Evaluating the Black-Scholes Model for Pricing Nigerian Exchange Call Options: Addressing Cost of Carry Bias

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Abstract

This study examines the Black-Scholes model and its limitations in accurately pricing call options on five Nigerian stocks listed on the Nigerian Exchange. The model shows systematic pricing errors, with calculated prices differing from market prices. Observations were made that future prices are traded at a discount to spot prices, causing a negative cost of carry bias. To address this, we replace the spot price (S) in the Black-Scholes model with the discounted value of the future price (DVFP), as suggested by Black. Our results show that the original Black-Scholes model produces significant pricing errors, but the modified model using DVFP shows improved accuracy.

Key Words: Black-Scholes' Model, Cost of Carry, Discounted Value of the Future Price

I. **Introduction**

The Black-Scholes model, introduced by Fischer Black and Myron Scholes in 1973, has been the basis in option pricing theory, transforming the way financial markets operate. Its widespread adoption and application nevertheless, the model's assumptions and limitations have been subject to extensive examination, particularly in emerging markets like Nigeria.

This study evaluates the Black-Scholes model performance in predicting 2437 call option prices written on underlying stocks of five companies on the Nigerian Exchange (NGX). from 1st of May 2022 to 30th of April 2025. We will examine the model's pricing accuracy and investigate the cost of carry bias by comparing stock futures prices to spot prices. We will also explore an alternative approach by using the discounted value of future prices instead of spot prices.

An option gives the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined price. There are two main types: calls (right to buy) and puts (right to sell). Options are used for hedging, speculation, and income generation.

The Nigerian Exchange (NGX), formerly known as the Nigerian Stock Exchange, is a key player in West Africa's financial landscape, with a growing derivatives market. It is a platform where companies raise capital, and investors buy and sell shares, bonds, and other securities. It was established in 1961 as the Lagos Stock Exchange, later renamed the Nigerian Stock Exchange in 1977 and rebranded to Nigerian Exchange (NGX) Group in 2021.

However, the applicability and accuracy of the Black-Scholes model in Nigerian Exchange remain underexplored. The model's assumptions, such as constant volatility, risk-free interest rates, and no dividends, often diverge from real-market conditions, potentially leading to mispricing.

Some studies have been made by different researchers to ascertain the pricing accuracy of the Black-Scholes' model. Onyegbuchulem et al. (2021) carried out a study that replaces spot price in the original Black-Scholes' model by the Future price as proposed by Black (1976) and the pricing accuracy of the models were tested. Rice Forward prices were estimated using both Black's and Black-Scholes' models and comparison made on their outputs. The result of the analysis revealed that Black-Scholes' model produced high pricing error than Black's model.
Rinalin, (2023) study on the Indian Stock market revealed that the Black-Scholes' model has several limitations. They discovered that modifying the Black-Scholes' Model did not yield better results for NIFTY index options, especially for at-the-money, out-of-the-money, and deep out-of-the-money options. The study applied Corrado-Su corrections to the Black-Scholes Model using Gram-Charlier expressions to price options in the Indian market. Results revealed that Corrado-Su modified formula performed better for equity options than the original Black-Scholes' model with implied volatility but did not improve NIFTY index option pricing significantly. Basically, Corrado-Su adjustments performed better for equity options but not as much for index options.

Ukah & Okiro (2022) compared the Black-Scholes model with the Feedforward Networks Model using S&P 500 option Index data from September 2019 to August 2021. They showed that the Black-Scholes' Model tends to misprice options, particularly deeper out-of-the-money options, compared to near out-of-the-money options. This error gets worse as volatility increases. In contrast, feedforward networks tend to have less pricing error than the Black-Scholes model for these deeper out-of-the-money options.

Inani (2017) studied the Black-Scholes' model and gave detail analysis of the assumptions of the model saying that the weakness of Black-Scholes is that it is based on simplistic assumptions such as constant volatility and a normal distribution function for the given asset return. This led to substantial discrepancies between market prices and prices calculated under the Black-Scholes' model which are most evident in the observation of different implied volatilities based on the strike price and maturity.

