% critical values are Bootstrap simulated as in Table 1 and Table 3. These are -6.0373 (ADF), -1.8059 (ARNN-ADF), -6.6542 (SUR-ADF), and -0.4995 (SUR-ARNN-ADF).

Table 4 shows the findings for ADF, ANN-ADF, SUR- ADF and SUR-ANN-ADF. The ADF test in column one indicates the GDP hypothesis is rejected in only Equatorial Guinea (see the result in Table 4). The ANN-ADF test result shows 19 out of 20 reject the unit root in the GDP per capita. The SUR versions of the ADF and ANN-ADF tests was added to compare them to the univariate versions. SUR-ANN-ADF rejected the GDP hypothesis (GPH) in almost all country except Namibia. Thus, the ANN-ADF and SUR-ANN-ADF tests demonstrated better powerful than the tests without the incorporation of ANN and their inclusion addressed the gap between the Neural network-based tests and their non- Neural network counterparts found in the single-equation tests.

The ANN-ADF test outperforms the ADF test, highlighting the importance of accounting for nonlinear dynamics and using machine learning techniques in time series analysis. The SUR-ANN-ADF test outperforms the SUR-ADF test by addressing its inconsistent findings. The improvement illustrates the significance of combining cross-sectional dependency and nonlinear approaches. When ADF and SUR-ADF approaches are extended, such as ANN-ADF and SUR-ANN-ADF, the majority of countries show stationarity. Traditional ADF tests may perform poorly in determining stationarity in complex or nonlinear data. Countries like Equatorial Guinea, South Africa, and Egypt have strong evidence of GDP per capita stationarity, which suggests long-term economic stability. Others, like as Rwanda and Zambia, show persistence in non-stationary behaviour, indicating that external shocks or structural breaks influence growth trends.

4.0 Conclusion

This study examines the unit root properties of the GDP per capita in twenty (20) African countries from 1991 to 2023, to ascertain whether the series exhibit mean-reverting behavior. We employed a battery of unit root tests, which include the convectional univariate and proposed SUR panel unit root test namely SUR-ANN-ADF. Having employed the traditional ADF unit root test, we found that the decisions based only on traditional ADF might be misleading as we found in more 95% of the cases, where we failed to reject the hypothesis. That is ADF test rejects the unit root hypothesis only for Equatorial Guinea. Given the possibility of structural changes in the time dynamics of GDP per capita, which could have resulted in various types of non-linearities, the ADF unit root test is unable to adequately ascertain the stationarity properties of GDP per capita in the African nations under investigation. In light of the aforementioned, the study too into account additional three unit root testing frameworks. These include ANN-ADF, SUR-ADF and SUR-ANN-ADF which account for 95%, 0% and 95% cases of the rejections of the unit root hypothesis of GDP per capita, respectively. Thus, we may conclude that the GDP per capita in the majority of African nations appears to be mean reverting for the time period examined. Nonetheless, on the basis of the most preferred unit root testing frameworks, we find that SUR-ANN-ADF test to be the most reliable among the contending models, outperforming all others in majority of the African countries studied. Conclusively, the mean reversion holds in nineteen of twenty African countries studied and they include Algeria, Benin, Botswana, Cameroon, Congo, Egypt, Equatorial Guinea, Ethiopia, Gabon, Ghana, Kenya, Lesotho, Mali, Morocco, Nigeria, South Africa, Tunisia, Uganda, and Zambia. GDP per capita in these countries revert back to their mean levels and by implication temporary shock recover to its long-run growth path. This has important implications for economic forecasting, the effectiveness of policies, and regional convergence, and it encourages the use of long-term planning strategies aimed at sustainable development.

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