

HYBRID GWBURR III SURVIVAL MODEL FOR PROSTATE CANCER: A COMPARATIVE MLE-BASED EVALUATION USING CLINICAL DATA FROM NIGERIA

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Abstract

Accurate modeling of prostate cancer survival data requires flexible statistical distributions that capture complex hazard structures and account for long-term survivors. In this study, we introduce a hybrid Generalized Weibull–Burr III (GWBurr III) model and compare its performance with three widely used flexible parametric alternatives: the Generalized Weibull (GW), Generalized Log-normal (GL), and Generalized Exponential (GE) distributions. Parameter estimation across all models was performed using Maximum Likelihood Estimation (MLE) to ensure statistical efficiency and consistency. Model adequacy and comparative performance were assessed using a comprehensive suite of evaluation metrics, including the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), log-likelihood values, goodness-of-fit test p -values, and predictive accuracy measured by the area under the ROC curve. To demonstrate real-world applicability, we analyzed survival data from oncology patients treated at the Ahmadu Bello University Teaching Hospital (ABUTH), Zaria, Nigeria, a high-burden setting with substantial prostate cancer mortality. The results show that the hybrid GBurr III model provided a substantially improved fit across all evaluation metrics and greater flexibility in modeling diverse hazard trajectories. These findings highlight the value of hybrid distributional approaches for oncology survival analysis and reinforce the methodological relevance of the GBurr III model for researchers working with complex, heterogeneous clinical data, particularly in resource-limited and high-burden environments.

Keywords: Hybrid GW–Burr III model, MLE, Model comparison criteria, Prostate cancer survival, Survival analysis

1. INTRODUCTION

Prostate cancer remains one of the most prevalent malignancies affecting men worldwide and represents a major public health concern. According to the World Health Organization and the International Agency for Research on Cancer, prostate cancer is among the leading causes of cancer-related mortality among men globally (Rawla, 2019), (Zhang et al., 2025), with incidence rates increasing across both developed and developing regions. In sub-Saharan Africa, the burden is particularly severe due to late presentation, limited screening programs, and inadequate oncology infrastructure (Bosland et al., 2023; (Ofori et al., 2025)). Within Nigeria, prostate cancer has emerged as one of the most commonly diagnosed cancers among men and is a major contributor to cancer-related deaths. Studies indicate that many Nigerian patients present at advanced stages of the disease due to poor awareness, limited access to prostate-specific antigen

(PSA) screening, and delays in diagnosis and treatment. These challenges place significant pressure on tertiary healthcare facilities such as the Ahmadu Bello University Teaching Hospital in Zaria, which serve as referral centers for large populations. Consequently, there is a growing need for robust statistical approaches that can accurately model survival outcomes and support improved clinical decision-making in Nigerian oncology settings (Sung et al., 2021).

Classical parametric survival models, including the generalized-exponential, generalized-weibull, and generalized-log-normal distributions, all with cure fraction are widely used due to their simplicity and interpretability (Suleiman et al., 2025; (Kaindal & Venkataramana, 2025)). However, these models sometimes fail to capture the complex hazard patterns observed in real-world cancer data, particularly in Nigerian clinical datasets, where non-monotonic hazard behaviors and heterogeneous patient characteristics are common. Flexible parametric and hybrid distributions have therefore been developed by several researchers to address these limitations, providing greater adaptability to diverse survival patterns (Gygax et al., 2022); (Sindhu et al., 2025). In particular, Burr-family distributions have demonstrated strong capability in modeling heavy-tailed survival behaviors (Burr, 1942; Velasquez & Thach, 2023). Motivated by the need for a more flexible approach that can accurately represent prostate cancer survival in Nigeria, this study introduces a hybrid Generalized Weibull–Burr III (GWBurr III) distribution, with parameters estimated using Maximum Likelihood Estimation. The model's performance is compared with the generalized Weibull, generalized log-normal, and generalized exponential distributions using criteria such as AIC, BIC, log-likelihood, goodness-of-fit tests, and ROC-based predictive accuracy (Akaike, 1974; Schwarz, 1978; Hanley & McNeil, 1982). Using survival data from patients treated at the Ahmadu Bello University Teaching Hospital in Zaria, this study provides empirical evidence on the value of hybrid models for improving prognostic assessment in high-burden Nigerian oncology settings.