Onyegbuchulem, et al (2020) investigated the pricing accuracy of the Black-Scholes' model in an unstructured over-the-counter Nigerian agricultural commodity market. Rice prices were valued using the Black-Scholes' equation. The model's predicted prices were compared with the market observed prices. The result of the study revealed that there are cases where Black-Scholes' model performs well, because the prices produced by the Black-Scholes' model provided a good match with the market observed prices. The study also revealed a positive and significant correlation between the Black-Scholes' model and market observed prices.

Since most of the reviewed literature revealed that the Black-Scholes' model exhibits pricing errors, hence there is need to evaluate its performance in predicting call option prices on the Nigerian Exchange (NGX). This study aims to investigate the Black-Scholes model's performance in pricing NGX options, considering the local market's unique characteristics, such as liquidity constraints, regulatory frameworks, and underlying asset dynamics. Specifically, it will

i. compare the NGX futures prices with corresponding spot prices to identify the existence of negative cost of carry problem.

ii. compare the NGX Discounted value of futures prices with corresponding spot prices to address the negative cost of carry problem.

iii investigate the NGX call options pricing errors calculated under the Black-Scholes Model.

iv. investigate the NGX call options pricing errors calculated under the Discounting Value of Futures Prices

v. investigate the Money and Maturity biasness of NGX call options calculated under the Black-Scholes Model.

The study seeks to contribute to existing literature by providing insights into the Black-Scholes model's efficacy in an emerging market context and shedding light on the cost of carry problem's impact on option pricing. The findings will have implications for investors, policymakers, and market participants in Nigeria and similar markets.

2. Methodology

2.1 The Black-Scholes' Model

The price of a call option $C(S,t)$ follows the Black-Scholes-Merton partial differential equation given by:

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0 \tag{1}$$

satisfying terminal condition

$$C(S,T)=(S-K)^+ \tag{2}$$

and boundary conditions

$$C(0,t)=0,\ \text{ for }\ 0(\le t\le T) \tag{3}$$

$$C(S,t)\to S,\ \text{ as }\ S\to\infty \tag{4}$$

The following formulas give Black-Scholes price of a European call at time *t*

$$C(S,K,T)=Se^{-q(T-t)}N(d_1)-Ke^{-r(T-t)}N(d_2), \tag{5}$$

where

$$d_1=\frac{\log\left(\frac{Se^{-r(T-t)}}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2=\frac{\log\left(\frac{Se^{-r(T-t)}}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}=d_1-\sigma\sqrt{(T-t)}$$

and $N(.)$ is the standard normal cumulative distribution function,

$$N(d)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d}e^{\frac{-t^2}{2}}dt$$

The stock price follows the Geometric Brownian Motion. In the risk neutral world, it has the following dynamics:

$$\partial S_t=(r-q)S_t dt+\sigma S_t\partial W_t \tag{6}$$

where

S_t is the stock price at time t, t is current time, r is the risk free interest rate, q is the dividend yield, assumed to be constant, σ is the volatility of the asset's price, W_t is a Brownian motion, $N(.)$ is the cumulative distribution function of a standard normal variable, K is the strike or exercise price, $(T-t)$ is the time to maturity. All these parameters are easily obtainable from a standard market except volatility parameter (σ). The volatility parameter (σ) is assumed to be constant while calculating option prices. σ is calculated through two approaches which are historical volatility and implied volatility. The historical volatility is calculated by the annualized standard deviation of historical daily returns. The historical approach is much simpler than the other one. The implied volatility looks more on the future movements.

2.2 Cost of Carry and Black Model

The relationship between future price and spot price can be summarized in the terms of the cost of carry. The cost of carry is basically the cost of holding a position in an underlying security until a futures contract expires. It can be positive or negative, depending on whether the futures price is higher or lower than the spot price. For financial assets, the cost of carry includes storage costs, interest, and income received. In commodity markets, cost of carry is used to describe these costs.