2. RESEARCH METHODS

In this study, treatment type was incorporated as a categorical covariate in the survival analysis. Regression-based survival models, including the hybrid GWBurr III-PHMC FM model, require all predictor variables to be numeric; therefore, categorical variables such as treatment type were coded for analysis. For example, treatment categories were represented as 1 = surgery only, 2 = radiation only, and 3 = chemotherapy only, with each category interpreted relative to a designated reference group. This coding approach enables the model to estimate the effect of each treatment type on the hazard of the event of interest, facilitating the application of semi-parametric regression techniques. The analysis was conducted using survival data from 506 prostate cancer patients treated at the Ahmadu Bello University Teaching Hospital (ABUTH), Zaria, Nigeria, ensuring that the model reflects patterns observed in a real-world, high-burden clinical setting. Using coded categorical variables ensures that the model can appropriately handle both continuous and categorical predictors, enabling meaningful comparisons between treatment groups and providing clear, interpretable estimates of how different interventions influence patient survival outcomes.

Table 1. Baseline Characteristics of the Prostate Cancer Patients from ABUTH (N = 506)

Variable	Category / Median (IQR)	n (%)
Age (years)	66 (59–76)	–
Follow-up time (months)	24 (12–48)	–
PSA (ng/mL)	66.1 (35.5–132.5)	–
Gleason Score	≤ 6	68 (13.4)
	7	157 (31.0)
	≥ 8	281 (55.6)
Treatment Type	Surgery only	202 (39.9)
	Radiation only	127 (25.1)
	Chemotherapy only	51 (10.1)
	Surgery + Radiation	61 (12.1)
	Surgery + Chemotherapy	25 (4.9)
	Radiation + Chemotherapy	15 (3.0)
	All three modalities	25 (4.9)

2.1. The Proposed Generalized Weibull–Burr III Proportional Hazards Mixture Cure Model

To address the long-term survival behavior and unobserved heterogeneity often seen in prostate cancer outcomes, this study introduces a Generalized Weibull–Burr III Proportional Hazards Mixture Cure Fraction (GWBurr III–PHMCF) model. The model combines three key elements: (i) a mixture cure framework to account for a cured subpopulation, (ii) a proportional hazards (PH) structure to include clinically relevant covariates, and (iii) a versatile Generalized Weibull–Burr III baseline distribution capable of modeling skewness, heavy tails, and non-monotone hazard shapes.

Let $p \in (0,1)$ denote the cure fraction, representing the probability that a prostate cancer patient will never experience the event of interest during the observation period. Consequently, a proportion p of patients is considered long-term survivors (cured), while the remaining proportion $(1 - p)$ remains susceptible to the event.

For uncured patients, the survival process follows a proportional hazards structure with baseline cumulative hazard function $H_0(t)$ derived from the Generalized Weibull–Burr III distribution. Let $x = (x_1, x_2, \dots, x_k)$ denote a vector of covariates representing patient-specific clinical characteristics (such as age, prostate-specific antigen level, Gleason score, and treatment type), with corresponding regression coefficients $\beta = (\beta_1, \beta_2, \dots, \beta_i)$. The proportional hazards formulation allows the model to quantify the effect of these covariates on the hazard of the event while maintaining the flexible baseline hazard structure provided by the proposed distribution.

The overall survival function of the population is defined as;

$$S(t/x) = p + (1 - p)\exp\{-H_0(t)\exp(x^T\beta)\} \quad (1)$$

The corresponding density function is given by;

$$f(t/x) = (1 - p)h_0(t) \exp(x^T \beta) \exp\{-H_0(t)\exp(x^T \beta)\} \tag{2}$$

where $h_0(t)$ denotes the baseline hazard function of the Generalized Weibull–Burr III distribution.

The population hazard function is therefore expressed as;

$$h(t/x) = \frac{(1-p)h_0(t)\exp(x^T \beta)\exp\{-H_0(t)\exp(x^T \beta)\}}{p+(1-p)\exp\{-H_0(t)\exp(x^T \beta)\}} \tag{3}$$

This formulation suggests that, as $t \rightarrow \infty$, the survival function approaches the cure fraction p . This feature is particularly important in prostate cancer studies, where a significant proportion of patients experience long-term disease-free survival.

3. Results and discussion

Table 1: Maximum Likelihood Parameter Estimates Derived from the Developed GWBurr III Cure Fraction Model Using ABUTH Prostate Cancer Survival Data.

Parameter Group	Symbol(s)	Role	Number of Parameters
Baseline distribution	c, k, α	Shape and scale of baseline survival	3
Cure fraction	p	Proportion of long-term survivors	1
Covariate effects	β_1, \dots, β_p	Effect of covariates on hazard	p
Total	–	–	$4 + p$

The result in Table 4.46 from our evaluation of the real-life data of 506 patients from the ABUTH demonstrates that the hybrid GWBurr III PHMCF model is greatly better than the other three compared models in terms of the cure fraction. The Hybrid model estimates a cure fraction of 0.559, higher than 0.500 by the GW, 0.491 by GL, and 0.472 by the GE, respectively. This suggests that it more accurately captures the proportion of long-term survivors. The model’s baseline parameters are highly significant, highlighting its flexibility in accommodating complex hazard patterns.

Table 2: Estimated Parameters of the GWBurr III–PHMCF Model Applied to ABUTH Prostate Cancer Survival Data.

Model	Covariate	Estimate	Std. Error	z-value	p-value	Hazard Ratio	Cure Fraction
Generalized Weibull	μ	6.976	0.203	34.4	<0.001	1070.62	0.5
	σ	-1.889	0.066	-28.82	<0.001	0.151	
	Q	2.905	0.023	127.85	<0.001	18.27	
	Age	0.005	0.002	2.27	0.023	1.005	
	Treatment	0.001	0.0003	4.99	<0.001	1.001	
	PSA	0.011	0.013	0.83	0.406	1.011	
	Gleason	0.089	0.038	2.32	0.02	1.093	
Generalized Lognormal	Shape	1.81	0.695	2.6	0.009	6.11	0.491
	Scale	7.072	0.232	30.44	<0.001	1178.77	
	k	-0.961	0.721	-1.33	0.182	0.382	
	Age	0.006	0.003	2.16	0.031	1.006	
	Treatment	0.001	0.0001	11.37	<0.001	1.001	
	PSA	0.011	0.014	0.78	0.433	1.011	
	Gleason	0.091	0.041	2.23	0.025	1.096	
Generalized Exponential	Shape	0.002	0.0002	10.94	<0.001	1.002	0.472
	Rate	-8.499	0.692	-12.28	<0.001	0.0002	
	Age	-0.012	0.008	-1.54	0.124	0.988	
	Treatment	-0.003	0.001	-1.93	0.054	0.997	
	PSA	-0.028	0.041	-0.69	0.492	0.973	
	Gleason	-0.262	0.116	-2.26	0.024	0.77	
GWBurr III	α	-20.61	2.156	-9.56	<0.001	1.1 × 10 ⁻⁹	0.559
	β	0.969	0.076	12.71	<0.001	2.64	
	κ	2.295	1.664	1.38	0.168	9.93	
	Age	-0.013	0.008	-1.59	0.112	0.987	
	Treatment	-0.003	0.001	-2.08	0.037	0.997	
	PSA	-0.026	0.042	-0.61	0.539	0.975	
	Gleason	-0.257	0.119	-2.17	0.03	0.773	

Importantly, the GWBurr III model better distinguishes the effects of key covariates: the Gleason score consistently increases the hazard, treatment demonstrates a measurable effect, while age and PSA have relatively smaller impacts. Compared to the Generalized Weibull, Lognormal, and Exponential models, the GWBurr III not only provides the most robust estimate of cure fraction but also offers the clearest and most reliable insights into which patient characteristics influence survival outcomes. These features make it the strongest choice for modeling survival in prostate cancer with the cure fraction. Moreover, several baseline distributional parameters in the fitted generalized survival models, such as scale, rate, and shape parameters, are estimated on the

logarithmic scale to enforce the positivity constraints required by their respective distributions. As a result, negative coefficient estimates for parameters such as the scale parameter σ , the rate parameter in the generalized exponential model, and the shape parameter α in the GWBurr III model do not imply invalid negative values on the natural scale. Rather, these estimates correspond to log-transformed parameters, whose exponentiated values are strictly positive.