Cost of carry (c) in commodity market according to Hull, (2007) in Onyegbuchulem, et al. (2021) can be defined as:

$$F = Se^{(c-y)} \tag{7}$$

for a consumption asset, where, y is the convenience yield which is found in commodity market and not found in stock market.

$$F = Se^{cT} \tag{8}$$

for an investment asset.

For an investment asset, cost of carry (c) becomes 'r', such that:

$$F = Se^{rT} \tag{9}$$

For an investment asset any known yield on the stock, like dividend, should be subtracted from 'r' in equation (9). Hence,

$$F = Se^{(r-d)T} \tag{10}$$

$$S = \frac{F}{e^{(r-d)T}} \tag{11}$$

The Black model, developed from the Black-Scholes model, uses future prices (which factor in cost of carry) to value options, especially for physical commodities.

The Black's formula to calculate call option price of physical commodity is given by the following equation:

$$\begin{aligned} C(S,K,T) &= Fe^{-r(T-t)}.N(d_1) - K.e^{-r(T-t)}.N(d_2) \\ &= e^{-rt}\left[F.N(d_1) - K.N(d_2)\right] \end{aligned} \tag{12}$$

where

$$d_1 = \frac{\log\left(\frac{F}{K}\right) + \left(r + \frac{\sigma 2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\log\left(\frac{F}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{(T-t)}$$

In equation (12), F is the Future price of the asset and other input parameters are same as the inputs used in the Black-Scholes' model. According to Black, Future prices provide valuable information for the market participants who produce, store and sell commodities. The Future prices observed during the various transaction months, help the producers and traders to decide on the best times to plant, harvest, buy and sell the physical commodity. The Future price of a commodity therefore reflects the anticipated distribution of spot prices at the time of maturity of the Future contract. Black observed that changes in spot price and change in Future price are usually correlated. Both spot prices and Future prices are governed by the general shifts in cost of producing a commodity and the general shifts of demand of the commodity. Shifts in demands and supply due to fresh harvesting can create difference between spot and future price.

2.3 Data Compilation

Data on 2437 call option prices for stocks of five companies listed on the Nigerian Exchange (NGX) were collected from the exchange's website (ngxgroup.com) for the period May 1, 2022, to April 30, 2025. Closing prices of 2437 Call options were observed and collected. Since only closing prices were available, intra-day prices were not compared, which may likely lead to errors for thinly traded options due to timing mismatches. To minimize this, only highly traded options which are expected to trade until the last moment on the concern stock exchange were considered.

Table 1: Number of Observations from Ist of May 2022 to 30th April 2025

Observation Year	*May 2022-April 2023*	*May 2023-April 2024*	*May 2024-April 2025*	*Total No. of Observations*
No. of Observation	**683**	**898**	**856**	**2437**

2.4 Input Parameters used in Option Pricing

Other input parameters for estimating call option prices with the formulas are obtained as stated in Onyegbuchulem, et al (2021)

(i) **Volatility (σ) Parameter:** Among input parameters required in the models, the volatility parameter or standard deviation (σ) of the returns for the duration of the option are not observable from the market, so an estimate is required.

Volatility is the standard deviation and can be determined by two methods, one is using the direct formula, which is given as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(X_i - \mu\right)^2}{n-1}} \tag{13}$$

where,
σ = Standard Deviation, X = Monthly price change of the security, μ = Mean or Average of the security, n = Number of values in the dataset. Daily volatility can be annualized by multiplying it by the square root of the number of trading days (usually 250 or 252). In this paper, we have used the MATLAB Simulink approach as it gives the standard deviation directly in its data statistics.

(ii) **Time to Expiry "t":** The term e^{-rt} has been used, where 't' is the time left for the option to expire. Here, calendar days has been used to calculate 't', irrespective of intervening holidays. Further, 't' is annualized by dividing 't' by 365 days.

(iii) **Interest Rate (r) Parameter:** The monetary policy rate (MPR) in Nigeria has been adjusted several times to control inflation and stabilize economy. However, the MPR was increased from 11.5% in 2022 to 27.5% in 2024 to curb rising inflation. Hence, the value of risk-free rate of interest was taken at 27.5%.

2.5 The Pricing Accuracy

To check how accurate the model is compared to real market prices, the following error measures were used:

(i). **Mean Error (ME)**: Mean Error adds all errors and divides total error by the number of observations. $Mean\ Error = \frac{1}{N}\sum_{n=1}^{N} e_n$ The result is acceptable when all error data have the same sign (either all are positive, or all are negative).

(ii) Mean Absolute Error (MAE): The mean absolute error value is the average absolute error value. The closer this value is to zero, the better is the estimate. The neutralization of positive errors by negative errors can be avoided by applying mean absolute error MAE is

computed using the formula- $Mean\ Absolute\ Error = \frac{1}{N}\sum_{n=1}^{N} |e_n|$

(iii) **Mean Squared Error (MSE):** Mean Squared Error is computed as the average of the squared error values. This is the commonly used error indicator in statistical fitting procedures. As compare to the Mean Absolute Error value, this measure is very sensitive to large outlier as it places more penalties on large errors than Mean Absolute Error. MSE is computed using the following formula

$$Mean\ Squared\ Error = \left(\frac{1}{N}\sum_{n=1}^{N} e_n\right)^2$$

(iv) **Root-Mean-Squared Error (RMSE)** is the square root of mean squared Root Mean Squared Error (RMSE) measures the average magnitude of the error. It is the square root of the average of squared differences between prediction and actual observation.

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N} (e_n)^2}$$

(v) **Thiel‟s U Statistic (Inequality Coefficient)** is an inequality coefficient for measuring the degree to which one time series differs from another. Theil‟s U statistic is computed as:

$$U = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(y_n - f_n\right)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}{y_n}^2} + \sqrt{\frac{1}{N}\sum_{n=1}^{N}{f_n}^2}\sqrt{\frac{1}{N}}}$$

Here, the two-time series in question are (a) the actual value of options y_n and (b) the value of option predicted by the models f_n. Thiel"s U will equal 1 if a forecasting method is found no better than using a naive forecast. If Thiel"s U is less than 1 it indicates that the method is superior to a naive forecast. A value close to zero indicates a good fit, whereas value greater than 1 indicates that the technique is worse than using a naive forecast.

3. **Results and Discussion**

3.1 Comparison of Errors using Option Pricing Models

Call option prices were estimated using Black-Scholes and Black's formulas, and their errors were compared. The formula with the lowest error was considered better. Errors were calculated by comparing calculated values with actual market prices.

Table 2- Comparison of Futures Prices with Corresponding Spot Prices

Period	*Total No. of Observations*	*No. of observations when FP<SP*	*No. of observations when FP>SP*	*No. of observations when FP<SP in Percentage (%)*
May 01 2022-April 30 2023	**683**	**274**	**409**	**41.08**
May 01,2023-April 30, 2024	**898**	**239**	**659**	**35.83**
May 01,2024-April 30, 2025	**856**	**154**	**702**	**23.09**
Total	**2437**	**667**	**1770**	**27.37**

Table 2 shows that 27.37% of NGX futures prices which represents 667 out of 2437 observation were quoted below their corresponding spot prices

Table 3: Errors of Black-Scholes Model for Pricing NGX Call Options

Total No. of Observations	***ME***	***MAE***	***MSE***	***RMSE***	***Thiel's U Statistic***
2437	-2.3631	11.2159	36.8209	16.3844	0.1312

The errors for call option calculated under the Black-Scholes model is shown in Table 3. The Table 3 shows that the Black-Scholes model tends to underprice call options on the NGX stocks, with a mean error of -2.3631. Analysis of 2437 observations shows high error levels: MAE of 11.2159, MSE of 36.8209, and RMSE of 16.3844. The Theil's inequality coefficient of 0.1912 (above 0.1) suggests that the model is not a good fit, likely due to the negative cost of carry.