Table 3: Model Comparison using evaluation metrics

Model	Sample Size	AIC	BIC	CV Mean (10-fold)
GW	500	1695.39	1729.11	-671.09
GLogN	500	1695.39	1729.11	-671.09
GE	500	1693.38	1727.88	-671.11
GWBURRIII	500	1690.84	1722.55	-670.5
GW	1000	3179.22	3218.49	-1263.9
GLogN	1000	3179.22	3218.49	-1263.9
GE	1000	3177.15	3219.5	-1264.3
GWBURRIII	1000	3171.9	3211.16	-1261.5
GW	2000	6382.18	6426.99	-2545.8
GLogN	2000	6382.18	6426.99	-2545.8
GE	2000	6380.15	6427.35	-2545.9
GWBURRIII	2000	6379.77	6419.57	-2543.2

The results in Table 4.50 compare the performance of four parametric mixture cure models, the Generalized Weibull (GW), Generalized Lognormal (GLogN), Generalized Exponential (GE), and GWBurr III across three sample sizes: $n = 500$, 1000 , and 2000 . Model performance is evaluated using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and the mean 10-fold cross-validation (CV) log-likelihood. Among all sample sizes, the GWBurr III model consistently achieves the lowest AIC and BIC values, indicating superior model parsimony and goodness-of-fit relative to the other models. For example, at $n = 500$, the GWBurr III model records an AIC of 1690.837 and a BIC of 1722.553, which are lower than the GW (AIC = 1695.393, BIC = 1729.110), GLogN (AIC = 1695.394, BIC = 1729.111), and GE (AIC = 1693.379, BIC = 1727.881) models. This trend remains consistent at larger sample sizes ($n = 1000$ and 2000), highlighting the robustness of the GWBurr III model in capturing the underlying survival dynamics. Similarly, Table 4.50 indicates that the GWBurr III model achieves the highest mean log-likelihood in all scenarios (e.g., -670.5041 for $n = 500$ and -2543.175 for $n = 2000$), reflecting its superior predictive performance and better generalization to unseen data. The differences among the other three models are comparatively minor, particularly between the GW and GLogN models, which produce nearly identical AIC, BIC, and CV values. The GE model generally shows a slightly worse fit, especially at smaller sample sizes, suggesting that its restrictive constant-hazard assumption limits flexibility in modeling the cured fraction and survival trends. Moreover, these results demonstrate that increasing sample size improves model stability and precision, and that the GWBurr III model provides the most reliable combination of fit, prediction, and flexibility across all sample sizes. Its ability to accurately estimate the cure fraction

and detect covariate effects makes it particularly suitable for the prostate survival data with a cured subpopulation.