Table 4: Comparison of Discounting Value of Futures Prices with Corresponding Spot Prices

Period	***Total No. of Observations***	***No. of observations when DVFP<SP***	***No. of observations when DVFP>SP***	***No. of observations when DVFP<SP in Percentage (%)***
May 01 2022-April 30 2023	**683**	**468**	**215**	**26.62**
May 01,2023-April 30, 2024	**898**	**636**	**262**	**36.18**
May 01,2024-April 30, 2025	**856**	**654**	**202**	**37.20**
Total	**2437**	**1758**	**679**	**72.14**

Table 4 displays the comparison of Discounting Value of Futures Prices with the corresponding spot prices. It was found that about 72.14% **of** NGX futures prices (1758 out of 2437 observations), were lower than the spot prices. This suggests that around 72.14% of the observations might be impacted by the negative cost of carry issue.

Table 5: Errors of Discounting Value of Futures Prices for Pricing NGX Call Options

Total No. of Observations	***ME***	***MAE***	***MSE***	***RMSE***	***Thiel's U Statistic***
2437	-0.35	8.6307	374.5089	14.6327	0.1603

The error for call option calculated after replacing Spot price by DVFP in the Black-Scholes model is shown in Table 5. The Mean Error (ME) improved by -1.7331, Mean Absolute Error (MAE) improved by 4.4736, Mean Squared Error (MSE) improved by 11.0528 while Theil's inequality coefficient; improved by 0.0181 compared to the original Black-Scholes model. (Tables 3 and 5 have the details).

Table 6: Moneyness bias of Black-Scholes Model for Pricing NGX Call Options

Moneyness	*Total No. of Observations*	***ME***	***MAE***	***MSE***	***RMSE***	***Thiel's U Statistic***
In the Money (ITM)	1035	8.35	27.71	356.77	59.12	0.0513
Out of Money (OTM)	1402	17.31	25.46	267.23	46.86	0.321

The Black-Scholes Model's accuracy in pricing NGX call options was tested based on moneyness. The results in Table 6 show that for ITM options 1035 observations were made. Mean Error (ME) is 8.35, Mean Absolute Error (MAE) gives 27.71, Mean Squared Error (MSE) is 356.77, Root Mean Squared Error (RMSE) is 59.12 while Thiel's U Statistic is 0.0513. For OTM options 1402 observations were made. Mean Error (ME) is 17.31, Mean Absolute Error (MAE) is 25.46, Mean Squared Error (MSE) is 267.23, Root Mean Squared Error (RMSE) is 46.86 while Thiel's U Statistic is 0.321. The model shows pricing errors for both ITM and OTM options. Ther is non-zero mean pricing bias. The error is more significant for OTM options based on Thiel's U statistic. No ATM options were found during the study period.

Table 7: Maturity bias of Black-Scholes Model for Pricing NGX Call Options

Moneyness	*Total No. of Observations*	*ME*	*MAE*	*MSE*	*RMSE*	*Thiel's U Statistic*
Near Month	791	5.87	24.01	357.70	78.93	0.102
NextMonth	801	17.11	25.63	251.76	31.56	0.021
FarMonth	845	26.37	41.37	445.03	53.77	0.123

The Black-Scholes model has errors in pricing NGX call options with different maturity periods which include near month, next month, and far month as shown in Table 7. The average pricing bias is not zero for these periods, indicating that the model is not accurate. Table 7 shows the breakdown of the errors: The results show the model's pricing errors vary across different maturities.

4. Conclusion

This study evaluates the Black-Scholes model's performance in predicting call options on five Nigerian stocks listed on the Nigerian Exchange market, compares stock futures prices to spot prices to check for negative cost of carry bias. The study also compares the Discounting Value of the Future Price (DVFP) to spot prices. The study found that Nigerian Exchange futures prices were mostly traded below spots (27.37% of the future prices), showing a negative cost of carry bias, and 72.14% of the time, futures prices were discounted. This led to significant pricing errors in the Black-Scholes model for call options on the NGX 2437 equity index. The Black-Scholes model mispriced Nigerian Exchange call options due to this bias. Replacing spot price (S) with DVFP in the Black-Scholes model improved the errors calculated using the error measures. The findings are crucial for investors, policymakers, and market participants in Nigeria and similar emerging markets, suggesting need for adjustments to the Black-Scholes model to account for local market characteristics.

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