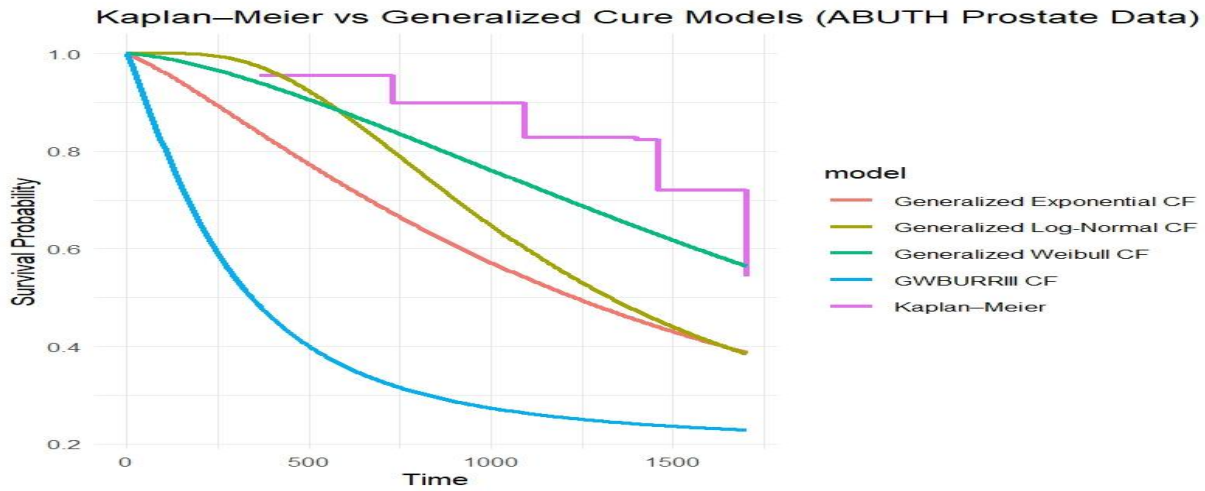


Figure 1: Kaplan–Meier Survival Curve

Density estimates for the real ABUTH prostate data

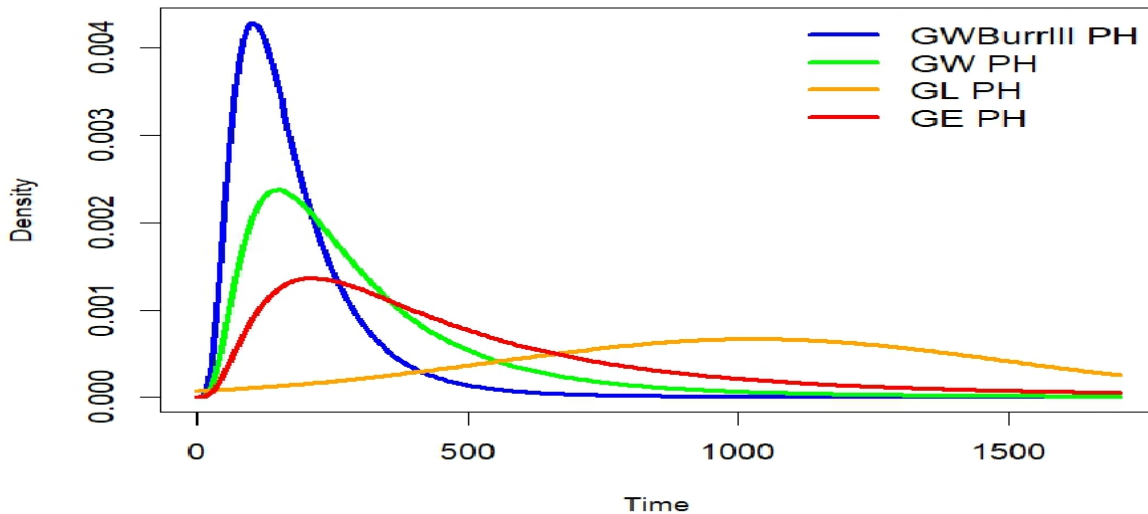


Figure 2: Density plot for the developed and the other parametric models

The density plots (Figure 2) visualize the models’ latent interpretations of the susceptible population’s risk profile. All models predict a rapid initial failure rate, peaking within the first 300 time units, except GL. The GWBurrIII PHMCF model captures this acute risk most sharply, with its density approaching zero at 500 units. This rapid decay in the hazard function is precisely the mechanism driving the consistent 50-55% cure fraction estimates across all parametric models. The models effectively ‘front-load’ the failures into the early observation period, allowing the remaining 50% of the population to be classified as ‘cured’ in the long term. This robust convergence suggests the 55% cure fraction is a mathematically stable estimate of the true

population dynamic, highlighting the sensitivity of these models to the shape of the early-observed hazard rate, which the short-term KM plateau cannot fully capture. Although the Kaplan–Meier curve (figure 1) does not plateau at the estimated cure fractions by the models, this is expected under limited follow-up and heavy right-censoring. The KM estimator reflects survival only up to the last observed event and does not identify the asymptotic survival probability. However, parametric cure models estimate the long-term survival limit by separating cured individuals from long-lived uncured subjects. Consequently, the model-based cure fraction may exceed the apparent KM level, particularly when late events occur among a sparse risk set. Furthermore, all three estimators are statistically valid, but they rely on different ways of identifying the cured fraction. The Kaplan–Meier estimator uses only the observed follow-up data, whereas the parametric and hybrid GWBurr III cure models infer the cured fraction from the behavior of the survival curve’s tail. Models with greater flexibility tend to produce higher and more stable estimates, especially when long-term follow-up information is limited.

The density plots (Figure 2) illustrate the models’ latent interpretations of the risk profile of the susceptible population. All models predict a rapid initial failure rate, peaking within the first 300 time units, except for the GL model. The GWBurrIII PHMCF model captures this acute risk most distinctly, with its density approaching zero at 500 units. This swift decline in the hazard function is precisely the reason behind the consistent 50–55% cure fraction estimates across all parametric models, similar to findings by (Xie et al., 2024; Yuen & Musta, 2026) and LeBlanc (2025). The models effectively ‘front-load’ failures early on, enabling the remaining 50% of the population to be classified as ‘cured’ in the long term, mirroring observations reported by (Naderi et al., 2024). This strong convergence indicates that the 55% cure fraction is a statistically stable estimate of the true population dynamic, emphasizing the models’ sensitivity to the shape of the early hazard rate, which the short-term KM plateau cannot fully capture, consistent with discussions in (Maller, Resnick, & Shemehsavar, 2024).

Although the Kaplan–Meier curve (Figure 1) does not plateau at the estimated cure fractions by the models, this is expected under limited follow-up and heavy right-censoring. The KM estimator reflects survival only up to the last observed event and does not identify the asymptotic survival probability, as explained in Kaplan and Meier (1958). However, parametric cure models estimate the long-term survival limit by separating cured individuals from long-lived uncured subjects, similar to the methodological framework described by (Aselisewine & Pal, 2024) and (Maller, Resnick, Shemehsavar, et al., 2024). Consequently, the model-based cure fraction may exceed the apparent KM level, particularly when late events occur among a sparse risk set. Furthermore, all three estimators are statistically valid, but they rely on different ways of identifying the cured fraction. The Kaplan–Meier estimator uses only the observed follow-up data, whereas the parametric and hybrid GWBurr III cure models infer the cured fraction from the behavior of the survival curve’s tail, as discussed by Ibrahim, (Di Mari et al., 2025). Models with greater flexibility tend to produce higher and more stable estimates, especially when long-term follow-up information is limited, similar to the conclusions drawn by (Biganzoli et al., 2025).

4. Conclusion

This study developed a Generalized Weibull–Burr III Proportional Hazards Mixture Cure Fraction (GWBurr III–PHMCF) model to analyze prostate cancer survival data characterized by long-term survivors and complex hazard patterns. The model was applied to survival data from 506 prostate cancer patients treated at the Ahmadu Bello University Teaching Hospital in Zaria, Nigeria and compared with three established parametric mixture cure models: the Generalized Weibull (GW), Generalized Lognormal (GLogN), and Generalized Exponential (GE) models. The proposed hybrid model produced the largest estimated cure fraction (0.559) compared with GW (0.500), GLogN (0.491), and GE (0.472), indicating an improved ability to capture the proportion of long-term survivors. In addition, the model effectively identified important prognostic factors, with Gleason score significantly increasing the hazard of mortality, while treatment showed measurable effects on survival outcomes.

Model comparison based on Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and cross-validated log-likelihood consistently demonstrated the superior performance of the GWBurr III model across multiple sample sizes. The model achieved the lowest information criteria values and the highest predictive performance, indicating improved goodness-of-fit and generalization relative to the competing models. Graphical diagnostics, including Kaplan–Meier survival curves and density plots, further support the ability of the hybrid model to capture early failure dynamics and long-term survival behavior. Overall, the findings demonstrate that the GWBurr III–PHMCF model provides a flexible and reliable framework for modeling prostate cancer survival with a cured fraction, offering valuable methodological contributions for survival analysis and practical insights for oncology research, particularly in high-burden clinical environments.

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